VŠB - Technical University of Ostrava, Faculty of Safety Engineering

Ing. Pavel Šenovský, Ph.D.

Modelling of Decission Processes

textbook

4^{th} edition

Ostrava, 2024

Modelling of Decission Processes
4^{th} edition
this text has not gone through edition process

Vysoká škola báňská - Technical University of Ostrava, Faculty of Safety Engineering

Contents

List of Figures

List of Tables

Source code listing

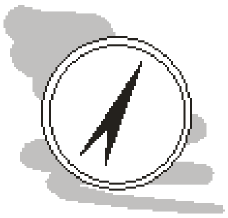
Introduction

Dear student, you are holding in your hands a textbook on the subject of modeling of the decision processes. Basic goal of this textbook is to supply you with various methods for decision support.

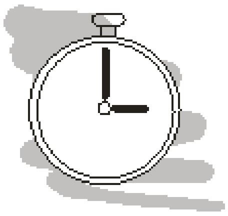
This textbook has been derived from 5-th edition of Czech version of the textbook, which has been published first in 2021 in Ostrava.

The textbook is intended as a companion study material to the lectures. So it is to be used together with your notes and presentations available from LMS (https://lms.vsb.cz).

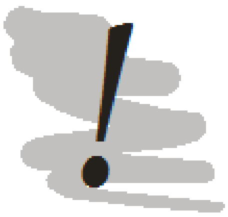
The text is accompanied by a set of icons to guide you through the learning process:



Study guide
Intends usually to set your expectations on the text organization including learning objectives for the chapters.



Time required to study
Is estimated time necessary to understand (learn) all of the concepts presented in the chapter. Time estimation should be looked upon only as rough time guidance to help you better organize your time. So do not be disturbed if the learning requires little bit more or less time.

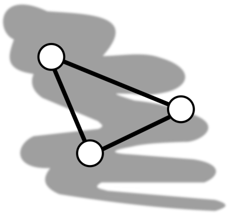


Definitions, notes and other important details
Accompanying this icon there will be text which the author finds so important, that he puts exclamation mark to it. Please pay close attention to it. It might be important :-).

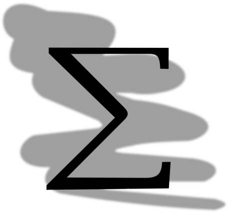
Test your knowledge
On the end of each chapter there are several questions or tasks you can use to self-evaluate your knowledge. These questions are not exhaustive to the main topic of the chapter, so if you know all answers to the presented questions . . . it is a good start :-), if not, you at least know where there is a gap in your knowledge.

Also this book has been typeset in LaTeX to introduce some features to improve your learning experience. The text has list of figures and tables, also it has the index with most important concepts with their location.

Since, this is 4th edition of the textbook, it should be relatively mature text. Still, it is possible that you will find some kind of error or maybe some part of the text that is unclear ... do not keep



Relation to other information
I sections of text with this icon, you will find how the discussed topic relates to other topics either in this textbook or in other courses.



Summary
Summarizes basic ideas of the chapter. This is the basics you absolutely must understand of the chapter.

Take a break
Well done, you worked hard, so it is time to take break.

Example or task
In this textbook, there are many examples but also many tasks available for you to compute by hand or using the computer. Take your time to compute them – it is a good indicator of whether you understand the problem or not.

this information for yourself a provide feedback. Amendments may not be in time for you, before you finish course, but it will help other students who are going to study it in the future.

You can report errors or any other type of feedback to my email: pavel.senovsky@vsb.cz. If you student of VSB - Technical University of Ostrava, you can also contact me via MS Teams and students of course Modeling of Decision Processes can contact me via LMS.

As the author I hope that you will like the textbook and find it beneficial in your further studies and also your future professional carrier.

Pavel Šenovský

What's new in 4th edition of the textbook
For 4th edition of the textbook many small changes have been made to improve the text. There also some added information:

- improved explanations on computation of stochastic decision trees
- Entropy Weight Method has been added as a alternative way of establishing weights for purposes of multi-criteria analyses
- better explanations of the MCA methods limitations
- MCDASupport package for R has been pushed to version 0.29
- added chapter for basics on Markdown and RMarkdown

- in Balance models added example computed in R (to the existing computation in MS Excel)
- corrected small error in prisoners dilema (Games Theory)
- corrected error in distance computation in Localization Models

What's new in 3rd edition of the textbook

3rd edition is basically rewrite of the textbook. Every part of it was edited for clarity. Multiple chapters have been added. Short, incomplete list of large changes is provided here:

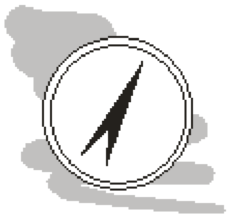
- revisited every chapter and brought it up to date with 5th edition of Czech version of textbook
- added chapter 1.2 to better state context of decision making
- added chapters 2.2 and 2.3 for topics of sensitivity analysis and extension of decision trees for multicriteria analysis
- replaced MCA case study
- added chapter for advanced multicriteria analysis methods including also more indepth information on various normalization methods and ways to estimate criteria weights.
- added chapter for AHP method
- added examples in R for various methods
- added chapter on realization and evaluation of the surveys
- rewritten section on network analysis and visualization in R (chapter 9.3)
- added section 9.4 to explain connections between network analysis and critical infrastructure
- added section 10.2 to explain relation of balance model to consequence analysis, critical infrastructure protection, etc.
- rewritten parts of chapter 11 to compute linear programs using R
- added public goods game to Games theory (relevant for explanation of behavior in society)
- added smaller chapters 14 and 15 to frame problematic of decision making into broader context of statistics and machine learning

What's new in 2nd edition of the textbook

- added introductory chapter one
- all subchapters now have questions to help you self-evaluate your knowledge on given problematic or tasks for chapters which are more practically oriented
- added practical software related chapter to project management
- added chapter for software support for Ford-Fulkerson and Dijkstra algorithms
- added chapter for software support of networks renderings based on the definition of the network structure
- added discussion of software tools for computation of linear programming problems
- many smaller corrections and changes.

Chapter 1

Introduction to decision making



Study guide
In this chapter we will learn something about decision making. Decision making on all things large and small in our personal or professional lives is something we do every single day. One would think that we must be good at it. Unfortunately it is not so. Humans are evolutionary "optimized" to make a fast decision with limited or even no supporting information, but such decision in not necessarily a good one.

Making good decision, or even optimal decision, is hard we need help of formal methods to guide us through it. Let's star slow by examining what a decision is.

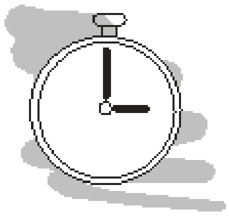
After finishing of study of this chapter you will

Know what

- is a decision
- decision context means

Be capable

- to distinguish between different types of decision situations



Time required to study chapter
For getting through this chapter you will need at least one hour ... you know only something small for starter.

1.1 Decision Making

Every day, every hour or perhaps even nearly every moment after we awake and before we go sleep we perform consciously and unconsciously countless decisions. Most of them are small and we make them automatically without even thinking of them. For example when we walk we automatically change the direction of our steps to go around obstacles, we greet people we know and such.

Some decisions are harder, for example we are hungry, we have to decide when and more importantly what to eat. While such decision can be very hard at the times it is not the decision for which we would need assistance of sophisticated decision support tools. But especially in our professional life, we often deal with much more complex decisions characterized by significant variability in decision outcomes - *variants of decision*.

Decisions are also characterized by *incomplete information* available to decision maker at time of decision. Also the outcomes of the decision are often guided by *stochastic* principles.

To deal with it we need a guidance of sophisticated decision support methods. Their employment often presents some kind of overhead by imposing on us strict requirements on data in needs to function and its quality. That in turn may require the analyst to have some specialized knowledge, or use some analytical software to facilitate the computation.

This overhead is balanced out by ability to provide actionable outcomes, which are easy to interpret.

In our exploration of decision making we will focus on such situations and the tools to help us deal with them.

We already specified some basis characteristics of the decision, but to properly understand the decision optimality we need do so again and in much greater detail.

The decision making is based on a choice of a decision variant. Such variants usually differ in outcomes, meaning that the outcomes will in *utility* they provide the beneficiary of the decision, or *optimality* of it.

We try to identify variant of the decision which leads to best outcomes by maximizing positive (beneficial) or mininizing negative (cost) properties of the decision outcome.

What we described in previous paragraphs clearly frames decision making as a problem of solving optimization problem by maximizing or nininizing one or more criteria characterizing the decision problem.

Predictability forms another property of the decision.

When we know exactly what the results of the decision will be, we call such situation as *deterministic*. Unfortunately the outcomes are not deterministic by nature. They are weighted by variants uncertainties leading to different outcomes with different probabilities. We call such situations as *decisions under uncertainty* or *stochastic* decisions.

There are many ways of how to express the *uncertainty*. We may be for example able to express the uncertainty by specifying variance in the outcome, i. e. expected profit 6 000 $\pm$ 50 EUR, or by specifying interval of the results such as [550, 650] for same example.

Stochastic approach to the problem would be to split possible outcomes into intervals, or choosing values representing such intervals and estimate the probability of such outcomes using ordinal scale (i. e. low, middle, high probability ... can use any number of options) or as percentages. There are also other ways of how to approach the problem.

The probability then is our tool to deal with uncertainty in quantitative manner.

What is the benefit of using decision support methods and tools? Well, using established method is like using highway instead of making your own way through the thick forest. Decision support methods are documented approaches some else used before us to solve the problem we are solving now. By using method we may reach goal more easily, with fewer resources and the outcome will come in expected format, that is if we use it as we should.

The method needs to be validated for the intended purpose. We also need to take into account all known constraints (limitations) of the method. After all no method is universally applicable. Good first step in decision making is to identify a good method to solve problem at hand.

Formally established decision support methods have benefit in that they usually have their limitations well understood and documented, which makes it easier for the practitioner to use them. Such information may include also set of problems the method is applicable on. Such information can speed up method selection for the analytic.

Another benefit is that the methods in general also enable us to make emotional distance between us and our results. Why? Because we all are biased in our decision making. Unfortunately *cognitive bias* is inherent property of human being. Our biases force us to take a known path or use known tool to solve the problem. It impacts directly our affinity to risk and even how we see the optimality of decision problem.

Kahneman described various forms of cognitive bias in his seminal book *Thinking Fast and Slow* [20]. While cognitive bias is well known it is still very hard to deal with it and impossible to completely evade.

If left unchecked, the cognitive bias will surely flaw the decision making potentially leading to suboptimal choices, which otherwise could be prevented. Using formal methods, understanding their process and properties will allow us to approach decision making more defensively, making our choices less error prone and more robust.

Some other guidance for decision methods support selection can be derived from the information we use to make the decision. In some decisions we have only single criterion to choose. Typical representative of such criterion is money as we are able to express a lot of different things using its monetary value.

Methods based on evaluation of single criterion are called *monocriterial methods*. Good example of such method would be *decision trees*. We will discuss the method in next chapter.

Analogically we have methods, which require many criteria to form their recommendation, we call such methods *multicriterial*. We will discuss these methods in chapters 2 - 6.

All of these methods (both mono- and multi- criterial) use *criteria* to characterize the decision and or its outcomes. Since the decision problems are very versatile the criteria need to be versatile too leading to different ways of how we can express these criteria. We can use:

1. numeric value – as in continuous numerical variable, for example age, charge time, price,
2. ordinal value – may be number or string, for example 0 – no, 1 – yes or
3. freeform answer – meaning that the criterion valuation for the variant will lead to paragraphs of text

We can also use fuzzy or grey numbers to encompass some kind of uncertainty directly to the expression of the criteria values.

It is only logical that different types of the criterion valuation will lead to different methods, which are capable to process them. For example numeric values are processable by statistical methods such as *regression analysis*, while ordinal values are better processed by data mining methods such as *decision trees*.

Decision Trees

In previous paragraphs we were discussing decision trees, please note that there are two different methods using this name. One, which will be discussed in next chapter, is monocriterial decision support tool, while the other can be used in data mining for classification purposes and works best with ordinal values. As this text is not focused on data mining, we will focus on first method only.

Also note that there are many other methods such as statistical methods or methods based on Artificial Intelligence (AI), which will also not be discussed in this text, even knowing that these methods can be very useful in supporting our decision. But perhaps you can study such methods in other courses on our Faculty (AI for Securty or Computer Science in Security) or abroad.

Portfolio of methods

In your professional life you will have to deal with many different decision situations requiring the usage of different approaches to reach successful solution – so build up portfolio of methods you are familiar with and capable of using them. The more different the methods will be the better. Healthy variety in the methods will allow you to solve much broader set of problems.

By the way this recommendation can be applied in any field of study and will still hold :-).

In this textbook, I try to supply the tools for many typologically different decision situations. So we can say, that the problem typology itself may be used as criterion to distinguish methods supporting our decision.

On the beginning of the chapter we were discussing classical choice – when deciding between different variants as in buying different product. What if the nature of decision (and the difference of the variants) is not in buying of new product, but rather in change of its operational parameters – setting up in different way. In such cases we talk about *optimization problems*.

Even for optimization very different approaches exist. Multicriterial Analysis (MCA) uses distance measures to theoretical optimum for recommendation formulation. But if problem is more geographically formulated, i. e. optimal location problems, there are *localization models* we can use to solve this kind of problem. Is the problem expressible by system of equations/inequations? Then we can use *mathematical programming methods*, such as *primary algorithm*, also known as so called *simplex method*.

Different types of problems require usage of different methods to solve them. So for us to be efficient, we need to build whole portfolio of methods and choose between them as needed.

1.2 Hierarchy and Structure of Decision Making

We already know in general terms, what decision making is and that for us to do it effectively we need to strive to achieve some kind of *goal or goals* we set for ourselves. Looks simple, right? But the looks can be deceiving. The goals can be achieved by different ways. Some of them may be complementary to each other, while others may contradict or be even mutually exclusive.

The goals will also differ depending on who does make the decision and in what *decision context* is the decision being made. Decisions, with same goal, can be made on different levels of decision hierarchy.

Consider following example: We have a company producing various machinery. The company aims to improve its workplace security and safety. Obviously, there are many options of how to approach this problem to choose from. All available options will to certain degree improve the security situation, but their contribution to the overall goal of improving the security will be different.

We may decide to implement some protective measure at the level of company as whole by for example deciding to implement new management system and new policies accompanying it. We may also choose to implement physical security measures in place, or change used procedures and processes.

To better understand decision situation, we can express it graphically using *goal hierarchy*.

Let's define another problem to demonstrate creation of such hierarchy: *We will be deciding on measures for improving security in road traffic.*

Goal hierarchy as well as the tools for achieving such goals is available on fig. 1.1.

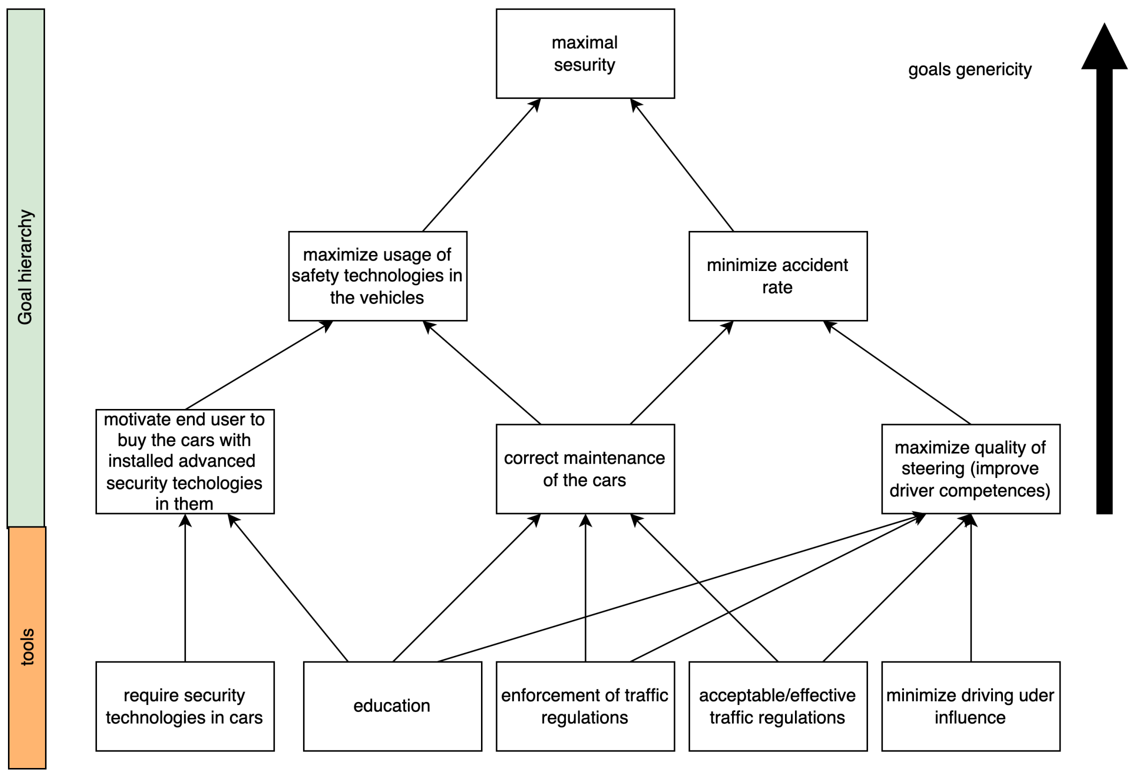


Figure 1.1: Goals hierarchy and the tools available to achieve them (adapted from [1])

Goals hierarchy on the figure shows differences in perceptions of the goals. The higher in the hierarchy we are, the more generic the goals are. That corresponds to management system in general, the higher in leadership hierarchy of the enterprise we are the more it is focused on "big picture" rather then mundae details.

Differences in goals can also be related to practitioners groups, in other words the people who will be responsible for decision making at the decision level in question. So in our example we can identify following decision makers:

1. common citizen, wishing to maximize his or hers personal safety
2. car manufacturer, who chooses between possible safety characteristics for his new car to be successful on the market
3. Ministry of Transport, which is responsible for legislative and regulation of road traffic
4. driving school
5. technical inspection stations
6. etc.

Each of these groups will have slightly different goals all under general umbrella of maximization of the traffic safety/security. That means that the preferences and the tools these group will have at their disposal and that they will be inclined to use will differ significantly even if the wording of the goal will remain same. This is important, as this very moment illustrates necessity to understand the *decision making context.*

Even if we limit ourselves to context of decision making in the enterprises, it is necessary to consider various positions on different levels of the management and their influence on decision making process. The decision itself is always in the competence of somebody. This competence is usually derived from job description or can be stated by some kind of inner regulation in the enterprise. In case of state and local governments such competence may be defined by legislation.

Competence to decide does not mean, that such person needs to perform whole decision process her/him-self. In fact, it is very unusual, if someone even tries to realize the decision in such way. The manager will decide and will hold managerial responsibility. But the materials on which the decision is based upon are usually prepared by one or more experts in the decision problem's domain. The more sophisticated the problem is, the larger team preparing such materials usually is.

Manager can delegate partial decision step on experts or other managers directly under him. This delegation does not diminish in any way overall managerial responsibility.

To improve chances to succeed, the manager evaluates proposed solutions to the decision problem, especially whether the proposition meets quality standards and is practically realizable in organization's environment. if (s-)he identifies any significant problem in the underlying analysis, such analysis is returned to the analyst to make appropriate amends to the analysis.

If no problem is detected, the manager chooses variant based on his/hers preferences using the analysis to support the decision and proceeds with its realization.

If the manager had to do the whole process by (him-)herself, the level of knowledge necessary for one person to have would be too high. So the manager needs to delegate. On the other hand, the manager also needs to have good enough knowledge in the problem domain and decision making methods to be capable of interpreting provided analytic results. Otherwise he or she will be completely reliant on the provided information, without the ability to judge quality of it, while taking full responsibility for outcomes of the realization of such decisions. That is clearly undesirable.

Length of the decision making and size of the team preparing supporting materials is proportional to complexity of the problem. Simple problems might require hours to find optimal solution (decision) using just normal office software, while complex problems might take large teams, months of work and use of specialized software.

Especially for complex decisions it is often necessary to better understand the factors influencing outcome of the decision. There are many methods usable for this purpose. You may already studied some them such as SWOT analysis or Ishikawa diagram. These are so notoriously well known, that we will not discuss them in this textbook. Other popular methods are brainstorming and mind maps, which we will discuss in later chapters. In this chapter we will focus on another method well suited to analyse decision situation from point of view of various factors influencing it. This method is called *influence diagram.*

As the name suggests, influence diagram is a graphical method allowing us to describe characteristics and connection between various factors we perceive as significant for the decision. This perception (believe) can be, but does not have to be accurate. So the influence may or may or may not be present. Formal specification of these influences allows us to set starting framework for the decision making. We can then focus on detailed analysis of the factors and exclude them (or not) at later time, based on additional information and better understanding of the problem.

Influence diagram with its simplicity, allow us to create good starting point for using specialized methods. That also means that by using influence diagrams we usually do not come to conclusion - identify variants of solution. Instead we learn more about the decision situation and various aspects of its solution.

Influence diagrams use just few graphically simple symbols (constructors) using shapes common in variety of software packages for graphics such as Dia [21], Microsoft Visio [22], SmartDraw [23], Diagrams.net[1] [24] and many more.

Constructors of influence diagrams are presented on fig. 1.2.

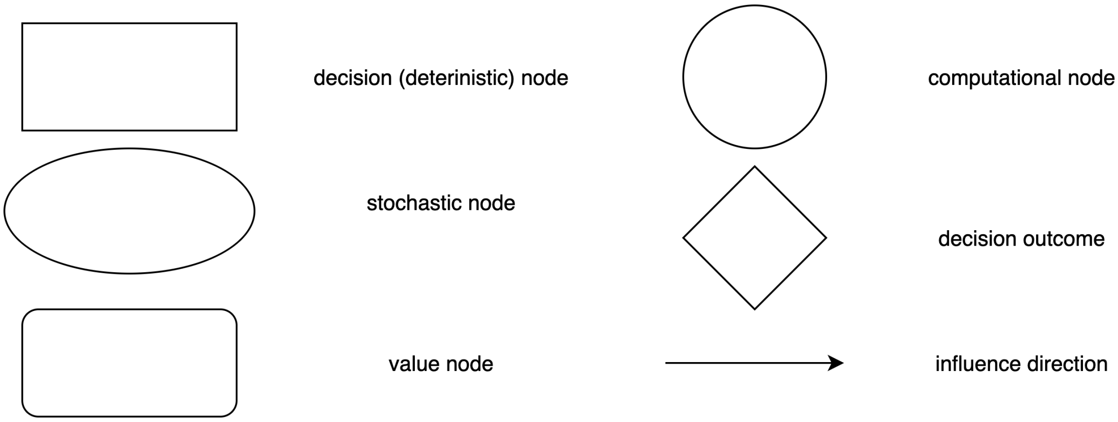


Figure 1.2: Basic constructors of influence diagrams

Let's look at the constructors in more detail. First is the *decision node*. This symbol is used for situations, when the nature of the (partial) decision is purely deterministic. In the other words we are sure how the outcome of this partial decision will play out.

Stochastic node allows us to describe the situations, when the outcomes of the patrial decision are not sure. That is useful in situations when the outcomes are not completely under our control, are probabilistic based, etc. As we already know, the majority of the decisions are stochastic in nature.

Value node serves for capture the measures used in computations. You can see simple example in fig. 1.3 for some decision based on profit.

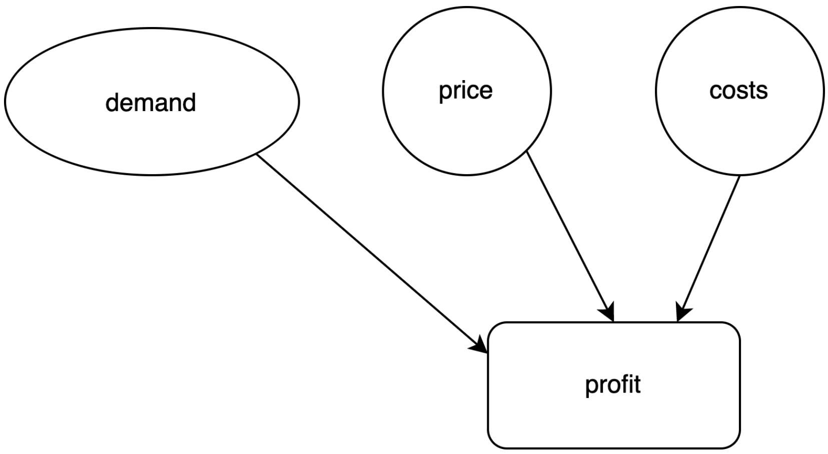


Figure 1.3: Simplistic diagram of profit computation using influence diagram

Mathematically we can express it quite simply as profit = demand * (price - profit), for unit of production. The equation itself may be directly written into the node or kept separately, depending on complexity of the computation. In the diagram more interesting are symbols used for demand (*stochastic node*) and for both price and costs (*computational node*). Both price and costs can be considered as constant for unit of production, while demand is variable depending on other factors (not explored in the diagram), thus expressed by stochastic node.

[1]the service is also known under the name Draw.io available both on `https://diagrams.net` and `https://draw.io`.

Fig. 1.3 represents only fragment of possible decision situation, to be complete it would need to have at least one deterministic node and also one, or realistically much more then one results of decision represented by *decision outcome node*.

Or consider influences on fig. 1.4. This time we are trying to better understand our decision space when fighting the terrorism.

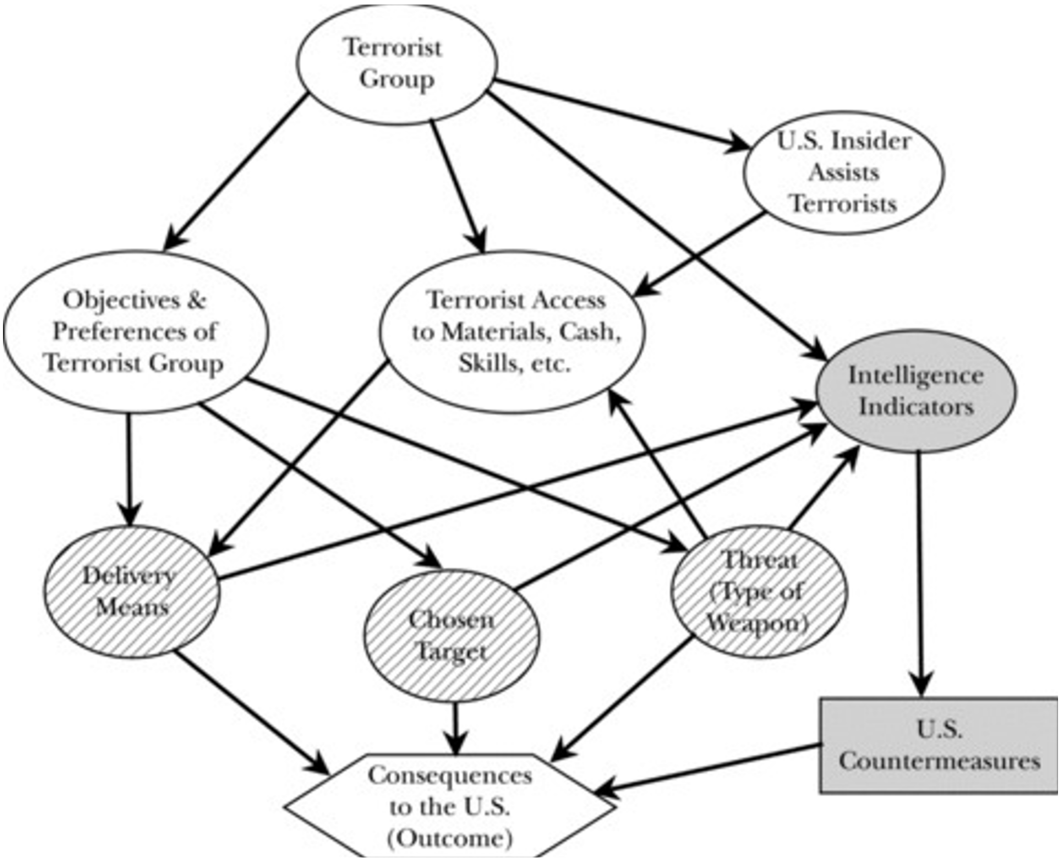


Figure 1.4: Influence diagram of terrorism consequences for the USA (courtesy of [2])

As you can see the influence diagrams are applicable on variety of decision problems and you even always do not need to use full portfolio of the symbols it supports for it to be useful.

Task - experiment with influence diagrams
Choose one of software packages mentioned in this chapter (or choose one on your own) and experiment with it to create some influence diagrams. First choose a problem to model and then decompose it and visualize using influence diagram.

Consider for example what influences your decision on course selection for next semester.

Summary

Process of decision making presumes existence of variant solutions, which differ by its utility for decision maker. The decision itself is always influenced by uncertainties regarding outcomes of decision making. Usually we are able to at least somewhat quantify this uncertainty, by establishing probabilities of the outcomes.

Decision making context play especially important role in decision making, it defines preference system of the decision maker, which will limit options for possible solutions (limit solution space).

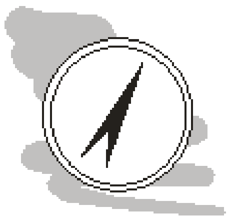
We can visualize decision situation and various influence in play for it using influence diagrams.

Questions

1. What characterizes decision situations?
2. What is difference between stochastic and deterministic decisions?
3. Specify types of criteria we use to describe utility of the solution variant (there are 3 of them).
4. Can nature of the problem, we are trying to decide, have impact on the methods effectively usable to solve it – can you give example?
5. Look back at your study plan and try to find methods usable as support for decision making, you already know.
6. What is influence diagram good for?
7. Try to specify reason to consider decision making context for effective decision making.

Chapter 2

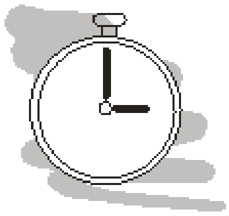
Decision trees



Study guide
In this chapter we will learn of how to use one of simplest tools for decision support – *decision trees*.

After finishing of study of this chapter you will Know

- What decision trees are
- When and how to use them
- what is sensitivity analysis and how to use it



Time required to study chapter
For getting through this chapter you will need at least one hour and you will need about two hours for calculation of examples.

2.1 Monocriterial decision making

One of most simple tools available for decision support are *decision trees.* Decision trees are oriented graphs, which are by its look similar to the tree. As normal tree, decision tree has one root and branches itself to leafs.

The root of the tree represents a basic choice between variants. The branching then represents partial decisions we may do during the decision making. What do we mean by partial decisions? In our decision we usually do not perform single decision – but a series of small decisions we make one after another, together leading to certain outcome. For example we want to buy a car, the basic choice may be between the car producers – Ford, Škoda, Volkswagen but there would be also other decisions leading to our dream car – what type will it be – SUV, hatchback, limousine and what about the engine? To finish the choice we must perform all these small decisions (branching on the decision tree). Then finally we reach the outcome of our decision – *leaf node.*

Writing decision situation into form of the decision tree helps us by visualizing the situation in the easy to understand way so we may focus on partial decisions while not loosing the biger picture of the decision situation.

Decision trees are monocriterial method, so each variant of the choice is characterized by single criterion. The criterion itself must be common over the whole tree regardless of the nature of partial decisions. It is logical requirement, otherwise we would not be able to compute the value of the decision as whole.

Such universal criterion is usually only one – money. We are capable to translate many different variables to financial value they represent. For some of them there are even available the tables to make the process easier (damages on fauna and flora, valuation of injuries, cost of building cubic meter of certain type of building and many more). The optimal solution to our problem then would be the variant with minimal o maximal value of the criterion depending on nature of the decision.

Even with money as common denominator we may face both maximalization and minimalization problems. For example the we usually like to minimalize the costs while maximalizing the benefits or profits.

When making decision or partial decision we may or may not know the outcome of that decision. Decision in situation under certainty means, that we fully know what will happen, if we make the choice. Unfortunately that is usually not the case, so we have to deal with some kind of uncertainty most of the times.

Decision in situation under uncertainty means, that we cannot state with 100% certainty, what exactly will happen, if we make that choice. But usually we are at least capable to establish probabilities of the outcome.

Both situations are solvable by decision trees, but of different type. In situation of certainty we use *deterministic decision trees*, when we need to consider probabilities we use *stochastic decision trees*. General example of both types of the trees is on the fig. 2.1.

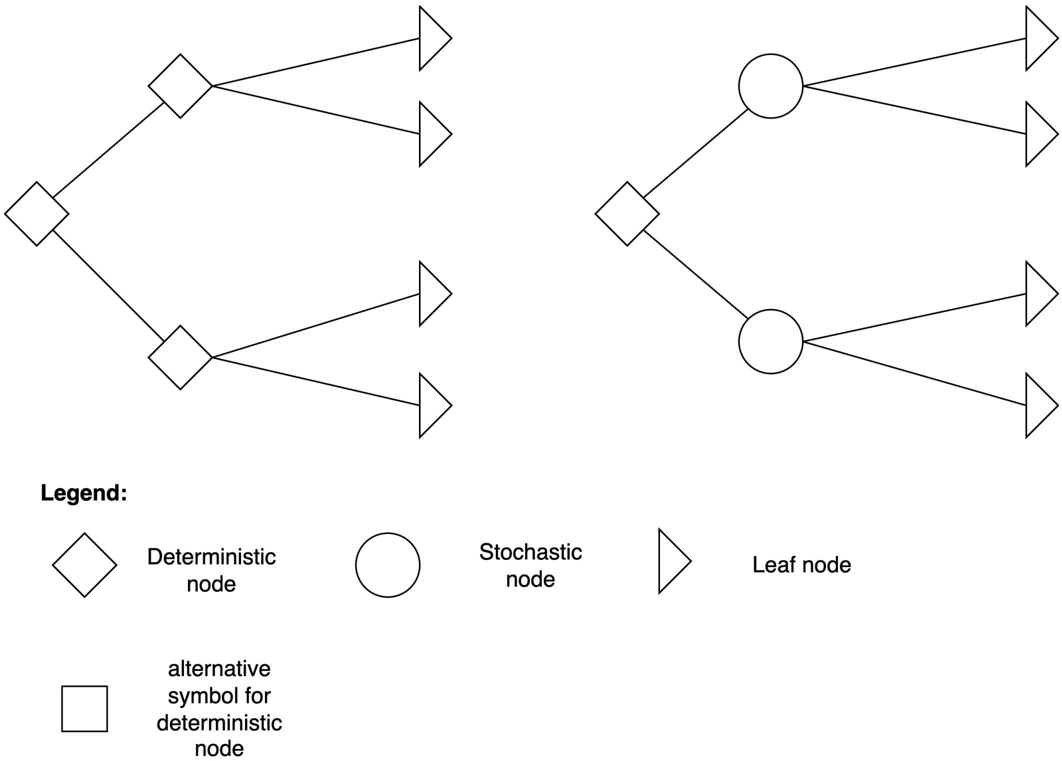


Figure 2.1: Deterministic (left) and stochastic (right) decision trees

Decision trees consist of the nodes and edges. The nodes represent decision points we make during whole decision process, leading us to the outcome of the decision represented by leaf node.

As you can see, in decision trees we use 3 types of nodes:

- deterministic,
- stochastic and
- leaf nodes

Deterministic node allows us to traverse the graph forward choosing one of the paths leading from it. *Leaf node* represents the overall outcome of the decision expressed as a single value in unit of the criteruim we used, i.e. EUR or CZK.

Stochastic nodes represent decision under uncertainty. Possible outcomes of such decision are realized as a patch forward from the node weighted by probability associated with it.

So, while in deterministic nodes *we* choose actively which path to progress, in stochastic node this decision in *probability based*. Instead of single deterministic value we are working with geometric mean of all possible outcomes, where the probability is used ar a weight.

Decision trees constructed only from deterministic and leaf nodes are called *deterministic decision trees.* If the tee has at least one stochastic node, we call it *stochastic decision tree.*

Deterministic trees can be considered special cases of a stochastic tree, where all stochastic nodes have only one possible outcome happening with probability $P = 100\%$.

The root node is usually deterministic as it represents the decision between basic variants of the decision, which are then more developed further down the tree. Also it makes little sense to create the decision tree using stochastic nodes only – in such situation, our decision would have absolutely no value, as we would have been fully in the hands of the chance.

Position of the node in the X axis of the graph should also correspond to the time of the decision, the node represents. The nodes representing partial decisions made farther in future should be farther right then those made in near future. Position of the nodes along X axis is for sake of better readability of the graph, the computation of the tree is affected by the structure of tree only, not by its graphical representation.

Also to improve readability of the decision tree, we are often numbering the nodes. Numbering enables us to more easily document the tree or perform more elaborate computations outside of the tree, while making it easy to identify part of the tree we are working with.

We compute deterministic and stochastic decision trees in slightly different way. Deterministic trees are more straightforward, so we start with these:

1. specify a root node
2. identify basic path - these will form our first edges in the graph. Don't forget to document meaning of each edge
3. at the end of the edge decide on a type of the node the path leads to (deterministic or leaf)
4. did we make all decisions along the path? If yes, then use leaf node, otherwise use deterministic or stochastic node and repeat from step 2)
5. in this manner develop whole decision tree until it is structurally complete
6. assign values of the criterium to the edges
7. sum these values along the path from the root to the leaf node to compute utility of the decision represented by leaf node
8. chose leaf node with highest utility - this is our optimum
9. edges between root and leaf node then represent partial decisions we need to take to reach this optimum.

Please note that the utility may mean different things depending on decision context. From utility point of view we may want to maximize or minimize the criterium.

Example 1: Deterministic decision tree
Company XYZ must decide whether or not to reconstruct its business premises. At present time the company is capable to sell its products for 2,5 mil. EUR, during reconstruction the company would have to temporally move into alternative buildings, from which only 1 mil. EUR worth of its products could be sold.

The reconstruction is expected to take one year and will cost 10 mil. EUR. In new business premises the company hopes to sell its products worth 3 mil. EUR per year.

Recommend whether to or not to reconstruct, considering 10 years time horizon.

Let's solve the example together. Considering the situation described higher, we may construct the decision tree similarly to fig. 2.2.

The decision tree on fig. 2.2 is rather simple – that is not so surprising as we already know, that the deterministic decision trees do not describe real decision situations well. So the example on fig. 2.2 is typical school problem just to demonstrate the way of computation.

Lets look at the tree. Its node 1 represents our decision to or not to reconstruct business premises. Node 3 only marks one year reconstruction period and does not represent the decision by itself.

We can calculate the net profit in the leaf nodes for 10 years period: **Node 2: we did not reconstruct**

Revenue: 2,5 mil EUR * 10 = **25 mil. EUR**

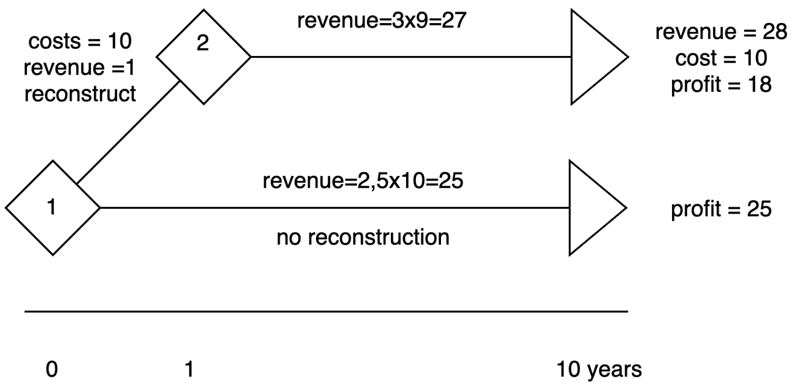


Figure 2.2: Deterministic decision tree construction

Node 4: we reconstructed

Profit:
reconstruction period: 1 mil. EUR
Post reconstruction: 3 * 9 mil. EUR = 27 mil. EUR
All in all: **28 mil. EUR**

Costs:
Reconstruction: 10 mil. EUR
Profit: 27 − 10 = 17 mil. EUR

So if we reconstruct, the profit will be 17 mil. EUR, while if we will not reconstruct, the profit will be 25 mil. EUR. Since its usually better to have 25 mil. then 17 mil., we must recommend for company not to reconstruct its premises while specified financial constrains remain.

Computation of a stochastic decision tree is much more complicated. First we need to derive basic structure of the tree in same manner we did for deterministic tree. Difference is, that we also use stochastic nodes where needed.

Edges going out of the stochastic node will need to have assigned probability $p_i \in < 0; 1 >$, where p_i is the probability of outcome i. For every stochastic node we need to develop full probabilities fan covering 100 % of possible outcome, so $\sum p_i = 1$.

Another difference in stochastic tree is that we can't directly choose what will happen as outcome of the decision, so the steps 7 and 8 of deterministic tree computation make no sense in context of stochastic tree, Instead we need to take the computed utility values of the leaf nodes and then compute the tree backwards to evaluate probabilistic nature of these outcomes.

We will need to evaluate utility of every single node in the tree including the root node. For deterministic nodes we choose best possible outcome available to the node, from the utilities of the nodes it directly leads to.

For stochastic nodes we compute geometric mean of all values the node can have weighted by the probability, so the utility U of such node can be computed (2.1).

$$U = \sum_{i=1}^{n} p_i w_i \tag{2.1}$$

where U is the utility of the node, p_i is the probability of the outcome i, u_i is a utility of the node representing outcome i and m is number of possible outcomes.

The result will represent all partial deterministic decisions we have done in the tree. Value (utility) the the node will represent a average outcome we experience when realizing the decision.

Lets try example 2 to demonstrate simple stochastic decision tree problem and its solution.

Well, the example 2 is also "shoolary" so there are some simplifications to make the computation easy. The largest one is that the vaccination will be 100% effective and could be realized "overnight" for whole population and will function immediately. Real vaccination requires realization of long

Types of problems

From the general description og computation process you should start to see some limitations of the approach. Se for example equation (2.1). To reach utility U as an average outcome we need to repeat the process several times. If our decision would be for "one time only" then we would get more representative result by choosing most probable outcome in the stochastic node, rather then the average represented by U in the computation.

We can reach the average in about 5 - 8 runs of the decision with reasonable confidence. (From statistics course you should know why.)

Example 2: Stochastic decision tree

Government committee considers economical usefulness of preventive influenza vaccination program. For such program the committee considers "early warning" system, which will cost 120 mil EUR and will permit to detect early start of influenza epidemic. After the early warning, the vaccination program will start.

If the vaccination program will not be realized, we estimate the cost of medication 280 mil. EUR with probability of 0,1, 400 mil EUR with probability of 0,3 and 600 mil. EUR with probability of 0,6. The vaccination program will cost 400 mil EUR and probability of influenza coming is 0,75.

We consider the vaccine to function immediately and be 100% effective.

(usually over October and November) vaccination campaign, only approximately 36% of the Europe's population is vaccinated (approximately 5% in Czech republic) and after the vaccination the body needs few weeks to create enough antibodies to protect itself effectively. Also the effectiveness of the vaccination is not 100%, but slightly above 60% - so do not take the example too seriously.

The construction process should be pretty obvious from the fig. 2.3, so let's start computing the gains of variants. The computation process, at least in its first phase will be exactly same as in deterministic tree – so we will ignore the probabilities for now.

Costs:
Not vaccinating and no epidemic: 0EUR (1-2-5)
Not vaccination and epidemic with low costs: -280EUR (1-2-3-6)
Not vaccination and epidemic with middle costs: -400EUR (1-2-3-7)
Not vaccination and epidemic with high costs: -600EUR (1-2-3-8)
Early warning system and no epidemic: -120EUR (1-4-9)
Early warning system and epidemic: -520EUR (1-4-9)

If the decision tree would have been deterministic, we would end at this point evaluating the costs. Unfortunately we have to consider the probabilities too, as we do not have the whole process under our control.

We have to compute the costs of stochastic nodes all the way back to node 1. We will compute the cost of such nodes in such way that we will multiply the costs of branches leaving the node by its probabilities and summarize them.

- node 3: (-280 * 0,1) + (-400 * 0,3) + (- 600 * 0,6) = -508 mil. EUR
- node 2: 0 * 0,25 + (-508 * 0,75) = -381 mil. EUR
- node 4: (-120 * 0,25) + (-520 * 0,75) = -420 mil. EUR
- node 1: max(-508, -381, -420) = -381 mil. EUR

Considering the nodes 2 a 4 we decide that we should not apply the vaccination program, as the costs connected to it are way larger (-420 mil EUR), then the costs of medication (-381 mil EUR).

Decision tree for example 2 is unrealistically simple. It has a single deterministic decision node leading to two possible recommendations:

- no vaccination
- realize early warning system and vaccinate if star of the epidemic is detected.

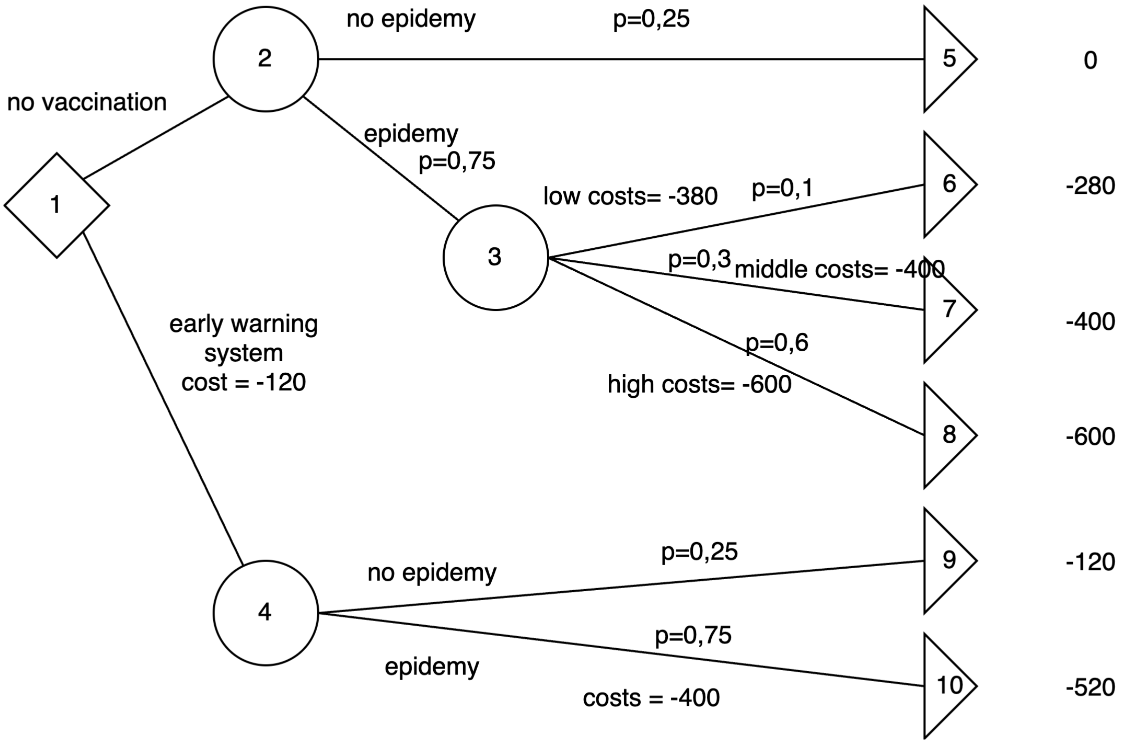


Figure 2.3: Stochastic tree construction

That is very easy to interpret. Consider different tree, see fig. 2.4.

Compute utility of leaf nodes in this tree first and then compute the tree back to the root node to apply probabilities.

The computation itself is usually not a problem for students, the interpretation is. C on the edges of the graph represents value of the criterium. For example value of the node 3 would be -100 + 600 = 500.

Value of the node 1 is 180. I provide computed values of the example in the box on the pages somewhere nearby for you to check your computations. In case you made error try to understand the structure of the tree. First check utility of leaf nodes. After you got right values for these start going back in the tree.

What was your solution and what decision did you mad based on it? Optimal correct solution for the tree is path 1 - 6, if 6 [+] then 6 - 7 - 9, else (6 [-]) 6 - 12 - 13. Did you come to same conclusion? If yes I congratulate you because you clearly understand the inner workings od decision trees and because of that you may go directly to section 2.2. If you did not succeed then I also congratulate you, you are in good company of almost all students of this course.

So why is our recommendation so complicated? Reason is that the optimal solution of our problem goes through the stochastic node 6. We know that based on computing utility of the nodes N2 = 170, N5 = 0 and N6 = 180. Since N1 is a deterministic node, we choose best value of 180 presented by N6. But N6 is a stochastic node, so we do not have control over the outcome, only probability of it.

Also note that the nodes N7 and N12 are again deterministic, for them we need to again choose optimal path of graph traversal.Our recommendation needs to take that into account.

There are two possible outcomes of node N6 denoted as [+] and [-] leading to nodes N7 and N12. When we reach these nodes then what? We again need to choose how to proceed, leading us to paths N7 - N9 and N12 - N13.

I hope that the interpretation of the results is clearer now. I provide also some other tasks you can train on in the boxes around.

You may try to compute some tasks yourself.

The Examples and task has been taken from [26].

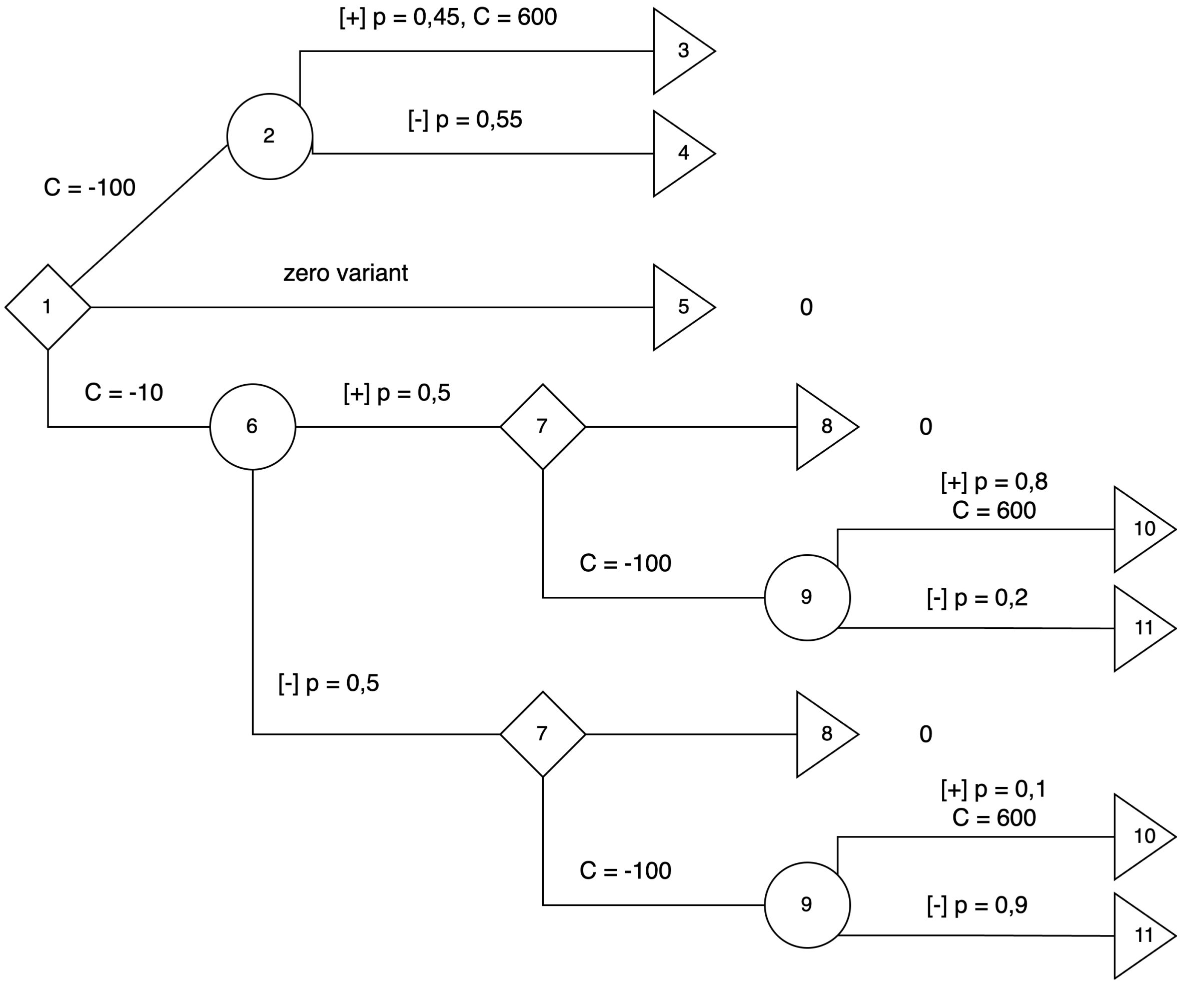


Figure 2.4: More sophisticated stochastic tree

Solution for Example 3, see fig. 2.4
leaf nodes: N3 = 500, N4 = -100, N5 = 0, N8 = -10, N10 = 490, N11 = -110, N13 = -10, N15 = 490, N16 = -110.
other nodes: N1 = 180, N2 = 170, N6 = 180, N7 = 370, N9 = 370, N12 = -10, N14 = -50.

2.2 Sensitivity analysis

One of methods to identify whether the variable has some influence is to perform so called *sensitivity analysis*. This type of analyzes is generic, so it can be use together with basically any method including decision trees. Basic principle is simple, change one variable at time and examine how are the results affected.

The analytic process follows these steps:

1. identify variables
2. choose variable from yet not analyzed ones
3. find possible range of the variable (upper and lower limit)
4. substitute actual value of the variable with lower and upper limit and compute the model
5. go back to step 2 and choose different variable

optional visualize the results using tornado graph (or any other type of graph)

6. interpret the results

Task 1
MDS is company oriented onto geologic evaluation of the land. MDG has the possibility to buy a land for 3 mil EUR. If MDS buys the land, it will perform geologic evaluation of it. Such evaluation costs about 1 mil. EUR and will result:

- Manganese 1% chance
- Gold 0,05% chance
- Silver 0,2% chance

If the one metal is found, no other metal may be found. If the manganese is found, the land may be sold for 30 mil EUR, if gold for 250 mil EUR and silver for 150 mil EUR.

MDG may also pay 750 000EUR for right to realize 3 day preliminary evaluation. Past experience show, that 3 day evaluation will cost 250 000EUR and increases possibility, that there is metal deposit to 50%. Chances to find the metals is:

- Manganese 3%
- Gold 2%
- Silver 1%

If 3 day test does not show deposit existence, chances that there actually is metal deposit is very low.

- Manganese 0,75%
- Gold 0,04%
- Silver 0,175%

What will you recommend?

Solution: task 1
3 day test and if it is successful buy

Task 2
Company decides, if it is going to take part in selection procedure. They estimate, that only the preparation for selection will cost 10 000EUR. If it is going to participate, they estimate that there is 50% chance that they will advance to second list. In second list it will be necessary to supply more information (estimated cost 5000EUR).

Company estimates, that the cost of the work and material for realization of the contract will be 127 000EUR. Company thinks about three variants of offer: 155 000EUR, 170 000EUR, 190 000EUR. Probability of success will be: 0.9, 0.75, 0.35.

Should company take part in selection procedure and if yes at what price.

Solution: task 2
take part in selection procedure, if selected into short list go for middle price (170 000EUR)

So how does the actual sensitivity analysis look like? Let's go back to simple example of stochastic tree we computed in example 2 in previous section of the text. In tree we have quantified all edges of the graph in order to be able to compute the tree. To do that we needed to establish both utility and

Other tasks you may want to practice on
On Moodle supporting course "Modeling of Decision Processes" there are also available other tasks for computations. See if you can solve them all.
The module is available at `https://lms.vsb.cz` [25].

probabilities connected to the graph.

Since decision tree is single criterion method, we used costs as this criterion. Formally from statistical point of view we estimated mean value of expected costs. In reality the true value will be higher or lower for each run if the decision as the stochastic nodes resolve themselves, as the events they represent happen. Average utility value will be achieved in long term by repeated decision making.

In sensitivity analysis we try to better understand influence our uncertainty has on the proposed solution provided by the method, in our case a decision tree.

What exactly can be uncertain on the decision tree? Probabilities ar rough estimation usually. So is the the estimation of the costs, revenue, etc.

Each source of uncertainty provides us with a range of possible values. If we have a robust datasets, we might even be capable to derive these ranges purely by statistical means.

We often use 95 % percentile for that. In other words we choose the limits to cover 95 % of variable occurrences in the dataset.

Full statistical analysis requires us to identify probability distribution for the variable. To do that we need robust dataset, from which we can derive required information. With that we have two problems. Data availability may be problematic, so in the real-world situations we often do not have available *large* enough and *good* enough dataset. That is exactly the case for our example. When no hard data is available, we have to make our own, usually based on expert estimation. See tab. 2.1 for our example.

Expert estimations are notoriously imprecise, so in case we have data, its always better to derive required information from it. That was not an option in our case.

Table 2.1: Estimate of upper and lower limit for the variables used in task 2 for purposes of sensitivity analysis

label	**Variable**	**Mean** [mil. EUR]	**lower lim.** . [mil.]EUR]	**upper limit** [mil EUR]
A	epidemic (small costs)	280	250	310
B	epidemic (medium size)	400	360	440
C	epidemic (large costs)	600	520	680
D	early warning system	120	100	140
E	vaccination (after warning)	400	360	440

Using lower and upper limits from tab. 2.1 we compute the decision thee. First for values of nodes 2 and 4 (their utility will determine the optimality of the solution, see fig 2.3. We performed this type of computation in previous section, so you can just copy these results: node 2: 381 mil. EUR and node 4: 420 mil EUR.

These values lead from interpretation perspective to decision not to use early warning system for influenza (we are minimizing the costs). We will also use these values to test sensitivity of the solution. For variables A - C we will try to determine if the costs are not greater then 420 mil. EUR. For variables D and E we will try to determine, whether the costs do not fall under 381 mil. EUR.

We will star with computation of the upper and lower limit for variant A in eq. (2.2).

$$costs = 0.75 \cdot (0.1x + 0.3 \cdot 400 + 0.6 \cdot 600) \tag{2.2}$$

In equation (2.2) the x is the analyzed variable. In case of variant A we substitute the x with values 310 and 250 mil EUR. And after computation we can clearly see, that the results are not sensitive to changes in treatment costs when the epidemics is small, because the resulting we in both cases lower then 420 mil. EUR.

Using our tree on fig. 2.3, we can derive equations similar to (2.2) for all other analyzed variables. You can find the results in tab. 2.2. For later use there is also added column with difference between upper and lower limit of the costs.

With use of the software you can even simulate reaction of the overall result of the model to even more granular changes in analyzed variable to derive even more information from the model.

You can use MS Excel for this purpose, but using other software such as MathLab or R has potential pro provide better results and this type of the analysis is also easier to realize in these.

Table 2.2: Computation of the costs when analyzing sensitivity of the variable for decision tree from task 2

label	**Variable**	**costs lower lim.** [mil. EUR]	**cost upper limit** . [mil.]EUR]	**costs Δ** [mil EUR]
A	epidemic (small costs)	378.75	383.25	4,5
B	epidemic (medium size)	372	390	18
C	epidemic (large costs)	345	417	72
D	early warning system	400	440	40
E	vaccination (after warning)	390	435	45

From tab. 2.2 we can see that the decision situation represented by tree from task 2 is not overly sensitive to changes in variables. If the task was real, as opposed to scholarly, we would have better confidence in the validity of the results the tree provided.

If we found out, that the solution is actually significantly sensitive to changes in certain variables changing, we would be motivated to better understand these variables and sources of uncertainties about them and try to keep them under better control, if possible. Otherwise we also could fail to reach desired outcomes of the solution.

It is possible to present results from tab. 2.2 in graphical form. See influence of changes in treatment costs on fig. 2.5. We used R [27] to generate the graph. Script is provided in Appendix 1 of this textbook. Similarly we could visualize all analyzed variables, but I suppose that for demonstration purposes it is not required.

Graph on fig. 2.5 is not the only possibility we have. If we need to visualize sensitivities of multiple or even all variables the *tornado graph* is often used. The name comes from typical "tornado cone" shape. Formally tornado graph is sub type of bar chart, where on y axis there are variables and on x axis value of analyzed variable. The variables in the graph are ordered by size of the difference between model results for upper and lower limit from largest differences to smallest. The ordering leads to typical shape.

We will be using pre-prepared values of Δ from tab. 2.2.

For our example the tornado graph is available of fig. 2.6.

We could also compute sensitivity for various probabilities to get complete picture for the topic. Since we would be basically repeating already demonstrated steps, I will leave the computation to you.

If you want to learn more, we can also recommend the book from Clemen & Reilly [1], which is considered authoritative for this topic.

2.3 Using Decision Trees for Multiple Criteria Problem Solution

Technically we can use decision trees also for solving multiple criteria decision problems, but we do not recommend it. Using decision trees for finding solution has a benefit in relative easiness of use, and straight forward interpretation of the results. If we introduce multiple criteria, these benefits vanish.

That's why we will touch on this topic only briefly. Start of the process is same as in single criterion decision trees:

1. derive structure of decision tree
2. quantify probabilities of partial decisions
3. quantify utility of *all criteria* (one at time)

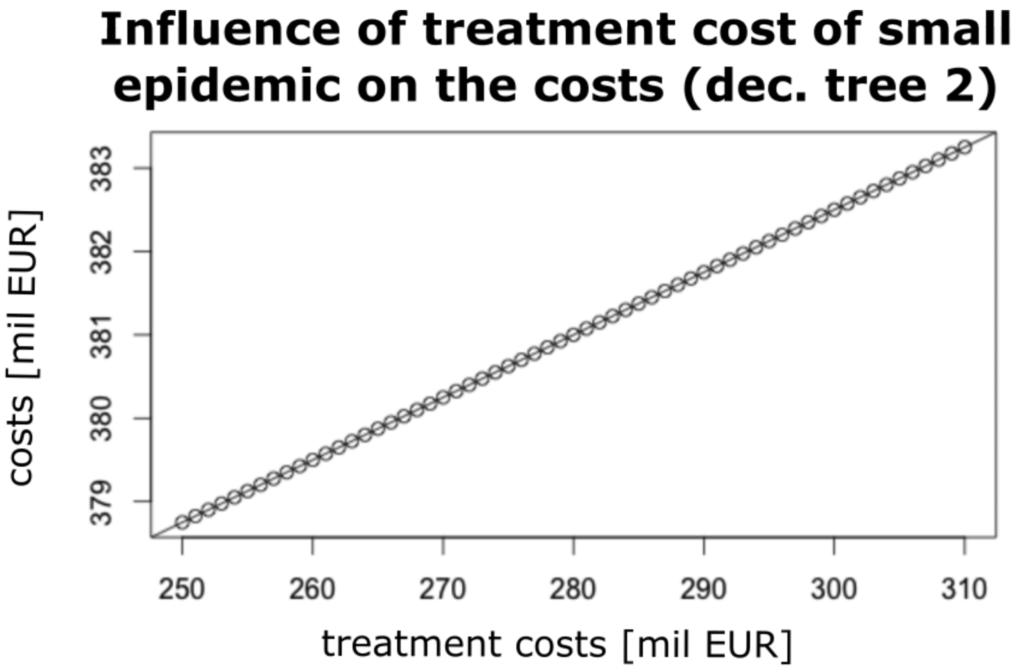


Figure 2.5: Stochastic tree construction

4. compute utility in leaf nodes for *all criteria* (one at time)
5. *establish weight system for the criteria to describe, which of them is more significant for the decision*
6. *normalize utility of the criteria originally expressed in its natural units (so we can compare them)*
7. *apply the weights in normalized utility of the criteria and aggregate them*
8. with this result, we can work as in single criterium situation (compute as standard decision tree)

Differences in comparison to single criterion computation are emphasized using italics. In step 3, when we compute the tree separately for every single criterium, we are basically working with a separate tree for every single criterium. The structure of the tree remains same, but its utility values change.

If we evaluated these trees separately, we would come to very different conclusions and recommendations regarding optimal choice. But we can't do that the the all these criteria characterize single decision problem. We need to evaluate all of them in integrated manner to come to relevant conclusions.

In other words we need an integration strategy to aggregate separate tree into a single one, which we then can evaluate and compute a "normal" tree.

Such aggregation process is not so straightforward, as each criterium has its own unit, which in turn means that we can't aggregate them directly. First we need to normalize them, so they are at least on same scale.

When normalizing, it is important to understand, that it will transform the scale in certain way, depending on chosen normalization method. It will have impact on the computation itself (what will be recommendation in the end).

Normalization methods are discussed in more detail in following chapters.

We also need to establish weight system as basis of aggregation of previously incomparable criteria values. Again there are many ways of how to establish weight and the choice of the process will again influence the results. Without weights we would be presuming, that all criteria are equal in their importance, which is not realistic. Some methods for establishing weights will be introduced in next chapter.

From perspective of ease of use, considering last few paragraphs, we recommend **not to use** decision trees for multiple criteria decision problems. MCA methods are specialized on this type of problems and provide much more elegant solution, as we will show in next chapter.

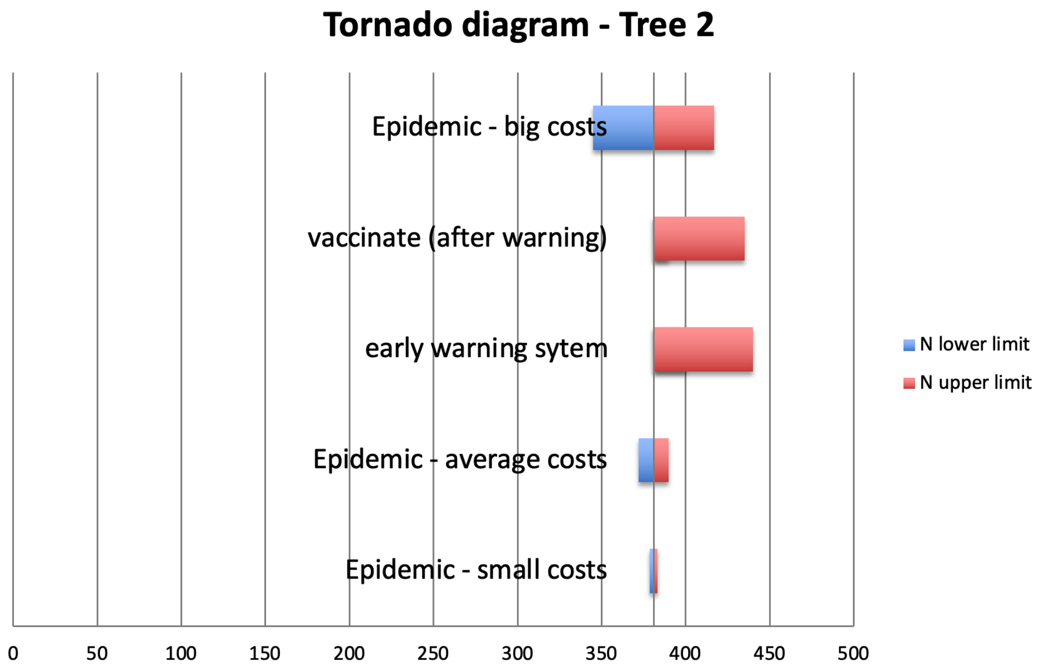


Figure 2.6: Tornado diagram for differences between the variables in decision tree 2

Computation and visualization of the variable's sensitivity
Choose one of "unsolved" variables (B - E) and perform computation of the sensibility using limits from tab. 2.1. Check, that the results correspond to those presented in tab. 2.2.

Software support for sensitivity analysis
Sensitivity analysis does not require usage of sophisticated software, you can perform basics in any spreadsheet, such as MS Excel, LibreOffice Calc or any other. That being said, using more advanced software packages such as R, MathLab, SciLab or other has it own challenges, but also benefits. While there is higher learning curve associated with them, they also allow for much better control on the results and are generally more versatile. We recommend choosing one (or two) and invest time to learning at least basics of them.

In the course we will be using R, so it is logical choice :-). It has also large support community, thousands of libraries for solving various types of problems and is open source, so you do not have to pay for it.

Summary

It is possible to transform some decision problems into shape of tree. We call such tree as *decision tree*. Solving practical problems we use more often "stochastic" variant of decision tree, allowing us to valuate probabilities of various outcomes of partial decision.

Although we can use decision trees to solve multiple criteria decision problems, we use it usually in single criterion decision situations, with criterium often being some monetary value.

After conducting the analysis it is often useful to also evaluate how is the proposed solution sensitive to changes in input variables. If the solution is not overly sensitive to such changes, it increases our confidence in robustness of proposed solution. For such purposes we use *sensitivity analysis*. We can visualize its results using *tornado graph*.

Questions

1. What are the rules for the edges going from stochastic nodes?
2. What are rules for choosing whether the node should be deterministic or stochastic?
3. What is tornado graph and what is its purpose?
4. What is the purpose of sensitivity analysis?
5. In textbook there is described graph of dependence of the result on changes of one variable (model input parameter). Do you thing that it is possible to similarly visualize such dependence for multiple variables at same time? How would such graph look like?

Chapter 3

Multiple – Criteria analysis

Study guide
In this chapter you will learn something about various methods of analysis of the decision situation, when we are optimizing decision based on multiple criteria. We will focus on decision analysis, which we will examine both theoretically and practically.

After reading this chapter you will understand how to

- Create decision analysis

know

- several approaches to multiple criteria analysis
- benefits and limitations of them
- some approaches to weight establishment
- basic idea on working of variable normalization

Time required to study chapter
This chapter is relatively hard. You will be using MCA in one form or another for your semestral project. We will also build on the knowledge from this chapter in next two chapters (MCA - Case Study and Advanced MCA). Take your time, go slowly and make sure you understand the topic.

Advanced multicriteria decision methods
In this chapter, only basic outline of multicriteria analysis is presented to introduce you to the decision making framework. For that we use weighted sum method and simplistic way of expressing values of criteria, weight, etc. With use of computers, there are many more advanced methods available. Some of these are discussed in chapter *Advanced multicriteria decision methods.*

Decision tree, we were discussing in previous chapter, is without any doubt very useful tool, but it is capable to use single criterion only. What if we actually need to use other decision criteria too, perhaps even many more criteria? And what if these criteria are measured using different units? The various criteria themselves may also differ in usability for decision making – some of them may be more important then the others.

Under such circumstances usage of decision trees is out of the question. We have to use specialized type of analysis called MCA.

So what benefits do we get in general, when using MCA?

- we may compare criteria measured in different units,
- we may assign different importance to the criterions,
- we may evaluate both benefits and risks connected to variants both independently and together.
- MCA allows us to easier achieve objectivity in decision making – to diminish effects of our bias.

So how exactly MCA achieves such different goals?

3.1 Decision analysis

Main goal of decision analysis is to process available information into tabular form and if possible to objectively analyze the problem. There are usually two main goals we can have when using this type of methods:

- identify best/worst option - in general choose one of many different variants or
- rank variants

If we consider these goals, they are actually similar, but not same. If the goal is to rank the variants, we need the method to be capable to accurately "understand" the criteria and decision maker's preferences regarding them. So it is more complex problem then the problem of finding just one optimum. Since all methods compute some kind metric usable for distinguishing between the variants, you may find last remark surprising, as we can assign rank to variant based on such metric.

The question is, *is the provided metric accurate enough to be capable distinguish between the variants*. Good analogy is the the one of distance. With MCA methods we measure their distance to optimal variant. In this regard variant closest to the optimum is the best one. But what about the others?

If our goal is to rank all variants, that distance to optimum is not enough, we need to also consider distances between the variants and that complicates things, especially if some of the criteria are expressed by ordinal values, other criteria may be expressed as continuous variables.

Each criterium has some uncertainty attached to it and we also need to transform them to deal with different units and apply different weights, which have their own sources of uncertainties. When you look at it like that, saying with confidence that one variant is better that the others suddenly seems much harder.

MCA in its written form is a supporting document for manager's decision. Manager doesn't usually directly participate in the analysis other than setting the decision problem, allocating resources needed by the analytic team and providing managerial guidance when needed.

Analytic team, especially for complex problem is usually assembled ad hoc, so the team members have large portfolio of specializations to cover whole problem's domain.

Manager needs to have general knowledge allowing him/her to interpret the results of the analysis, evaluate its quality, identify the gaps in information it provides and must be able to act on it.

The result – the analysis, has to general enough for manager to understand it and agree with its conclusions (and possibly also constrains of the decision itself).

The analysis itself is made in several steps:

1. Definition of the problem
2. Information analysis
3. Evaluating benefits of variants
4. Evaluating risks of variants
5. Final evaluation – recommendation of variants.

This is a general ambition of the analysis. There might be slight variations in it from method to method. For example lot of methods evaluate utility a risks in integrated manner, while provided steps addressed these separately. Generally speaking the steps outline works well across majority of MCA methods.

Our first task is to *define the problem*. Correct definition of the problem allows us to specify the criteria, which we can use to evaluate the variants. Definition of the problem may be problematic itself as the original definition is performed by manager outside of analytic team. The analytical team has to make sure that its members understand the problem same way as the manager does, they are aligned in intend.

If there is no such agreement/alignment or is questionable – the problem must be redefined to resolve all controversies and questions regarding the problem. To do that analytical team needs to communicate problems with the manager. The redefinition itself is performed my the manager alone again.

Failing to resolve such problems may result in the rejection of the results of the analysis as whole or the need to remake large parts of it at later time – which requires time, human resources and presents additional costs of the decision, we would like to evade whenever possible.

By *analysing of information* we mean gathering all available information significant to problem solution and its evaluation. There are different types of information we can use – *analytical information* describing various interesting properties of the variants and decision situation and *thematic information.*

Analytical information are the information adding to our understanding of the problem. They may consist of the values, catalogs, need specification, description of present state, business offers and other information. All of these support us on our way to reach the goal the manager set for the analytic team.

Thematic information is typologically same as analytical with single, but very important difference – they do not help us in reaching specified goal, at least not directly as stated by manager. So what are these information good for? They map alternative routes to similar goals.

For example the manager sets the goal to choose the new car to buy under some specified budget constrains. Analytical information would be on prices, car producers, type of the engine, size of the car, fuel consumption, safety and other information aligning with se goal. But thematic information might be information on alternative way of financing – perhaps it would be financially interesting not to buy the car "cash", but lease it or even rent it. What if getting new car was not necessary at all. We might thing about overhauling old car.

So the thematic information basically describes the road not taken during decision process. Such information may be useful in case the original goal is or becomes unachievable. Even if the analysis is good and its result accepted by the manager, the optimal variant may not be realizable (purchasable, rentable, ...) due to many different circumstances.

The company may change its orientation due to different situation on the market, leaving original goal behind. The company may not have enough money to "make it happen" or the negotiations with bank, producer, contractor, ... may fail. At that moment the company stands before the choice not to pursue the goal at all or choose different approach.

Thematic information may become analytic information at that moment, forming base of the new decision situation, with slightly different goals.

We then start new analysis, but with relatively in-depth information. Which allows us to to move much faster and cheaper to later phases of the analysis.

We are basically doing another iteration of the MCA on the same topic. Adjustments of the goals will require us to develop information sources some more, but the required effort should be much lower then in previous iteration.

3.2 Evaluating Utility of the Variant

To evaluate variant utility we have to define evaluation criteria and quantify them in some form. Different criteria may be quantified using different units. Such is the nature of the world around us and the way we use to describe it.

To demonstrate the approach we are going to use one of aldest and also easiest MCA method available - *Weighted Sum Method (WSM)*

At start, every single criterium is expressed in its natural units. For example for criterium battery stand-by time for mobile phone we can find such time from product page of its producer telling us that the battery will hold for X minutes. Other examples of criteria for such decision problem could be screen resolution in pixels or selling price in Czech Crowns (CZK) or Euros (EUR), or any other currency.

Since the criteriua are expressed in their own units, they are not directly comparable. One of first steps would need to remedy this problem by transforming scales of criteria to a common scale.

First we have to create table (matrix) with criteria in its natural units (see table 3.1). To increase the objectivity of the evaluation process, we use neutral labels C1 – Cn instead of criterion names and V1 – Vn instead of variant names.

Strictly speaking at this point such anonymisation is not required as we need to collect all "measurements" of variants in single table. We are going to use our identified information sources, and there the "performance" of the variants in criterium will be clearly attributed to certain variant, there is no way around it.

But this table should by the last one to provide this information at least by the last one to provide this information at least to the point our analysis reaches some kind of conclusion and we need to formulate recommendation for the decision for the decision. At that point we need to be dealing with a real variant, so we need to deanonymize the variants again.

The anonymisation is a tool we can employ to gain some "cognitive distance" to the decision problem and should allow us to be more objective by minimizing some cognitive biases ... but only some of these biases and not fully.

Table 3.1: Quantification of the criteria of the variants in their natural units

Criteria	**Unit**	**V1**	**V2**	**...**	**Vn**
C1	[min]	10	12	...	9
C2	-	Good	Average	...	Excellent
...	...	...	...	...	...
Cn	[kg]	0,5	0,4	...	0,55

Criteria in tab. 3.1 are expressed in their original measures, we may call these *natural units* of the criterion. When we search for information on the criterion for the variant, we usually find it in these units. Measures in the table express performance of the variant in the criterium. That is why we call this table and matrix derivable from it, the *performance matrix*.

3.2.1 Miking criterion's scales comparable

The criterium can also be expressed in entirely different type of scale. For example C1 and Cn are continuous variables, but C2 uses ordinal scale. Natural units and differences in scale types and ranges make the criteria, at this point of time, directly incomparable. It is clear from logic of MCA, that we need to compare them. To make such comparison possible, we need to transform all of the criteria into single (comparable) scale.

Basic method presented in this chapter does not provide any guidance on how to do such transformation. That means, that we are responsible to choosing such method by ourselves, and make sure, that this transformation is both mathematically sound and interpretation-wise makes sense. There are well understood and documented mathematical methods for this task called *normalization methods*. You can find some examples of such methods in chapter *Advanced multicriteria decision methods*.

Please note that this transformation, regardless of using normalization methods or more intuitive approach described in following paragraphs, will have significant impact on the results of the analysis.That means that you, as analyst, need to really think through this transformation. Won't it blur the information in the original scale? Remember, the transformation need to maintain maximum from original transformation, otherwise our recommendation based on transformed data would be misleading.

The analyst also needs to document the transformation and manager needs to review this documentation to assure that the transformations are compatible with his perception of the decision problem.

In this chapter we will choose more intuitive approach this transformation and perform it manually based on measure of our "satisfaction" with the value, expressed as percentage. Benefit of this approach is that we usually can do such transformation quickly, yet the transformation can have complex thought process behind it.

Unfortunately there are also some disadvantages to this approach. We, as humans, often choose such transformation arbitrary, without in-depth reasoning behind it. The result of such transformation is basically expert estimation of the value. As experts (including us) are human beings, we tend to be imprecise, or even wrong at times. We all are also biased. Considering all this, the transformation step can introduce easily huge errors into decision making, which the method has no means to deal with. It presumes, that these estimations are precise.

Another problem is, that the transformation process might be nontransparent for decision maker if the analytic did not describe his/hers thought process in the analysis text.It may be complicated for decision maker to decide, whether the analysis can be trusted.

Some advanced methods (see later chapters of the textbook) have incorporated means to deal with this problem to some extend, but not this one.

Transformation to percentages also solves another problem, we did not mention yet. The criteria scales may have different orientation. So some of the criteria we might need to maximize, while the other need to be minimized.

Let's say that for our example C1 may represent benchmark for some task performed by computer. 9 minutes as best time will represent 100 %, while 12 minutes (variant 2) will be only 10 %. In this case we evaluated 3 minutes differences in the criterium between the variants as highly significant. This would be justifiable in situation when an action resulting in these 3 minutes difference is repeated often.

Consider different situations with different numbers. Say we do the measured action 3x per day and we will compare it with situation when we repeat the action only once per month.

In 3x per day situation, we can compute easily amount of time difference as number of workdays, multiplied by 3 (we do task 3x per day) and multiplied again by 3 (3 minutes difference). A year has approximately 260 workdays, which leads us to: $260 \cdot 3 \cdot 3 = 2340$ minutes, which is 39 hours, which is about one workweek, assuming 8-hour work day.

For our second scenario, we do the action once per month. We have 12 moths in year, leading us to yearly time difference: $12 \cdot 3 = 36$ minutes.

When we compare these two numbers, they represent significantly different picture. Would the 100 % vs 10 % valuation still be justifiable in circumstances of second scenario? My feeling is that it would not, but it is up to you as an analyst to provide the interpretation for these number. This interpretation should be part of the analysis, so it needs to be formally expressed in textual form and put into the analysis.

Establishing of the percentage of satisfaction will enable us to compare the criteria, as the their unit will be same.

With introducing of percent values, we may transform table 3.1 into form of simple utility (see table 3.2).

Table 3.2: Table of simple utility

Criteria	**V1**	**V2**	**...**	**Vn**
C1	80	10	...	100
C2	75	50	...	100
...	...	...	...	...
Cn	60	100	...	50

Now we have criteria with exactly same unit (percentage), so theoretically comparing these values is possible. That is under presumption that the importance of all these criteria is exactly the same. In other words, if we believe that all criteria contribute to finding solution to the problem in exact same way, we may just start to comparing them. Needless to say, this is not exactly real world scenario, as we usually regard some criteria more important than the others. We have to find a way to evaluate criteria importance and incorporate this knowledge into computational process.

Fig. 3.1 provides some guidelines for some of possible transformations.

- linear,
- marginal utility function,
- custom function.

Linear growing function (and analogically linear decreasing function) represents linear relation between the criterium's value and perceived utility. With increasing value of the criteria the utility increases with constant "tempo". Fig. 3.1 only depicts linear growing function applicable on benefit criteria, but very same principle can be applied to cost criteria. We would use linear decreasing function to accommodate this type of criteria.

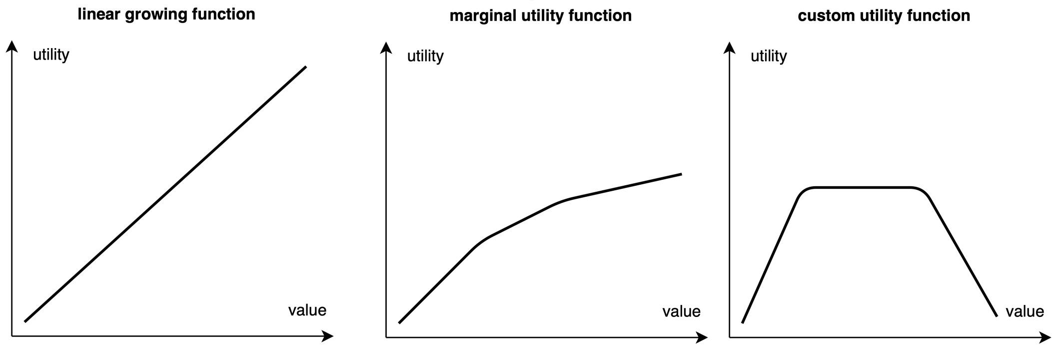


Figure 3.1: Possible transformations for a value to utility

Marginal utility function is well known approach from economy. It describes the situation when with increasing value of criteria the utility grows, but with each unit of the value, the increasing in utility is smaller and smaller eventually stabilizing at some level. Imagine situation, that that you are really hungry. You have no food, so your utility is zero. Then you are provided with portion of food, you are really hungry so you eat it immediately a it really helps, your utility significantly increases. Then you are provided with another portion of the food. You were really hungry so you might want to eat it right away too, but since you were not that hungry anymore, your utility increase would be much smaller. And you are provided with another portion, and another. Eventually regardless of how much you can eat, you will become full and will not eat any more portions of food provided.

In this moment the improvement in utility value would be zero and would nut change with any additional portion of food.

Custom utility function can be constructed in any way the analyst sees as justifiable. In fig. 3.1 we have the function split into 3 intervals, in first interval the utility grows, in second it remains stable and in third it decreases.

Imagine that you are deciding on your new mobile phone. One of the criteria would be a screen size. With larger screen the utility for the user increases, as there is more "real estate" to use your apps. But ar certain point this increase is size will not improve your utility, as even at lower sizes you were able to conveniently use the phone. In third function's interval we see the situation when increases in size are actually lowering the utility as the phone becomes too large to be conveniently used as a phone.

3.2.2 Weight of the criteria

There are many ways of how we may establish a weight system for the criteria. For this chapter we will use simple *binary pair-wise comparison* method called *Fuller's triangle*, different methods to derive weights are available in later chapters of this textbook.

Binary pair-wise comparison means that we will derive weights based on direct comparison of each pair of the criteria, deciding which of these is important. Binary means that we are answering only a question of importance but not a question of how much more important it is.

We presume, that the criteria preferred more often are more important than these which have been preferred less often or have not been preferred at all. Number of preferences will be in this method used to derive importance ranking for the criteria, which in turn may be used as basis to establish weight system.

It is entirely possible, that the criteria preferences (weights) will be part of problem documentation – the team got from manager, but if it is not, we will have to find another way to explore significance of the criteria. Before we do that let me point out one very important thing regarding criteria evaluating. **The criteria need to be independent**. This requirement is actually very hard to achieve for real world decision situations.

We usually take as satisfactory if the criteria are at maximum *weakly dependent*. Strong inter dependencies cannot be allowed to exist among the criteria in the decision analysis as they have a tendency to inflate influence of certain aspects of the problem or hide other relevant aspects.

Strongly dependent criteria need to be excluded from decision analysis, or aggregated by one of methods available for *dimension reduction*, such as Principal Component Analysis (PCA).

You can mathematically prove independence of the criteria by statistical methods such as Analysis of Variance (ANOVA).

Existence of dependencies in criteria system for decision problem may disproportionaly inflate influence of these criteria on overall results. So we can say, that inter-dependent criteria "support" each other.

Weak dependency means that, there is interdependence between some criteria, but the resulting influence change is very small and we presume, that is not significant from point of view of final recommendation.

We can also to some degree minimize undesirable influences of such dependent criteria by creating *criteria hierarchy*. Idea behind using such hierarchy us that is criteria will be leaf nodes of such hierarchy and we will put interdependent criteria under same higher level node.

Please note, that for complex problems with large number of criteria there may be multiple groups of interdependent criteria. We need to put these groups in separate part of the hierarchy. But to do that, we need to first know, that we have this problem.

OK, so we have a hierarchy now, what is it good for? Using it we will limit unwanted influence to bottom layer of the hierarchy. If we manage to properly establish weights in the upper levels of the hierarchy. In that way the influence of interdependent criteria has only limited ability to "spill" into other parts of the hierarchy.

MCA method, we are using in this chapter is WSM. This method does not support hierarchies out of the box. It can be modified to allow them by basically breaking normalized weighted performance matrix into separate submatrices, which are then computed separately and in the end aggregated up in the hierarchy.

General recommendation is - *if you identified hierarchy in your decision problem, and cannot simplify it to avoid it, use decision support method, which supports hierarchies out of the box.* Good go to methods are Analytic Hierarchy Process (AHP) or Analytic Network Process (ANP). We will have separate chapter for the AHP later in the textbook.

So how exactly does such dependence look like? Lets consider following scenario: we are considering buying of the new monitor to our desktop computer or notebook. Among other criteria we would like to evaluate also ecological aspect of such device. When searching for possible criteria we found following candidate criteria:

- energy star certification,
- TCO certification
- easiness of disassembly
- used materials
- environmentally friendly

Now when we know, that we are trying to demonstrate dependence between the criteria, it is rather easy to identify the problem – the monitor is environmentally friendly if it has low power consumption (energy star), low radiation emissions (TCO), is easy to disassemble and its produces is using materials which encumber the environment less then other materials we could legally use.

We may express the situation graphically by forming the hierarchy of the criteria used in analysis, see figure 3.2.

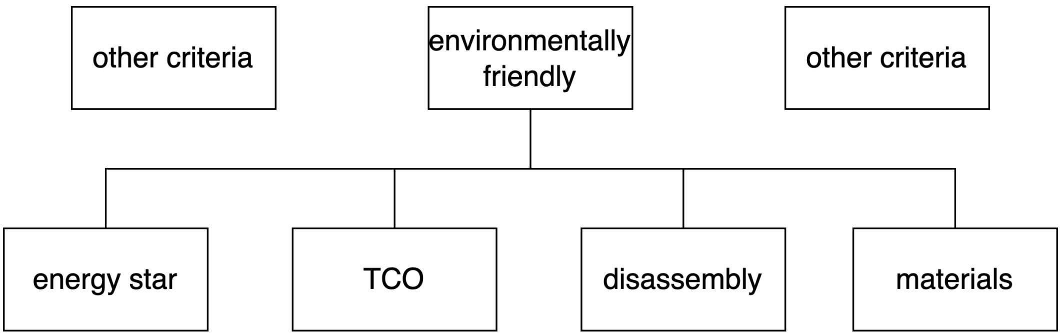


Figure 3.2: Forming up of the criteria hierarchies

To evade the problem, we have to resolve interdependencies, so we use either environmentally friendly or any other substitute for these four subordinate criteria.

To demonstrate problem of dependencies consider following. We can imagine that the criteria will form index, based on which we will formulate our recommendation. (Which is by the way exactly what we are doing).

To each criterium we assign a weight. WSM presumes a single layer of the criteria. Pair-wise comparison means that we will compare every possible pair of the criteria and establish based on expert opinion, which of these two is more important. Now imagine, that we consider ecological aspects of the purchase as very important. In context of criteria proposed in hierarchy on fig. 3.2 (minus the hierarchy itself), it is very easy to imagine, that we would prefer these 4 criteria quite often, which would lead us to assign them high weights to them. Environmental aspects of the decision then would dominate the decision regardless of differences between the variants described by other criteria.

Clearly such effects are highly undesirable.

Demonstrate inflation of influence of interdependent criteria
You can try it easily yourself by using criteria from hierarchy on fig. 3.2 using procedure described in following paragraphs. Add additional criteria like screen size, resolution, you can add other criteria as well. Compute weights using Fuller's triangle. Then compute alternative weights for same situation, with only change of aggregating energy star, TCO, disassembly and materials into single criterium - environmentally friendly.

Compare computed weights.

Let's go back to the problem of establishing criteria importance. There are many different ways of how to approach the problem. Perhaps similar problem has been solved by somebody in the company or the report is available in some journal or over the Internet. So the weights already may exist, we only need to find them. If that is not the case, we need to derive them using available information.

For this purpose we often use services of domain expert(s), see for exaple *Delphi method* discussed in latter chapters of this textbook. One of easiest ways of how to estimate weights is to use *Fuller triangle*.

Let's compare criteria C1 – C5. To do that we need to construct the comparison triangle.

Table 3.3: Fuller triangle for pair-wise comparison

C1				
	C2			
C2		C1		
	C2		C4	
C3		C2		C1
	C4		C4	
C4		C3		
	C5			
C5				

We construct the triangle column after column. Firs column has all criteria in it and we write it down without any evaluation. In the second we compare criteria next to each other, so we compare C1 and C2 – we prefer C2, C2 and C3 – we prefer C2, C3 and C4 – we prefer C4, C4 and C5 – we prefer C5. In third column we compare first criterion with third, second with fourth, third with fifth ... In the fourth column we compare first with fourth, second with fifth and lastly in fifth column we compare first with last.

This approach is generally usable for any number of criteria. Evaluation will always result in shape of triangle, always ending with comparing first and last criterion.

There are some presumptions for the method. For example we presume, that when comparing the pair, there always is one criterion which is better. In reality that is not always the case. If we identify such situation during evaluation, we will need to simply use different method for establishing weight system.

Having constructed the comparison triangle, we have to interpret the results. Let's make a table for it. We will count number of times we preferred the criterium. We may count occurrences in the first column, but nothing happens if we don't, as it has exactly one occurrence of each criterion and that's why it will not influence the final order of criteria ordered by importance, see table 3.4.

Table 3.4: Computation of the weights

Criteria	Preferred	Order	Weights
C1	2	2	2
C2	3	1	3
C3	1	3	1
C4	3	1	3
C5	1	3	1

In table 3.4, there are weights computed as inverse order to criteria order by importance. Most preferred criterion will get highest weight. In some cases, this way weights estimation may be too rigid, so we can modify it to get larger (or smaller) gap between the weights. What we can't, or better said – we shouldn't do is to modify the order itself – so that more important criterion can't have lower weights then those behind it.

In case some of the criteria share rank, in our case for example C2 and C4, we can use more granular evaluation. We can set weight of C2 to 3.1 and C4 to 2.9. The explanation could be that when comparing these two, we preferred C2 over C4.

As rule of thumb we need to make sure that out weight modification will not change the ranking of the criteria. Again in our case we must ensure, that the weight of C2 will remain greater then C1, as C2 was preferred more times and thus is more important for the decision making.

Fuller's triangle method puts together par-wise comparison and weight computation. This perspective is available in tab. 3.5. When you compare results presented in tab. 3.5 and tab. 3.4 you will see, that the results are exactly same. So we can perceive it more as an issue of layout then anything else, and choose between these based on personal preference.

Table 3.5: Fuller triangle for pair-wise comparison with weight estimation

Criteria comparison				criterium	occurrences	rank	weight
1	**1**	1	**1**	C1	2	2	2
2	3	**4**	5				
	2	**2**	**2**	C2	4	1	3
	3	4	5				
		3	**3**	C3	1	3	1
		4	5				
			4	C4	2	1	2
			5				
				C5	1	3	1

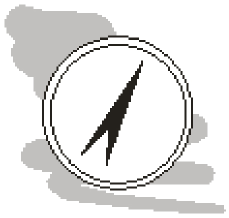
Preferences in tab. 3.5 are written in bold.

Applying the weights on matrix of simple utility we get the matrix of weighted utility (see table 3.6).

Table 3.6: Table of weighted utility

Criteria	Weights	V1	V2	M
C1	2	80 x 2 = 160	20	200
C2	3	75 x 3 = 225	150	300
C3	1	...	...	...
C4	3	...	...	...
C5	1	...	...	...
$\sum$		385	170	500
U	%	77	34	100

Alternative solution for tab. 3.6 is to use normalized weight coefficients. Simplest way of how to normalize them is to take coefficients from tab. 3.5 and divide them by sum of all weights. Resulting weight coefficient would automatically be $w \in <0; 1>$, with $\sum w = 1$.If the normalized criteria are expressed as % - $U \in <0; 100>$, then $\sum wU \in <0; 100>$ for all variants, which allows us to interpret it as % and we do not need to compute theoretical optimum M as used in next step.



Controversies around weight coefficients
In tabs. 3.4 and 3.5 the weights all $w \in <1; 3>$, this is not the only way of how to compute them. The choice of scales is arbitrary provided that we preserve proportionality of the weights. In our case for example that the criterium with original weight of 3 will retain weight 3 times that of weight of criterion with original weight 1.

For practical reasons it is often recommended to use weights $w \in <0; 1>$, which corresponds to possibility to interpret it as a percentage. Which is usually perceived as easier to interpret.

When choosing weights we also need to consider the way we are going to use them.

We compute the weighted utility as product of weight and simple utility for criterion of given variant. We also construct new column representing theoretical maximum variant M, which represents an optimal variant. Optimal variant fulfils the criterion (performance wise) on 100%, weighted gains then are simply $w \cdot 100$ for this optimal variant.

We need this information as WSM is a method which evaluates optimum in sense of "distance" of this ideal variant to the variants we are evaluating.

Alternatively to M as a theoretical maximums of the utility in the variant, we may use maximal value of the criterion for M variant instead. The ideal variant will fulfil our expectations to 100%, we can expect that the real variant, we are evaluating will not reach this value, only get close to it. In we use existing maximum, we ensure, that this ideal variant will be constructed brom best properties of evaluated variants and because of that, it will be closer to them.

Both approaches will lead to same order of evaluated variant, but with different valuation, which may be relevant, if you are going in similar way also compute risks and then try to aggregate results from these two evaluations.

For its properties we sometimes call results presented in this table as *preference matrix*. Reasoning behind it is that it holds information on the preferences of the decision maker in form of weights, which were applied on normalized performance of variants in these criteria.

3.3 Risk evaluation

For evaluation of risk we will basically perform same steps as for utility evaluation. We again start with performance of the variants in identified risks. We usually specify risk as a function of probability of occurrence and expected impact such occurrence will have. While it may seem straightforward, there are some caveats, which need to be discussed.

First we need to consider time horizon for the risk - it needs to be same for all the risks. Consider risk R1 with yearly probability of occurrence of 10 % and risk R2 with 10 % probability of occurrence in 10 years. We can expect R1 to occur ten times more often then R2.

Performance matrix for the risk is presented in tab. 3.7.

Table 3.7: Matrix of simple risks (performance matrix for risk)

Risk	**V1**	**V2**	**Vn**
R1	10%	8%	...
R2	15%	25%	...
Rn	...	...	...
...	...	...	...

Similarly to criteria utility evaluation, the risks may be weighted. If we perceive risk performance matrix in tab. 3.7as a measure of probability specified for set time period, we may perceive the weight as a measure of impact, forming together simple model of risk. See tab. 3.8 for example. The weights for risks in this case were established in similar way as the weights for the benefits.

Table 3.8: Matrix of weighted risks

Risk	**Weights**	**V1**		**V2**		Vmax	
		P	**GT**	**P**	**GT**	**P**	**GT**
R1	1	0,1	0,1	0,08	0,08	1	1
R2	2	0,15	0,3	0,25	0,5	1	2
$\sum$			0,4		0,58		3
GT	%		13,3		19,3		100

As opposed to utility evaluation, where we could have criteria to be maximized and other criteria which must be minimized, when evaluating risks, we always minimize them.

Similarly to utility preference matrix, we introduce new variant Vmax, with probability of occurrence 100%. We use this variant as etalon to compare our proposed variants.

Again Vmax may be also constructed based on worst available scenario, using highest value for given risk.

Analogically to utility evaluation we measure the risk as a distance to the Vmax variant, but this time around we are preferring variants as far as possible from this variant as such variant has lowest risk associated with it.

3.4 Results and their interpretation

By constructing utility and risk performance matrices we gathered all data required to formulate recommendation.

When choosing of what to recommend we usually follow one of strategies:

1. Maximal
2. Minimal
3. Optimal

Maximal strategy prefers maximization of the utility regardless of possible risks connected to the variant. Our recommendation in this strategy is derived solely from results computed in table of weighted benefits. We are searching for the variant with highest score.

Minimal strategy represents the situation, when we are completely averse to the risk – so we are minimizing risks regardless of possible utility we could get from realizing the variants. Our recommendation is based solely on results of the table of weighted risks.

Also remember, that for the risk evaluation using WSM we seek a minimal value. It seem probably obvious for you now, but my experience with students works is that this minor detail is often forgotten. And if that happens wrong conclusions are being derived from the data.

Lastly we can compare risks and utility optimizing our decision, see table 3.10.

Table 3.9: Utility vs. Risks

Order	**1st**	**2nd**
Utility	V1	V2
Risks	V1	V2

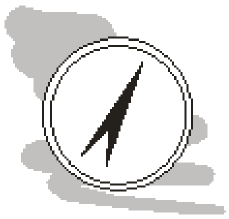
Table 3.10: Final Effect

	V1	**V2**
Utility (U)	77	34
Risk (R)	13,3	19,3
Final Effect (U-R)	63,7	14,7
Final Effect (U/R)	5,79	1,76

As we can see both minimal and maximal strategy did recommend same variant (V1), optimal strategy may only result into recommendation of this variant too, otherwise we would have to compute final effect of the variant (see tab. 3.10).

As final effect is computed in two different way. In our case both approaches lead to recommending of V1, but in cases when maximal and minimal strategies differ in its recommendations it may lead to different recommendations. In such cases we recommend one variant as optimal and the other one as its alternative.

Also it is good practice to write down executive summary for management, which consist of recapitulation of whole analysis and its recommendations.



Mistakes?
Well if you look back at the chapter, the process of the MCA is pretty strain forward. The analysis structure is firm and we use only basic mathematical operations, so we can not make a mistake, can we?

Unfortunately my experiences say that you can and it is not hard at all. It is rather easy to make mistakes both logical and numerical. Typical logical mistake would be to presume that we are maximizing all criteria, another one would be to maximize the risks.

These kind of mistakes come from lack of focus and automatic writing without thinking. Best defense against it is to thing logically proofread parts of the analysis to make sure to constrains hold.

Numeric problems come from bad translation of the criterion's natural units into matrix of simple benefits or simply making a mistake during the tables computation. Best help is to use spreadsheet such as MS Excel, LibreOffice Calc or any other to support you with computations. Using such programs will automatically recompute all tables as required, leaving to you only the task to copy them to the report.

Both types of mistakes come from heavy underestimating of the complexity of the analysis – so do not underestimate complexity of MCA.

Now that we went through the process of WSM, we need to also discuss some limitations of the approach. We touched some of them briefly when discussing parts of the WSM procedure, for example when we were talking about the requirement on independence of the criteria and criteria hierarchies or rather when we discussed absence of support of these hierarchies in WSM

But there are also different limitations. Let's play a little game. Take a piece of paper, a pen and based of what you already know of method take few guesses on nature of these limitations. Then continue reading. You should be able to identify at least some of them.

Spoilers:
One problem are the *uncertainties*. Consider information provided in tables 3.1 - 3.10. Do you see, how we incorporated uncertainty into our analysis? You don't, well that's fair as we were doing no such thing. We were simply presuming that information provided is precise and logically from that, no uncertainty sources do exist in our decision, at least if we use this method.

Another limitation is that we presume that all existing differences between the variants as measured by criteria valuation signals significant difference in utility between the variants. At least for purposes of identifying optimal variant of solution. In real-world scenarios, this is also often not the case.

We had example with time difference of 3 minutes. Another simple example could be a price difference 1 EUR vs 10 EUR. Is it a lot? Sure, if we are buying breakfast, but what if we are buying notebook or a car? Both of these numbers would present absolutely insignificant difference from utility point of view.

WSM has no ability to derive this information from data it processes. That means that we, as analysts need to manually provide this information by adjusting the data in such way, that proper conclusion may be derived from it by the method.

WSM is very simple method. This simplicity is weighted out by requirements on analyst using the method for preparing inputs and interpreting the results.

These limitations are inherent property of the method, or more specifically the limitations are derived from mathematics it uses to compute the results. These limitations are also more impactfull, then you may realize, as WSM is often used by various index methods for integrating information provided by various indicators.

We routinely encounter such approaches in risk analyses (especially qualitative ones), economy and variety of other study fields and problems. Be careful when using these and always try to evaluate how these limitations apply to your decision problem, or some other task you are using WSM to solve.

Summary

In MCA we process in following way:

1. collect analytic and thematic information
2. identify possible criteria and risks, which are going to be used in the analysis to measure performance of the variants
3. construct performance matrix for utility of the variants in criteria's natural units
4. normalize it to get comparable measures
5. derive wights for criteria and apply them to get preference matrix for utility
6. specify performance of risks using risk model of choice.
7. compute preference matrix for the risks
8. compute final effect
9. interpret the results and formulate recommendations

Quiz:

1. Define problem with the hierarchies of criteria and how can we resolve it.
2. How can we estimate weights of the benefits?
3. How is working pairwise comparison? (describe the principle)
4. What are limitations of WSM?

Chapter 4

MCA Case Study

Case study
in this chapter we will try to choose best e-ink e-book reader to demonstrate process of WSM for solving real decision situation.

The reader should will be used by heavy reader requiring maximal comfort when reading. The reader should be using e-ink display, which provides maximal contrast when reading even on direst sun. Physical buttons on the device are preferred for page turning to minimize grease deposit on the display.

Note to organization of the solution
As this task is relatively simple and for demonstration purposes criteria system has been simplified. Also note that all technologies age, chances are, that the readers we are using in the example have been replaced by some newer and better models, so they might seem archaic at this point. For purposes of demonstration that is irrelevant, as basic principles demonstrated here still hold.

Text of the analysis is accompanied by notes, which should provide context for the steps and commentary for you to better understand it. These notes are in this chapter emphasized using italics. These notes are not part of the standard analysis.

4.1 Information

At present time there are many devices allowing the reader to consume the books. Even for e-ink devices there are lot of options which differ significantly by the size of the reader, but also other properties. Most reader use following size of the display:

1. small - diagonal 5 inches
2. standard - diagonal 6 inches
3. large - diagonal 9.7 inches

For purposes of the analysis size of e-book readers will be limited to standard size only as it provides best ratio between the usability and reading comfort. 8 inches diagonal, would be ideal as it provides same size of electronic paper as A5 paper format, but there are no such devices on the market[1]. Large devices with diagonal of 9.7 are significantly heavier and also use older display technologies and do not allow to use front light to improve further contrast of the text on the display.

Differentiation is also on display technologies used. Especially older devices use Vizplex technology which is inferior to newer Peal and Carta displays. Carta displays provide best properties but at the time their availability is limited to only Amazon Kindle devices.

[1]well at least the were not available in 2013-2014, when the study has originally been developed.

As Visplex displays are utilized by outdated devices, the analysis will focus on devices with more modern Perl or Carta display.

From usability point of view, ergonomy of user interface is very important. More complex operations are better handled by touch displays, but for reading itself, using physical buttons for page turning is better, to limit display smudges. The reading is also better if the display is illuminated at least somewhat using LED, which subjectively increases text contrast and allows for reading in dim lit environment.

Another important feature of the e-book reader is support of various e-book file formats. Lack in support of certain format limit usability of certain e-book stores, which use such format for book distribution. Workaround for limitation would be usage of external conversion tool performed on computer. This conversion may be further complicated by usage of Digital Rights Management (DRM) by certain e-book publishers.

While there are technical means of how to break such protective measures, doing so is usually in violation with license terms of both publisher and the store you are using for your purchases. In EU and many other places around the world circumvention of such protection measures is even illegal.

If the e-book reader supports more formats, it provides the reader with freedom of the choice of what store to use. On the other hand many high quality conversion utilities exist, so the format support does not need to be considered such a large problem. Introduction of another utility to the e-book workflow presents some inconvenience and additional work, which could be otherwise avoided.

Format support criterium will be used as supporting (low weight) one.

If the device has audio-jack, it can also be used for consuming audio books or music. The quality of sound is questionable for this type of devices, but audiobooks typically do not require high fidelity audio support, to be enjoyable. Availability of the feature may improve usability of the device.

Slot for SD cards, can have its uses as well as it allows to physically transfer whole libraries between the devices, but for normal use, the available memory is good enough for most uses.

Audio-jack and SD card support will also serve as supporting criteria. On the contrary to that, ability to connect to the Internet via Wi-Fi has significant benefits, as it allows the device to update over the air, synchronize itself with computer without the need to physically connect the device using USB cable. It also allows tighter integration with e-book e-shop to improve user experience of finding and purchasing of the e-books. Consumption of on-line text content is also enabled by Wi-Fi, but refresh speeds of the e-ink is so low, that the surging is very inconvenient on such device. So this functionality is considered as secondary at best, as well.

Based on the information we gathered, we choose following devices:

- V1 - Kobo Glo
- V2 - Sony Reader PRS-T3
- V3 - Kindle Paperwhite 2
- V4 - Pocketbook Touch Lux
- V5 - Bookeen Cybook Odyssey HD FrontLight

Note commentaries in previous paragraphs. Specifications of parameters used in decision making is largely dependent on purpose of the analysis. Task formulation itself may be not well formulated, allowing multiple explanations. In such case analyst has only two options. (S-)He can interpret it best as (s-)he can, or inquire on mean meaning, leading to adjusting of problem formulation. The decision making process does not need to be one way process.

Also note, that the analyst commentaries to criteria and variants serves also a purpose. It informs the decision maker on how analyst sees the problem. This is important signal the decision maker can use to quickly identify possible problems with analysis. Proper step for decision maker would be to return the analysis to be amended.

In case the purchase of e-book reader will not be possible to complete, there is possibility to purchase another type of device - tablet. Tablets are more universal then e-book readers. They allow full fledged media consumption and use of large amount of applications, which are not available on e-book readers. This allow the tablet to be used for much broader portfolio of task and workflows. But there are also some negatives. They are often much pricier, heavier, and require recharging battery much more often then e-book readers.

Last paragraph was written in different way then the previous paragraphs. That's because it presents different type of information. While first paragraphs described rationale of task and criteria and variants, in other words analytic information, last paragraph explored alternative mindset. Purchase of tablet goes against the task as it clearly say that the device should be e-ink based. This type of information is thematic.

Now for the reason to change our mind. For example originally we planned to purchase both e-book reader and tablet, but we found that for economical reasons we no longer can purchase both. Suddenly the decision problem changes. Thematic information provides starting point for different decision analysis.

4.2 Utility

Table 4.1: Performance matrix (criteria in natural units)

Variants	**units**	**V1**	**V2**	**V3**	**V4**	**V5**
C1 Display	-	Pearl	Pearl	Carta	Pearl	Pearl
C2 weight	g	185	200	206	198	180
C3 touchscreen	A/N	A	A	A	A	A
C4 hardware buttons	A/N	N	A	N	A	A
C5 front light	A/N	A	N	A	A	A
C6 battery	no. of page turns	2 100	1 700	1 700	1 700	1 900
C7 Wi-Fi	A/N	A	A	A	A	A
C8 price	Kč	3 400	3 800	3 300	3 400	4 200

Legend

Display - newer Carta provides higher contrast in comparison with Pearl displays. We will evaluate Carta displays as 100 % and Pearl displays as 80 % utility. Generally speaking Pearl displays provide better reading experience.

Weight - is closely connected to convenience during long reading periods. The weight difference between lowest and highest values is only 26 g, which is very low value, almost insignificant. That's why we evaluate weight 206 g as 85 %.

Touchscreen - implemented in all devices we evaluates. We will exclude this criterion from rest of analysis as it does not contribute to our decision.

Hardware buttons - for page turns. In the buttons are not present on the device we evaluate it 50 % as it means significant discomfort in using the reader (because of smudges on the screen).

Front light - the display is illuminated by LEDs, which significantly increases the reading contrast, allowing reading in dim lit areas. The absence of front light is a major defect in usability, so such device is rated only 20 %.

Number of page turns (with Wi-Fi off): each e-book reader producer has its own methodology for computation of battry capacity. For example for Kobo Glo its manufacturer expects 70 hours, presuming 1 page turn per minute, which lead to estimated value 2100 pages. Other manufacturers do have their own methodologies on how to measure the battery capacity. E-ink displays require energy only to redraw the page. Page turns is ideal criterium to use to compare devices across different manufacturers and their own measurements.

Wi-fi - again is implemented on all devices and again we exclude it from further analysis as it does not contribute to the decision.

Price - is important factor fo the decision. Highest value 4 200 is considered to be only 50 %.

Please note, that the utility doesn't need to be in interval <0; 100>. Acceptance values are estimated solely from subjective point of view, just to allow us to identify differences between the devices and evaluating performance hit these differences make.

The utility curve may have but does not need to have linear character, any other function may by usable too. For example the price 4 200,- Kč is evaluated as 50 %, but somewhat higher price of 5 000, may be perceived as 10 % or even lower (if evaluated as prohibitively expensive). The analytic chooses transformation function arbitrary depending on his/hers knowledge and experience in problem domain. It is recommended for the analyst to describe his though process for these transformation as the transformation will significantly influence the results and it may be something which decision

Table 4.2: Normalized performance matrix

Variants	**V1**	**V2**	**V3**	**V4**	**V5**
C1 Display	80	80	100	80	80
C2 Wright	97	88	85	90	100
C4 hardware buttons	50	100	50	100	100
C5 front light	100	20	100	100	100
C6 battery	100	70	70	70	85
C8 price	94	70	100	94	50

maker might have hard time to digest.

Weight coefficients are estimated based on preference measurement using binary pair-wise method of Fuller's triangle. Pair-wise comparison is available in tab. 4.3.

Table 4.3: Binary pair-wise comparison - Fuller's triangle

C1 display					
	C1				
C2 weight		C1			
	C4		C5		
C4 hardware buttons		C5		C1	
	C5		C2		C1
C5 front light		C4		C2	
	C5		C4		
C6 battery		C5			
	C6				
C8 price					

Table 4.4: Weight coefficient estimation

Criterium	**no. of pref.**	**rank**	**weight**
C1 display	4	1	5
C2 weight	2	3	2
C4 hardware buttons	3	2	4
C5 front light	5	1	5
C6 battery	1	4	3
C8 price	0	5	1

Table 4.5: Utility preference matrix

Criteria	**weight**	**V1**	**V2**	**V3**	**V4**	**V5**	**M**
C1 display	5	400	400	500	400	400	500
C2 wright	2	194	176	170	180	200	200
C4 hardware buttons	4	200	400	200	400	400	400
C5 front light	5	500	100	500	500	500	500
C6 battery	3	300	210	210	210	255	300
C8 price	1	94	70	100	94	50	100
$\sum$		1 688	1 356	1 680	1784	1 805	2 000
utility (U)	%	84,4	67,8	84	89,2	90,25	100

4.3 Risk

For purchasing of e-book readers following risks have been identified:

- R1 - premature end of support for the reader
- R2 - rejecting reclamation for the display
- R3 - closing of the e-shop integrated in the reader

Table 4.6: Performance matrix for risks

Risk	**V1**	**V2**	**V3**	**V4**	**V5**
R1 end of support	40	30	5	10	10
R2 reclamation rejection	30	30	5	40	10
R3 e-shop's end	40	40	0	30	40

Table 4.7: Pair-wise comparison for risks

R1 end of support		
	R2	
R2 reclamation rejection		R3
	R2	
R3 e-shop's end		

Table 4.8: Estimation weight coefficients for risks

Risk	**no. of pref.**	**rank**	**weight**
R1 end of support	0	3	1
R2 reclamation rejection	2	1	3
R3 e-shop's end	1	2	2

Table 4.9: Preference matrix for risks

Risk	**weight**	**V1**		**V2**		**V3**		**V4**		**V5**		**Vmax**	
		p	**R**	**p**	**R**	**p**	**R**	**p**	**R**	**p**	**R**	**p**	**R**
R1 end of support	1	0,4	0,4	0,3	0,3	0,05	0,05	0,1	0,1	0,1	0,1	1	1
R2 reclamation rejection	3	0,3	0,9	0,3	0,9	0,05	0,05	0,4	1,2	0,1	0,3	1	3
R3 e-shop's end	2	0,4	0,8	0,4	0,8	0	0	0,3	0,6	0,4	1,2	1	2
$\sum$			2,1		2		0,1		1,9		1,6		6
R			35		33,3		1,7		31,7		26,7		100

4.4 Results

Table 4.10: Results - final effect

	V1	**V2**	**V3**	**V4**	**V5**
Utility (U)	84,4	67,8	84	89,2	90,25
Risk (R)	35	33,3	1,7	31,7	26,7
Final Effect (U-R)	49,4	34,5	82,3	57,5	63,3
Final Effect (U/R)	2,4	2	49,4	2,8	3,4

Considering values computed in this chapter variants V3 and V5 look as good candidates for the purchase. V3 is Amazon Kindle Paperwhite 2 and V5 is Bookeen Cybook Odyssey HD Frontlight.

Kindle can be considered as safe bet due to the size of its manufacturer Amazon and very good service record by both Amazon and e-book publishers. Cybook scores with innovative technical properties. Investment in it can be seen as slightly more risky as Bookeen is small French company in comparison with Amazon, leading to higher probabilities of problems associated with device service or reclamation.

Table 4.11: Ranking

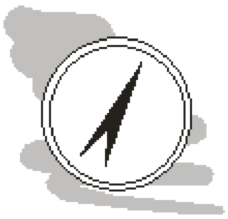
	1	**2**	**3**	**4**	**5**
per utility (maximal strategy)	V5	V4	V1	V3	V2
per risk (minimal strategy)	V3	V5	V4	V2	V1
optimal strategy (U-R)	V3	V5	V4	V1	V2
optimal strategy (U/R)	V3	V5	V4	V1	V2

Semestral project
Perform MCA on problem of your choice as your semester project. There must be minimally 5 variants characterized by 5 criteria. Consider also minimally 3 risks.

Upload result of your work on http://lms.vsb.cz course Modeling of Decision Processes for evaluation.

Chapter 5

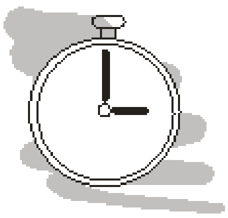
Advanced Multiple Criteria Analysis Methods



Study guide
In this chapter we will broaden our horizon with more advanced methods for MCA.

After reading this chapter you will know how to

1. apply various normalization methods
2. use alternative methods for weight coefficient estimation
3. compute formally defined decision problems using WSM (as in previous two chapters) and ELECTRE family methods with support of R.



Time for study
Time required for going through this chapter can be hugely different based on your expectations of it. It can take 20 minutes if you want to just go through it and get "strategic" view of the topic. On the other hand, if you are interested in topic, this rabbit hole goes really deep.

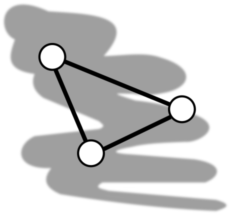
You can for example spend a lot of time on basics of R. Benefit would be to get competences to more effective data manipulation, support of statistical computations and advanced graphing. Variety of libraries exist for all sort of problems, form statistics to AI. In fact its currently impossible to outgrow R, as it is usable on everything from personal computers to supercomputers solving cutting edge research problems.

From perspective of decision support, you can also go deep. The library, we are going to use has implemented support for about a dozen of methods not described in this textbook. Armed with basics theoretical knowledge and user manual you can try them and learn a lot by doing so.

In this chapter we will use the basic outline of MCA, but discuss in greater detail some problems we encountered before with WSM and some strategies we can use to solve them, or at least minimize them.

First problem is the requirement on using comparable scales for all of the criteria. We need to transfer all criteria values to some common scale from its natural units to meet this requirement.

We are going to use normalization methods to address this problem.



Ranking, risk management, ...
Knowing the basics of decision making a good question can also be, where else can we use these principles? Turns out that what we learned is very broadly applicable. In risk management we use risk analysis methods to analyze identified risk for region, organization, unit, technology leading to ranking of the risks. Principles allowing us to compare variants of the decision allow us to compare risks as well.

Methods like WSM serve as base for various screening methods for example in risk mapping or critical infrastructure resilience evaluation. That allows us to better understand true strengths and weaknesses of those methods and better interpret its results.

5.1 Normalization

By *normalization* we understand transfer of numeric scale the criterion was expressed in (its natural units) to unified (normalized) scale often in interval $< 0; 1 >$. There are many normalization methods. On following pages we provide some more popular approaches for study.

Be careful when selecting a normalization method. There is lot of them and they are computed in different way, obviously. What is not so obvious is that they have significantly different statistical properties.

Normalization is a transformation of values after all and as such it will impact results of the analysis. That means that choosing inappropriate method disturb ability of gathered data explain the problem and analysis itself may lead to misleading results.

min-max
Simplest method and also possibly most popular one is so called *min-max* method. When using this method the normalized value is computed from ratio of minimum on the scale to difference between maximum and minimum. The equation is available in (5.1) for growing scales (for criteria we maximize)and in (5.2) for descending scales (for criteria we minimize). As apparent from the equation, the normalized value will always be $z \in < 0; 1 >$.

That means minimal value on used scale will be represented by 0, while maximal value will be represented by 1, at least for benefit (maximization) criteria. For cost (minimization) criteria it works analogically, but in reverse.

If we multiply z by 100, we can easily translate the scale to percentages.

$$z = \frac{x - min(x)}{max(x) - min(x)} \tag{5.1}$$

$$z = 1 - \frac{x - min(x)}{max(x) - min(x)} = \frac{max(x) - x}{max(x) - min(x)} \tag{5.2}$$

where z is normalized value and x is a value on original scale.

Comparison to mean value can be computed by dividing original scale value by mean of all values on the scale in dataset. This type of normalization is much less widely used as it does not guarantee transformation to certain specific range type of transformation. That basically defeats basic purpose of the normalization.

This transformation will center the scale around mean in the dataset, which will be equal to 1, with values lower then this average: < 1 and values higher then it will be > 1.

For purposes of MCA it is recommended to use means of vector normalization or comparing the value to the maximum. Another reason to avoid this normalization is that this method does not distinguish between maximizing and minimizing criteria, see eq. (5.3).

$$z = \frac{x}{\bar{x}} \tag{5.3}$$

Logarithmic normalization can be used for nonlinear relations on original scale. It is often used when the data are skewed to the right. Logarithm helps to make the data to look more like they belong to normal distribution by "compressing" larger values and stretching the small ones.

Most often we use natural logarithm for this purpose, but technically we can use any base for logarithm with very similar effect.

The equation is available for ascending scales in (5.4) and descending scales in (5.5).

$$z = \frac{\ln(x)}{\ln(\prod_{i=1}^{m} x_i)} \tag{5.4}$$

$$z = \frac{1 - \frac{\ln(x)}{\ln(\prod_{i=1}^{m} x_i)}}{m - 1} \tag{5.5}$$

Where m is number of values in dataset for the scale (criterium).

Vector normalization

We can also approach the normalization by geometric means. In this case we divide original value x by square root of the sums of the squares of the individual components, see (5.6) for ascending scales and (5.7) for descending scales.

$$z = \frac{x}{\sqrt{\sum_{i=1}^{m} x_i^2}} \tag{5.6}$$

$$z = \frac{\frac{1}{x}}{\sqrt{\sum_{i=1}^{m} \frac{1}{x_i^2}}} \tag{5.7}$$

Linear normalization

Another type of normalization is so called *linear normalization*. This form of normalization operates such, that the value on original scale is divided by sum of all values on that scale. Equation for ascending (5.8) and descending (5.9) scales are available.

$$z = \frac{x}{\sum x} \tag{5.8}$$

$$z = \frac{\frac{1}{x}}{\sum \frac{1}{x}} \tag{5.9}$$

We already mentioned this method when talking about normalization of weight coefficients in previous chapters.

For sake of completeness we also provide equation for *z-score* also known as *standard score*. This is a measure that quantifies a distance of the value to the mean of the dataset, or criterium in our case.

Mean value of x will be equal to 0, value of 1 will mean distance of one standard deviation of the dataset to that mean, see eq. (5.10).

$$z = \frac{x - \mu}{\sigma} \tag{5.10}$$

Where μ represents mean and σ standard deviation.

Since values of z-score don't have set range, using it is not recommended for purposes of MCA. But the method is being extensively used in various datamining approaches as a mean to pre-process the data.

All above mentioned methods allow us to re-scale values to new scale, but all of them will lead to different values. Let's demonstrate this on small dataset of $\{1, 2, 3\}$. Results of various transformations is available in tab. 5.1. Please note that since these transformations lead to different values, choice of normalization method will influence the results of MCA, which use these re-scaled values to formulate recommendations.

Table 5.1: Normalization of values 1, 2, 3 using various normalization methods

method	**1**	**2**	**3**
min-max	0	0,5	1
z-score	-1	0	1
to average	0,5	1	1,5
logarithmic	0	0,1155	0,1831
vector	0,2673	0,5345	0,8018
linear	0,1667	0,3333	0,5

There are many other normalization methods, each with its own set of benefits and limitations. Before applying these methods, or any other you find, be sure to understand mathematical properties of the transformation and that is will preserve all desirable statistical properties of the criteria.

Questions on what type of normalization should you use are unfortunately often dismissed both in literature and real world. That is gross simplification with potential to invalidate the results of the analysis.

Our recommendation is: be careful, think before using such method and precisely document your approach to problem solution including your reasoning behind it.

Since for explanation purposes we are using R language, we provide code listing used to compute these values. Normalization methods are available for easiness of use in package MCDASupport, which is available either in course on Learning Management System (LMS) or GitHub page: https://github.com/psenovsky/MCDASupport.

Provided code is functional in package up to version 0.29. Version 0.30, which is as of date of publishing this edition of the textbook, has a different approach to normalization.

Listing 5.1: Normalization of values in R using package MCDASupport v0.29

```
install.packages("MCDASupport_0.29.tar.gz", repos=NULL, type="source")
library("MCDASupport")
tonorm <- c(1,2,3)
norm_minmax(tonorm)
norm_zscore(tonorm)
norm_toaverage(tonorm)
norm_logarithm(tonorm)
norm_vector(tonorm)
norm_linearagreg(tonorm)
```

Listing 5.2: Normalization of values in R using package MCDASupport version 0.30+

```
# only examples of computation provided
mcda_norm(tonorm, method = "minmax", minmax = "max")
mcda_norm(tonorm, method = "zscore")
mcda_norm(tonorm, method = "toaverage")
mcda_norm(tonorm, method = "logarithm", minmax = "max")
mcda_norm(tonorm, method = "vector", minmax = "max")
mcda_norm(tonorm, method = "linear aggregation", minmax = "max")
```

Note that in version 0.30 the minmax parameter of mcda_norm function is implicitly set to "max", so for our example, it could be omitted without changes in functionality.

Listing presumes, that the MCDASupport package is downloaded in same folder as the script the installation is called from and that it is also set as working folder in R environment. Also please note, that the name of package may be slightly different, for example in version number.

We are installing local package, that not so common. The package itself is fine, it has been created for purposes of this course. It is open source licensed, but we decided not to upload it on CRAN as to make it much more available. Reason for this decision is that packages available for whole community require regular support, which is commitment we didn't want to make. That decision has one undesirable side effect. CRAN packages are able to download and install other packages automatically to resolve missing dependencies. When installing local packages, these dependencies must be resolved manually.

We provide script to install such packages, but note, that newer versions might have different dependencies. If that is the case, R will complain, so you should be able to quickly identify, what is missing and install it.

Listing 5.3: Installing packages MCDASupport 0.29 package depends on

```
install.packages("mathjax")
install.packages("graphics")
install.packages("igpraph")
install.packages("diagram")
install.packages("stats")
install.packages("dplyr")
install.packages("tidyr")
install.packages("visnetwork")
install.packages("plotly")
```

Documentation of the available functions of the package is available in PDF format [28]. Package itself is available in accompanying course in LMS or on GitHub pak of the package [29][1].

5.2 Weight Coefficient Estimation Methods

There are many methods we can use to estimate weight coefficient. All of them are based on some kind of either preference mapping or using some underlying presumption on how these preferences look like.

Apart from pairwise comparison (i.e. Fuller's triangle) we can use also philosophically very different methods, allowing us to be much more precise and also describe more complex relations between the criteria. In that regard the weight are measure of these relations.

It may be feasible to use externally stated weight for certain problems. If information exist (in sufficient precision and quality) to solve our problem, we can estimated the weight *directly*. We perceive as direct also cases, when the weights are stated in problem formulation (directly) by decision maker.

Alternative methods use scoring to evaluate the criteria based on perceived importance. We can think about two basic scenarios for weight estimation. In first we score the criterium using predefined scale. The evaluation will be performed on every single criterium independently on evaluation of other criteria. In this case the sum of scores is not set in advance. We call this method as *direct scoring*.

Second variant presumes, that we have set amount of point we can allocate to the criteria to estimate weight coefficients. Logically sum of the scores across the criteria will be constant in this case. In this case we can talk about the score budget to be allocated to criteria - hence the name *budget allocation*. This approach gives the analyst high flexibility as (s-)he can allocate whichever part of the budget to the criteria as necessary and bonify or penalize some criteria. In extreme case the analyst can allocate the budget to single criterion, transforming the multiple criteria decision problem to mono-criteria one as all other criteria would be effectively excluded from decision making.

From assigned scores we can easily derive weight coefficient by using linear normalization (5.8). We divide assigned number of points by sum of all points (scores) to compute the weight.

There is also a method which doesn't use preferences of any kind for weight establishment. This method is called Entropy Weight Method (EWM). This method derives weights from variance in performance of the variants in the criteria. It basically evaluates how much are the values different in the criterium.

Presumption here is that the criteria with higher variability have better chance to explain differences between the variants and because of that are more important for decision making. As result the criteria with higher variability will get assigned higher weights.

This presumption may have some merit to it, if we as analysts believe, that all the criteria are significant for the problem solution and its variability is representative to change in utility connected to values of these criteria. Method itself has no ability to mathematically prove, that the criteria are indeed relevant to the problem, or that its variability has such strong connection to utility.

[1] https://github.com/psenovsky/MCDASupport

Analyst must find supporting evidence outside of EWM.

EWM takes performance PM as its input. The values of PM do not need to be normalized, but must be numeric. Normalization is incorporated into computation procedure, see eq. (5.11).

Please note, that eq. (5.11) only works for criteria we are maximizing. We omitted minimization variant, but transformation of the equation to minimization form is trivial.

$$EM_{ij} = \frac{PM_{ij}}{max(PM_{ij})_j} \tag{5.11}$$

Then we compute probability of criteria value to occur, see (5.12).

$$p_{ij} = \frac{EM_{ij}}{\sum_{i=1}^{n} EM_{ij}} \tag{5.12}$$

Then we compute entropy E of criteria j, see (5.13).

$$E_j = -\frac{1}{ln(n)} \cdot \sum_{i=1}^{n} p_{ij} \cdot ln(p_{ij}) \tag{5.13}$$

Degree of divergence div is computed in eq (5.14).

$$div_j = \|1 - E_j\|) \tag{5.14}$$

Finally we may compute entropy weights Ew_j using eq. (5.15).

$$Ew_j = \frac{div_j}{\sum_{i=1}^{n} div_j} \tag{5.15}$$

Where PM_{ij} is performance matrix (performance of alternatives in criteria), i ... alternatives, j ... criteria, n ... number of alternatives, EM ... normalized performance matrix (normalized per criteria), E ... entropy, p ... probability of criteria value to occur, div ... degree of divergence, Ew ... entropy weights.

You can also use other methods to methods to compute weights. For example AHP allows to derive weights from preference measurement. This method uses pair-wise, but non-binary, comparison and also has a measure to identify randomness in preference statements. AHP method and its properties is being discussed in separate chaúter.

5.3 Weighted Sum and Product Methods

We discussed to certain degree *weighted sum method* (WSM). We used it for utility and risk preference matrices (see tabs. 4.5 and 4.9). Formally we can formulate the computation in following way (5.16).

$$U = \sum_{i=1}^{n} w_i \cdot u_i \tag{5.16}$$

Where U is utility of the alternative, u_i is the performance of criterium i, n is number of criteria used and finally w_i is the weight attributed to criterium c.

In equation (5.16) the utilities are used, but we can adapt it easily to compute risk $R = \sum_{i=1}^{m} w_i \cdot r_i$. We transformed U and u to overall risk R and risk r, with m being number of risks being evaluated. Significant difference is in interpretation of the results as for utility we maximize the result, while for risk we minimize it.

The WSM method is very easy method both from point of view of usage and results interpretation. Unfortunately there are also some limitations of the method. For example we need to normalized inputs for the method, before we use it. That brings burden of responsibility of these scale transformations to analyst. As we noted in previous chapters choice of normalization method will impact the results.

Another significant limitation, perhaps a most important one, is a presumption that all criteria do have from point of view of the contribution to the optimal can be substituted for each other. That means that bad performance of the alternative in one criterium can be compensated by better performance of the alternative in another criterium. If the presumption does not hold for decision problem, we can't use WSM method as the results would be misleading.

In such case Weighted Product Method (WPM) may be better option. Computation is very similar to WSM (5.16) but instead of sum we compute product of the weighted utility (5.17).

$$U = \prod_{i=1}^{n} w_i \cdot u_i \tag{5.17}$$

The result in this case will be product of weighted performances of alternative across the criteria. Practically, if the performance of the alternative in criterium is unacceptable → equal to 0, than the result itself will also be 0 as anything multiplied by zero equals to zero. That property of the method can be useful for evaluation of utility, but this property makes it unsuitable for risk evaluation. Since we are minimizing risk and work with positive values only, minimal value will be always 0. So be careful when using the method that your results actually make sense.

It is also clear, that the result will no longer be interpretable as a percentage. So the interpretation will be much less intuitive.

Requirement on normalizing inputs of the method before usage are same as for WSM. And again this burden is entirely on analyst.

Main strength of the method is in that it gives analyst a tool to effectively transform performance measure in natural units to utility by specifying threshold over/under which the alternative performance will be evaluated as unacceptable regardless of alternative's performance in all other criteria. That may be useful especially when computing using computer.

In this case we can leave all the alternatives inside the processed matrices. The method will clearly identify "excluded" alternatives by making their utility $U = 0$. That might not seem like much, but without it, the decision to exclude alternative from decision analysis must be made separately and must be well described. Another benefit is, that it is much easier for analyst to experiment with cut-off levels and thus better understand properties of decision.

In listing below there is demonstrated usage of MCDASupport package in R to solve our use case using WSM. This method is available using mcda_wsm() function.

Listing 5.4: E-book reader purchase using WSM in R

```r
library("MCDASupport") #attach package
#define decision problem
#choosing best e-book reader - evaluating utility only
KoboGlo <- c(80, 97, 50, 100, 100, 94)
SonyPRST3 <- c(80, 88, 100, 20, 70, 70)
KindlePaperwhite2 <- c(100, 85, 50, 100, 70, 100)
PBTouchLux <- c(80, 90, 100, 100, 70, 94)
BookeenCybookOdyssey <- c(80, 100, 100, 100, 85, 50)
criteria <- c("Display", "weight", "HWButtons", "FrontLight", "batery", "price")
PM <- as.data.frame(
  rbind(KoboGlo,
  SonyPRST3,
  KindlePaperwhite2,
  PBTouchLux,
  BookeenCybookOdyssey))
colnames(PM) <- criteria
w <- c(5, 3, 4, 5, 2, 1)
t <- mcda_wsm(PM, w, 'max') #use WSM method
t$preferences #print results
t$scoreM #visualize score of alternatives
dom <- mcda_get_dominated(PM) #adjancancy matrix
plot.prefM(dom) #network graph of alternative domination
mcda_del_dominated(PM) #simplify the problem by removing dominated alternatives
```

```
24 t$resultTable
```

The listing is self describing, so only short remarks on the process. First we define alternatives as vectors of its performance in the criteria, using vector creation function c(). Every vector member represents performance in single criterium. Order of criteria is clear from vector defining it on row 9. Order of criteria must be same for all alternative vectors. Next we construct performance matrix using alternative vectors as rows using rbind() function. The matrix will have alternatives in rows and criteria performances as columns.

Alternatively we could construct the matrix per columns using cbind() function. That would require the vectors to be representative of criteria performance in alternatives instead of alternative performance in criteria.

In both cases the resulting matrix will lack row and cols names. We can remedy the problem by assigning them using colnames() and rownames() functions. Please note, that we did not set row names in the listing.

The utility computation is performed by mcda_wsm() function. Which takes performance matrix PM as input, together with vector of weights w and information of whether the criteria are ascending (maximizing) or descending (minimizing). If all criteria have same direction of scale, we can use 'max' (like in our case) or 'min'. If we work with mixed scales we need to provide vector with information on the critera direction. Logically such vector must have number of elements equal to number of criteria. That is same for vector of weights as all criteria need to have some weight.

Method also provides graphical outputs. Firs of these is a network diagram showing domination of alternatives over other, see fig. 5.1. Second one is a bar chart, which shows contribution of the criteria to overall score of the alternative see fig. 5.2.

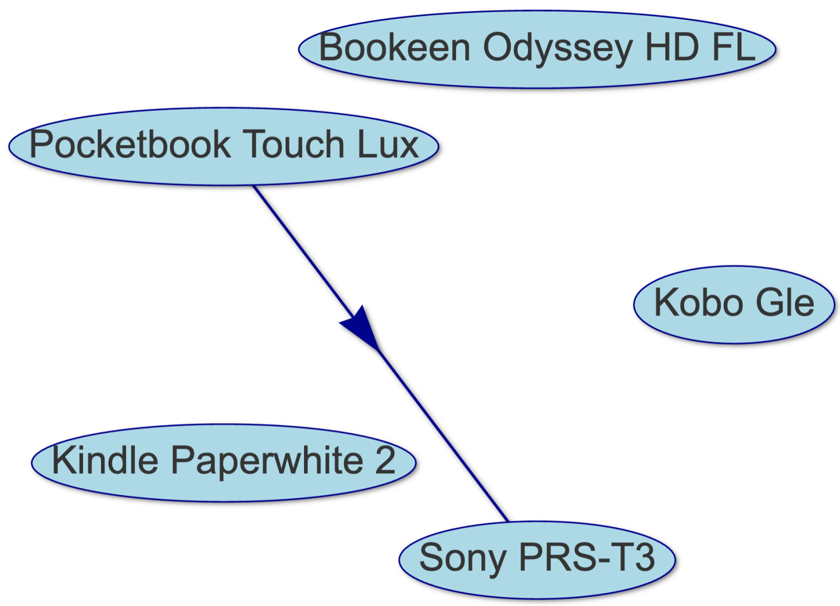


Figure 5.1: Network diagram for visualization of dominance between the alternatives

As you can see of fig. 5.1 the only clearly dominant relation exists between PocketBook Touch Lux and Sony PRS T3, where PocketBook dominates Sony PRST3. For remaining devices such dominance does not exist, which means that the ranking of these alternatives must be derived from overall utility score.

WSM uses concept of *simple dominance.* Such dominance means, that the alternative has better performance than other alternative in *all* criteria. In such case we can say, that such variant is dominates inferior variant. If our goal is to identify best alternative (as opposed to problems of ordering or scoring of the alternatives), we can clearly see, that the dominated variant cannot be best. We may even decide to exclude such alternatives from further evaluation.

This exclusion can also be performed in R using mcda_del_dominated() function from MCDA-Support package.

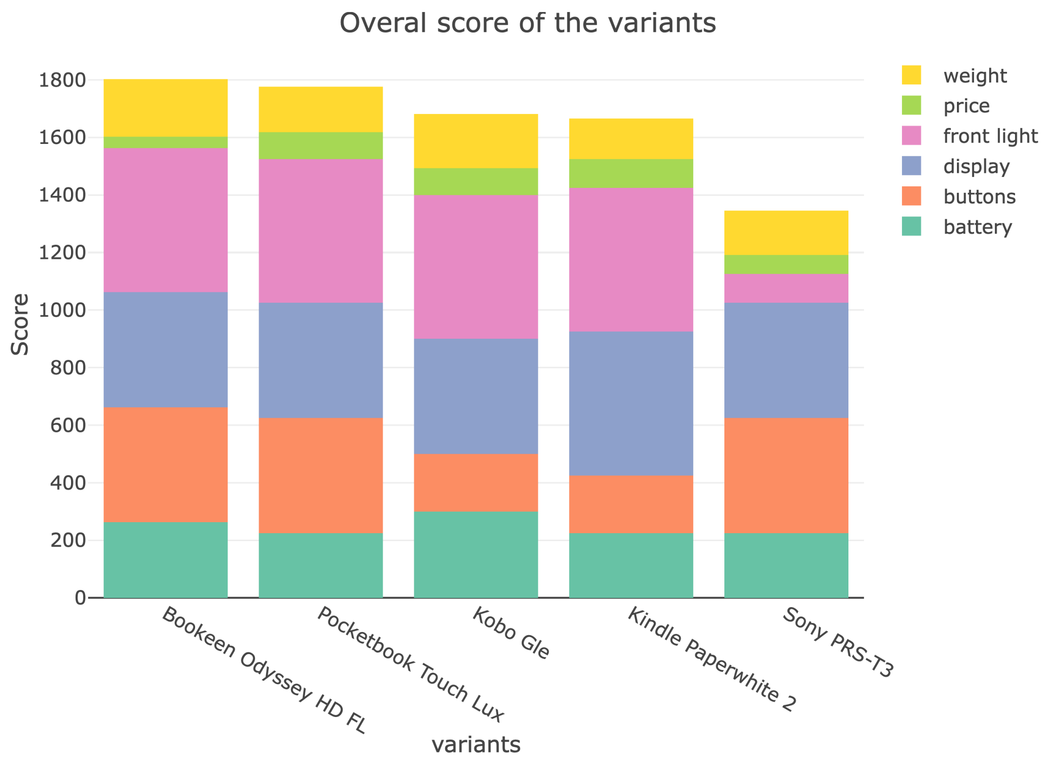


Figure 5.2: Contribution of criteria performance in alternatives to overall score of the alternative using WSM

We can also print results in tabular form, similarly as we used table of weighted preferences in previous chapter.

Listing of matrix of weighted preferences

```
                     Display weight HWButtons FrontLight batery price
KoboGlo              400     291    200       500        200    94
SonyPRST3            400     264    400       100        140    70
KindlePaperwhite2    500     255    200       500        140    100
PBTouchLux           400     270    400       500        140    94
BookeenCybookOdyssey 400     300    400       500        170    50
```

Our results in overall score:

```
KoboGlo SonyPRST3 KindlePaperwhite2 PBTouchLux BookeenCybookOdyssey
1685    1374      1695              1804       1820
```

Method also provides percentages for these results for better interpretability.

```
BookeenCybookOdyssey PBTouchLux KindlePaperwhite2 KoboGlo  SonyPRST3
100.00000            99.12088   93.13187          92.58242 75.49451
```

As you can see, even simple method can provide pretty and interesting results. From interpretation perspective, we always need to consider what is the core of the decision making and what kind of results are expected from us. By expectations we mean format of the decision analysis, not confirming some kind of expected result.

5.4 ELECTRE

WSM method was very simple, on the other hand there were a lot of limitations and requirement on input preparation for analyst connected to it. There are lot of other interesting methods, which might make our analyst lives more easier. There are actually whole families of methods with member methods focusing on different types of problems. For example we may want to use some measure for uncertainty of the criteria performance. If we talk about battery, and its duration say in hours, then WSM always takes a single value, which it considers being precise, i. e. 6 hours. But what if we are not entirely sure about that and estimate lifespan of battery to 6 hours ∓ 1 hour.

In real world situations we often can't come to precise estimations, so it would be beneficial if the method itself was at least somewhat aware of this uncertainty and was capable to work with it. As WSM, does not provide any support for it, we need to search for other methods. Out of all available methods we decided to present ELECTRE family of methods. These methods are relatively old, well understood and described in literature including use cases and limitations. Methods are also capable to avoid or limit lot of disadvantages we have to solve manually for WSM to work.

MCDASupport package supports currently following ELECTRE methods:

- ELECTRE I
- ELECTRE 1S
- ELECTRE II
- ELECTRE III
- ELECTRE IV
- ELECTRE TRI

Even as we listed six separate methods, it is not exhaustive list, just more popular methods in the family.

The methods differ significantly in its goals and parameters, whit they require for computation. For example ELECTRE I, 1S and II allow to identify co called *kernel solution*, but each of these approaches the solution differently. All methods have some common properties, like ability specify limits for alternatives performance in criteria, which need to be exceeded to consider performance of other alternative in this criterium as beet or worse. Methods differ in implementation of these thresholds. ELECTRE II and 1S define it using fuzzy logic, ELECTRE I defines it as single number.

ELECTRE II, III, IV and TRI then can be used to solve ranking problem (we search for order of the alternatives considering its performance in decision criteria). Even for this methods use different approaches. ELECTRE III allows us to specify limits (preference and veto) separately for every criterium. That is hugely beneficial as the performance can remain expressed in its natural units. Analyst then does not need to prepare input data so thoroughly. Also we know that the transformations used for normalization do influence the results.

ELECTRE IV then for computation does not require formal weight specification. ELECTRE TRI allows the criteria to have categorical values, so we deal with situation when continuous scale is divided into in advance defined number of intervals, with exactly specified limits.

Another benefit of these methods is that they have incorporated normalization into them, so we technically do not need perform data normalization before using the method. That being said, we may still want to perform data preprocessing to introduce our interpretation of the utility for the criteria.

But be aware, that such transformation is possible source of bias and may introduce errors to one computation (so no "free lunch", everything has its ups and downs).

Let's try at least shortly look at process of the computation for these methods.

5.4.1 ELECTRE I

Oldest and also simplest method in the family is *ELECTRE I* method. As already specified in previous section of text, it allows us to identify kernel of the solution as set of alternatives which can be the optimum.

That definition may seem little confusing to you as if we specify some alternatives, then we presume that such alternative could be the optimal one. So what exactly is the kernel. Think of it in context of dominance. For some alternatives it may be easy to computationally prove that they are being dominated by one or more other alternatives. Such alternatives are clearly suboptimal and will not be part of the kernel.

For alternatives in the kernel it doesn't mean that all of them are the optimum, just that we failed to prove, that these alternatives cannot be optimum and as such they cannot be removed from analysis. Method doesn't serve for ranking, because in ranking we are not trying to identify single optimal alternative, but order the alternatives based on their properties expressed in criteria.

Required inputs are very similar to those required by WSM method. In addition to that we also can specify *concordance threshold* (implicitly 1) and *discordance threshold* (implicitly 0).

These values are directly related to the need to compute so called *concordance* and *discordance matrixes*.

We start with creation of *concordance index* (5.18) as direct comparison of two alternatives a and b. Index's values are written into matrix $m : m$, where m is number of alternatives. To be clear comparing alternatives a, b is concordance index. All such comparisons written into the matrix will form *concordance matrix*.

$$C(a,b) = \frac{\sum_{\forall j : g_j(a) \geq g_j(b)} w_j}{\sum_{i=1}^{n} w_i} \tag{5.18}$$

Where n is number of criteria, j criteria, in which the alternative a is better then b, $g_j(a)$ is then performance of alternative a in criterium j.

If we look again on equation (5.18), we find out that the values of concordance index can only be in the interval of $C(a,b) \in < 0; 1 >$. The index is greater the greater the proportion of the weights of the criteria in which the alternative a performance was better then alternative b. Considering nature of comparison it can be proven, that values in concordance matrix must be always 1 as we will technically compute $C(a,a)$, so $\forall j : g_j(a) = g_j(a)$. In other words, in both the numerator and denominator of the formula will be the sum of all the weights, which means that $C(a,a) = 1$.

Discordance matrix is complementary matrix to concordance matrix. It uses different angle for alternative's comparison. We are interested in criteria, for which the alternative b performed better then alternative a. Computation at level of pairs of alternatives comparison is realize using *discordance index*, see equation (5.19).

$$D(a,b) = \begin{cases} 0 & \forall j : g_j(a) \geq g_j(b) \\ \max_j \frac{g_j(b) - g_j(a)}{\delta_j} & \text{for all other cases} \end{cases} \tag{5.19}$$

Where δ_j is maximum of difference in criterium j (max. - min. value in criterium: $\max_j - \min_j$).

Evaluating whether alternative a has generally better performance then alternative b needs to use both matrixes. We can say that matrix a is superior to b (aSb) if concordance index describing evaluated pair is higher then concordance threshold c: $C(a,b) \geq c$ and at same time its value of discordance index is lower then discordance threshold d: $D(a,b) \leq d$.

These conditions will help us construct *adjancancy matrix*. The fields representing alternative pairs comparisons will be equal to 1 for comparisons complying to these conditions and 0 otherwise. We can use the matrix to construct graph such as one on fig. 5.1. In this case for our example with decision on e-book readers, the result will be exactly same as for WSM method, so we do not provide it.

Simply said part of solution kernel are alternatives, which were not dominated, so the sum of values in the row of adjancancy matrix is equal to 0.

Using R, the script will be very similar to the one using WSM. If we use default thresholds $c = 1$ and $d = 0$, then all we need to do is change name of the used function to Electre_1() and run the script. Thou for convenience we provide full script.

Listing 5.5: Using ELECTRE I method to solve choosing e-book reader problem in R

```
library("MCDASupport") #attach package
#define decision problem
#describe e-book readers
PM <- as.data.frame(cbind(
  c(80, 80, 100, 80, 80),
  c(185, 200, 206, 198, 180),
  c(50, 100, 50, 100, 100),
  c(100, 20, 100, 100, 100),
```

```
  c(2100, 1700, 1700, 1700, 1900),
  c(3400, 3800, 3300, 3400, 4200)
))
colnames(PM) <- c('display', 'weight', 'buttons', 'front light', 'battery', 'price')
rownames(PM) <- c('Kobo Glo', 'Sony PRS-T3', 'Kindle Paperwhite 2',
  'Pocketbook Touch Lux', 'Bookeen Odyssey HD FL')
w <- c(5, 2, 4, 5, 3, 1)
w <- w/sum(w) # sum of w = 1
minmax <- c('max', 'min', 'max', 'max', 'max', 'min')
E_I <- Electre_1(PM, w, minmaxcriteria = minmax)
E_I$GraphResult # network diagram
E_I$Kernel # kernel of solution
E_I$Dominated # dominated alternatives
```

The results in tabular form:

"Concordance Matrix"

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	1.00	0.80	0.70	0.80	0.65
SonyPRST3	0.45	1.00	0.45	0.55	0.50
KindlePaperwhite2	0.75	0.65	1.00	0.65	0.55
PBTouchLux	0.75	1.00	0.70	1.00	0.75
BookeenCybookOdyssey	0.85	0.95	0.70	0.95	1.00

"Discordance Matrix"

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0.00	1.0	1	1.00	1.00
SonyPRST3	1.00	0.0	1	1.00	1.00
KindlePaperwhite2	1.00	1.0	0	1.00	1.00
PBTouchLux	1.00	0.0	1	0.00	0.67
BookeenCybookOdyssey	0.88	0.4	1	0.88	0.00

"Preference Matrix"

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0	0	0	0	0
SonyPRST3	0	0	0	0	0
KindlePaperwhite2	0	0	0	0	0
PBTouchLux	0	1	0	0	0
BookeenCybookOdyssey	0	0	0	0	0

"Kernel"

KoboGlo	SonyPRST3	KindlePaperwhite2	BookeenCybookOdyssey
1	2	3	5

ELECTRE I method produces same network diagram as WSM method with these input parameters (see fig. 5.1).

5.4.2 ELECTRE II

ELECTRE II method is extension of ELECTRE I to add capability to rank the alternatives. Both methods share computation of concordance, discordance matrices and kernel but ELECTRE II uses that information as a chance to establish ranking relation between the alternatives.

In comparison to ELECTRE I, method also defines differently concoradance and discordance limits. ELECTRE II uses fuzzy number for the purpose. Concordance threshold is defined by three parameters c^-, c_0 and c^+, where $c^- \leq c_0 \leq c^+$, for discordance threshold two parameters are used d^- and d^+, with $d^- \leq d^+$.

Using fuzzy parameters allows the method to distinguish between strong (S^F) and weak (S^f) dominance when comparing the alternatives. The difference between the two is intensity of our believe that alternative a performs better then b.

We say, that aS^Fb, if and only if:

$$C(a,b) \geq c^+ \tag{5.20}$$

$$g_j(a) - g_j(b) \leq d^- \text{ for } \forall j \tag{5.21}$$

$$\frac{P^+(a,b)}{P^-(a,b)} \geq 1 \tag{5.22}$$

$P^+(a,b)$ corresponds to sum of weights in criteria, where a performs better then b (aPb), analogically $P^-(a,b)$ correspond to sum of weights where b performs better then a (bPa).

These condition can be used for construction of matrices describing strong outranking (aS^Fb). If above conditions are met we set the comparison field to 1, otherwise 0.

To compute weak outranking (aS^fb) we use conditions (5.21, 5.22), but instead of condition (5.20) we use following condition:

$$C(a,b) \geq c^- \tag{5.23}$$

Since $c^- \leq c^+$ we ease up the rate we are able to say that a really outperforms b (aSb).

Formally in both cases we work with adjancancy matrices, which we can visualize and further work with.

In R using MCDASupport package you can visualize all abjacant matrices usingusing plot.prefM() function. This function takes only single parameter, the adjacancy matrix that you want to visualize. Fig. 5.1 has been generated using this function.

ELECTRE II uses integration procedure, allowing to use computed information for purposes of estimating alternatives order. Method computes separately two so called *pre-orders* and then derives final order from them.

- if aPb in both pre-orders, then aPb also in final order
- if $a = b$ in one pre-order, but aPb in second pre-order, then a precedes b in final order
- if aPb in one pre-order, but bPa in second, then the alternatives are not directly comparable

Again we can demonstrate method's usage easily in R using MCDASupport package for e-book reader example. And again if we use preset default values for the thresholds $c^- = 0,65$, $c_0 = 0,75$, $c^+ = 0,85$, $d^- = 0,25$, $d^+ = 0,5$ the computation only needs to change name of the used function to Electre_2().

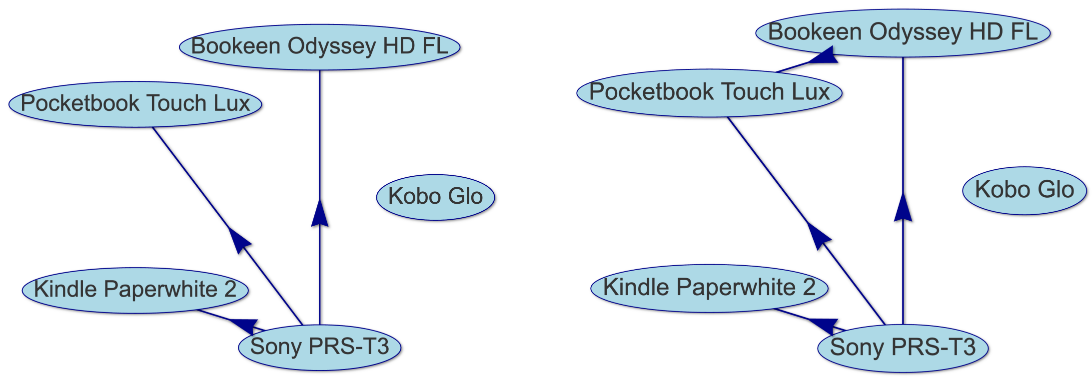
Listing 5.6: Using ELECTRE II to compute alternative orders of e-book reader recommentation in R

```
library("MCDASupport") #attach package
#define decision problem
KoboGlo <- c(80, 97, 50, 100, 100, 94)
SonyPRST3 <- c(80, 88, 100, 20, 70, 70)
KindlePaperwhite2 <- c(100, 85, 50, 100, 70, 100)
PBTouchLux <- c(80, 90, 100, 100, 70, 94)
BookeenCybookOdyssey <- c(80, 100, 100, 100, 85, 50)
criteria <- c("Display", "weight", "HWButtons", "FrontLight", "batery", "price")
PM <- as.data.frame(
  rbind(KoboGlo,
  SonyPRST3,
  KindlePaperwhite2,
  PBTouchLux,
  BookeenCybookOdyssey))
colnames(PM) <- criteria
w <- c(5, 3, 4, 5, 2, 1)
E_II <- Electre_2(PM, w, minmax)
E_II$finalPreorderSorted
E_II$graphResult
```

The listing uses default values for the concordance and discordance threshold, see left part of fig. 5.3a for resulting network. On right part of fig. ?? you can see influence of changing the thresholds. Otherwise the computation is same so we provide only changed lines of code for reference.

Listing 5.7: Using ELECTRE II to compute alternative orders of e-book reader recommentation in R, changing thresholds

```
E_II <- Electre_2(PM, w, minmax, c_minus = 0.8, c_zero = 0.88, c_plus = 0.95,
  d_minus = 0.1, d_plus = 0.25)
E_II$finalPreorderSorted
E_II$graphResult
```



(a) default thresholds (b) changed thresholds

Figure 5.3: Network diagram of outranking of alternatives using ELECTRE II

We provide results also in tabular form, without concordance and discordance matrices, as these are same as in ELECTRE I.

```
$StrongOutranking
                    KoboGlo SonyPRST3 KindlePaperwhite2 PBTouchLux BookeenCybookOdyssey
KoboGlo              0       0         0                 0          0
SonyPRST3            0       0         0                 0          0
KindlePaperwhite2    0       0         0                 0          0
PBTouchLux           0       1         0                 0          0
BookeenCybookOdyssey 1       1         0                 1          0

$WeakOutranking
                    KoboGlo SonyPRST3 KindlePaperwhite2 PBTouchLux BookeenCybookOdyssey
KoboGlo              0       0         0                 0          0
SonyPRST3            0       0         0                 0          0
KindlePaperwhite2    0       0         0                 0          0
PBTouchLux           0       1         0                 0          0
BookeenCybookOdyssey 1       1         0                 1          0

 $firstTotalPreorder #simplified and shortened
 [1] "KindlePaperwhite2"
 [2] "BookeenCybookOdyssey"
 [3] "KoboGlo"
 [4] "PBTouchLux"
 [5] "SonyPRST3"

 $secondTotalPreorder #simplified and shortened
 [1] "BookeenCybookOdyssey"
 [2] "PBTouchLux"
 [3] "KindlePaperwhite2"
```

```
 [4] "SonyPRST3"
 [5] "KoboGlo"

$finalPreorderMatrix
                     KoboGlo SonyPRST3 KindlePaperwhite2 PBTouchLux BookeenCybookOdyssey
KoboGlo                0       0         1                 0          1
SonyPRST3              0       0         1                 1          1
KindlePaperwhite2      0       0         0                 0          0
PBTouchLux             0       0         0                 0          1
BookeenCybookOdyssey   0       0         0                 0          0

$incomparableAlternatives
                     KoboGlo SonyPRST3 KindlePaperwhite2 PBTouchLux BookeenCybookOdyssey
KoboGlo                0       1         0                 1          0
SonyPRST3              1       0         0                 0          0
KindlePaperwhite2      0       0         0                 1          1
PBTouchLux             1       0         1                 0          0
BookeenCybookOdyssey   0       0         1                 0          0

 $finalPreorder
 KindlePaperwhite2 BookeenCybookOdyssey      PBTouchLux         KoboGlo        SonyPRST3
 1                    1                              2               3                4
```

5.4.3 ELECTRE III

ELECTRE III method approaches decision situation in different way. Similarly to ELECTRE II it uses fuzzy definition of the thresholds, but it uses three such thresholds, and also works differently with them:

- preference threshold - p - after going over this threshold we can say, that a is preferred to b: aPb
- indifference threshold - q - after going over this threshold, either of the alternatives cannot be preferred over one another, from comparison point of view we are indifferent to then: aIb
- veto threshold - v - after going over this threshold we **veto** thesis, that: aPb.

Thresholds p, q, v are set separately for each criterium. That allows us to leave criteria performance in its natural units, thus simplifying preparation phase of the analysis.

For the record in Electre I and II we also can leave the performance in criteria natural units, but for different reason. The methods first compute concordance and discordance matrices which basically normalizes the performance. After that, the methods compare these values to the thresholds.

Electre III works directly with the performances, that's why it needs separate thresholds for each and every criterium.

Apart of that it uses parameters α and β to compute so called *cut-off criteria*. Method uses these to estimate alternative's order.

As the thresholds are set separately for each criterium, we can't compute concordance matrix in single step. First we need to compute one concordance matrix for each criterium and then integrate matrices into overall concordance matrix.

$$c_j(a,b) = \begin{cases} 1 & \text{iff } g_j(a) + q_j \geq g_j(b) \\ 0 & \text{iff } g_j(a) + p_j < g_j(b) \\ \frac{g_j(a) - g_j(b) + p_j}{p_j - q_j} & \text{for all other cases} \end{cases} \tag{5.24}$$

$$C(a,b) = \frac{\sum_{j=1}^{n} w_j c_j(a,b)}{\sum_{j=1}^{n} w_j} \tag{5.25}$$

Discordance matrix is also computed separately for every criterium. Unlike concordance matrices, we do not perform integration step.

$$D_j(a,b) = \begin{cases} 1 & \text{iff } g_j(b) < g_j(a) + q_j \\ 0 & \text{iff } g_j(b) \leq g_j(a) + p_j \\ \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & \text{for all other cases} \end{cases} \quad (5.26)$$

Information gathered in concordance matrix and discordance matrices are then used to compute *credibility index*, from which we can construct *credibility matrix*.

$$S(a,b) = \begin{cases} C(a,b) & \text{iff } D_j(a,b) \leq C(a,b) \text{ for } \forall j \\ C(a,b) \prod_{j \in J(a,b)}^{n} \frac{1 - D_j(a,b)}{1 - C(a,b)} & \text{else} \end{cases} \quad (5.27)$$

Where $J(a,b)$ is criteria set for which holds condition $D_j(a,b) > C(a,b)$.

Credibility index is then used as base for ordering of alternatives.

Order is computed using two distillation procedures, which is then combined into final order. Ranking is derived from cut-off criterium $s(\lambda_k)$. Its initial value when distilling downward is equal to $s(\lambda_1) = maxS$. In following iterations then $s(\lambda_{k+1}) = \alpha \cdot \lambda_k + \beta$. Based on value of cut-off criterium alternatives are selected which are conforming to the criterium. Ordering is constructed iteratively.

Distillation upward work analogically, provided we use min instead of max function.

Usage of method is demonstrated in next listing. This time we compare five projects using five criteria.

Listing 5.8: Ranking of projects using ELECTRE III in R

```
# the performance table
PM <- cbind(
  c(-14,129,-10,44,-14),
  c(90,100,50,90,100),
  c(0,0,0,0,0),
  c(40,0,10,5,20),
  c(100,0,100,20,40))
rownames(PM) <- c("Project1","Project2","Project3","Project4","Project5")
colnames(PM) <- c( "CR1","CR2","CR3","CR4","CR5")
minmaxcriteria <- 'max'
Q <- c(25,16,0,12,10) #Indifference thresholds
P <- c(50,24,1,24,20) #Preference thresholds
V <- c(100,60,2,48,90) #Veto thresholds
w <- c(1,1,1,1,1) #weights
t <- Electre_3(PM, w, P, Q, V, minmaxcriteria)
t$final_ranking
t$graph #use t$ to access other interestig properties of the solution
```

```
[1] "Concordance matrix"
              Project1  Project2 Project3 Project4  Project5
Project1      1.0 0.6000000       0.6      0.60 0.6666667
Project2      0.8 1.0000000       0.6      0.80 0.8000000
Project3      1.0 0.8000000       1.0      0.80 0.8000000
Project4      0.8 0.8000000       0.6      1.00 0.8000000
Project5      1.0 0.6666667       0.8      0.75 1.0000000
[1] "Discordance matrixes of criteria"
 [[1]]
              Project1 Project2 Project3 Project4 Project5
 Project1        0       1.0        0      0.16        0
 Project2        0       0.0        0      0.00        0
 Project3        0       1.0        0      0.08        0
 Project4        0       0.7        0      0.00        0
 Project5        0       1.0        0      0.16        0

 [[2]]
         Project1  Project2 Project3  Project4  Project5
```

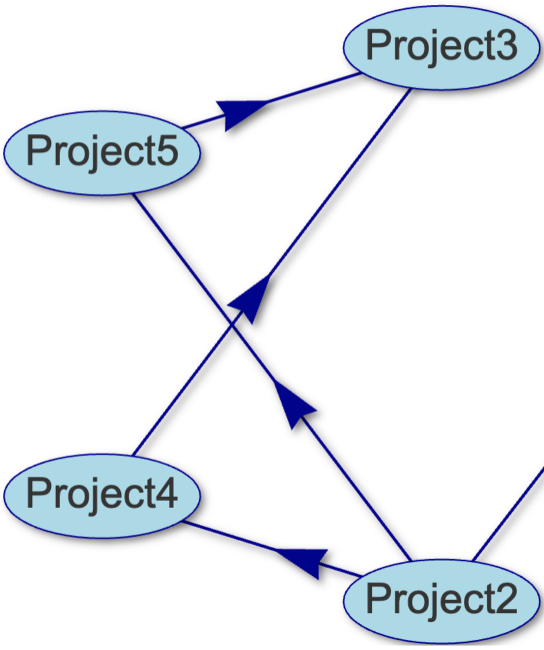


Figure 5.4: Network diagram of alternatives outranking using ELECTRE III method (project ranking example)

```
Project1 0.0000000 0.0000000        0 0.0000000 0.0000000
Project2 0.0000000 0.0000000        0 0.0000000 0.0000000
Project3 0.4444444 0.7222222        0 0.4444444 0.7222222
Project4 0.0000000 0.0000000        0 0.0000000 0.0000000
Project5 0.0000000 0.0000000        0 0.0000000 0.0000000

[[3]], [[4]] # shortened (all 0)

[[5]]
        Project1 Project2  Project3 Project4  Project5
Project1 0.0000000        0 0.0000000        0 0.0000000
Project2 1.0000000        0 1.0000000        0 0.2857143
Project3 0.0000000        0 0.0000000        0 0.0000000
Project4 0.8571429        0 0.8571429        0 0.0000000
Project5 0.5714286        0 0.5714286        0 0.0000000

[1] "Credibility matrix"
        Project1 Project2  Project3 Project4  Project5
Project1 1.0000000      0.0 1.0000000      0.8 1.0000000
Project2 0.0000000      1.0 0.0000000      0.8 0.6666667
Project3 0.6000000      0.0 1.0000000      0.6 0.8000000
Project4 0.2142857      0.8 0.5714286      1.0 0.7500000
Project5 0.6666667      0.0 0.8000000      0.8 1.0000000

[1] "First pre-order" #shortened to single table
 action rank     rank
(down)    (up)
1 Project1    1     1
2 Project5    2     3
```

```
3 Project2    3     1
4 Project3    3     3
5 Project4    3     2

[1] "total"
         Project1 Project2 Project3 Project4 Project5
Project1 "-"      "P+"     "P+"     "P+"     "P+"
Project2 "P-"     "-"      "I"      "I"      "P-"
Project3 "P-"     "I"      "-"      "I"      "P-"
Project4 "P-"     "I"      "I"      "-"      "P-"
Project5 "P-"     "P+"     "P+"     "P+"     "-"

[1] "Adjancancy matrix"
               Project1 Project5 Project2 Project3 Project4
Project1        0        1        0        0        0
Project5        0        0        1        1        1
Project2        0        0        0        0        0
Project3        0        0        0        0        0
Project4        0        0        0        0        0

[1] "Final ranking"
    alternative sum_outrank ranking
1    Project1            4       1
5    Project5            3       2
2    Project2            0       3
3    Project3            0       3
4    Project4            0       3
```

5.4.4 ELECTRE IV a TRI

For sake of completeness we also provide information on methods ELECTRE IV and TRI, in shortened form and without example. The MCDASupport package has tested examples available, so you can easily experiment on your own.

ELECTRE IV doesn't need weights for recommendations - that is probably it largest benefit. Method uses more complex system of preference mapping:

- $m_p(b,a)$ - number of criteria, for which b is strongly preferred to a
- $m_q(b,a)$ - number of criteria, for which b is weakly preferred to a
- $m_j(b,a)$ - number of criteria, for which there is no difference between a and b - indiference
- $m_o(b,a) = m_o(a,b)$ - number of criteria, for which performance of alternatives is same

Above mentioned variables are quasi-criteria, which are then used to construct credibility matrix, using following approach:

- quasi-dominance: $S(a,b) = 1$, if $m_p(a,b) + m_q(a,b) = 0$ and $m_i(a,b) < m_i(b,a) + m_q(b,a) + m_p(b,a)$
- Canonical dominance: $S(a,b) = 0,8$, if $m_p(a,b) = 0$ and $m_q(a,b) \leq m_q(b,a)$ and $m_q(a,b) + m_i(a,b) \leq m_i(b,a) + m_q(b,a) + m_p(b,a) + 1$
- Pseudo dominance: $S(a,b) = 0,6$, if $m_p(a,b) = 0$ and $m_q(a,b) \leq m_q(b,a) + m_p(b,a)$
- Sub dominance: $S(a,b) = 0,4$, if $m_p(a,b) = 0$
- Veto dominance: $S(a,b) = 0,2$, if $mp(a,b) = 1$ or if $(m_p(b,a) \geq \frac{m}{2}$ and $g_j(b) + v_j \geq g_j(a))$
- else $S(a,b) = 0$

We establish alternatives order from credibility matrix by using same procedure as in ELECTRE III (distillation downward, upward and final order derivation).

Last method in ELECTRE family, we are going to discuss in this chapter is ELECTRE TRI method. Method is specific by using categorical variables. By categorical variable we mean situation in which criterium us specified on continuous numeric scale, which si divided to intervals. We assign label to each interval. Uncertainty in this case is in question of how much sure are we, that performance of the criterium truly belongs to given category and not for example lower or higher category?

Concordance index is computed separately for every criterium.

$$c_j(a, b_h) = \begin{cases} 0 & \text{if } g_j(b_h) - g_j(a) \geq >= p_j \\ 1 & \text{if } g_j(b_h) - g_j(a) <= q_j \\ \frac{p_j + g_j(a) - g_j(b_h)}{p_j - q_j} & \text{else} \end{cases} \tag{5.28}$$

Where $g_j(bh)$ is the profile assigned to category of criterium v variant b

Then we compute c_j and also in inverse form: $c_j(b_h, a)$. We use equation (5.28) for this purpose, inly with switched a and b_h parameters.

Discordance index is also computed separately for each criterium, but the computation is simpler:

$$d_j(a, b_h) = \min(1, \max(0, \frac{g_j(b_h) - g_j(a) - p_j}{v_j - p_j})) \tag{5.29}$$

In this case we also compute inverse discordance index $d_j(bg, a)$.

Credibility index:

$$s(a, bh) = \begin{cases} c_j(a, b_h) \prod_{j \in V} \frac{1 - d_j(a, b_h)}{1 - c_j(a, b_h)} & \text{if } V \neq 0 \\ c_j(a, bh) & \text{else} \end{cases} \tag{5.30}$$

Where $V = j \in J : d_j(a, b_h) > c_j(a, b_h)$.

For dealing with fuzzy logic we use credibility index. We search for lowest value of credibility index, for which we can say, that a has better performance then b_h (aSb_h).

$$s(a, b_h) \geq \lambda => aSb_h \tag{5.31}$$

$$s(a, b_h) < \lambda => \neg aSb_h \tag{5.32}$$

$$s(b_h, a) \geq \lambda => b_h Sa \tag{5.33}$$

$$s(b_h, a) < \lambda => \neg b_h Sa \tag{5.34}$$

Comparing aSb_h and b_hSa we can distinguish relations between a and b_h:

- $aIb_h \iff aSb_h \wedge b_hSa$
- $a > b_h \iff aSb_h \wedge \neg b_hSa$
- $b_h > a \iff \neg aSb_h \wedge b_hSa$
- $aRb_h \iff \neg aSb_h \wedge \neg b_hSa$

Final order (ranking) is then derived using optimistic and pessimistic procedure.

When using *optimistic* procedure, the alternative a is compared with categories $b_k, b_{k+1}, ...$, one after another until $b_h > a$. In such case the a is assigned to worst category C_h, pro which condition $b_h > a$ holds.

When using *pessimistic* procedure the alternative a is compared with $b_k, b_{k+1}, ...$, one after another until aSb_{k-1}. In such case the a is assigned to best category C_h, for which aSb_{h-1} holds..

In both cases the alternatives are ordered by categories, they were assigned to, from which we derive rank. Since we have two methods for ranking, we get two orders. Unlike previous ranking ELECTRE methods ELECTRE TRI does not try to consolidate result to single order.

Summary

In this chapter we discussed in detail some aspects of advanced decision making. In section WSM and WPM we discussed problem of performance measurement additivity of the alternatives. We also discussed problem of compensation mechanism deeply integrated into WSM method. By compensation we mead using good performance in one criterium to compensate bad performance in other.

ELECTRE methods allow to certain degree compensate uncertainties is value estimation in performance matrices. Using fuzzy parameters we can lower imprecision in the computation process.

The methods are intended to solve variety of problem. Form point of vie of decision making we distinguish between two basic goals:

1. identifying solution kernel
2. ranking

Questions

1. What is normalization and for what purpose do we use it in decision making
2. Describe budget allocation method for weight coefficient estimation.
3. What does substitution effect mean (i.e. for WSM method)? When does such substitution present problem?
4. What is solution kernel? How different are alternatives in it compared to those which didn't make cut?
5. What is ranking?
6. What is difference between concordance and discordance index?
7. What do we mean be credibility (in ELECTRE methods)?
8. How are used the thresholds (preferential, indifference and veto) in ELECTRE methods?

Example

In this chapter we discussed lot of methods. We also presented variety of listings demonstrating them. Install R and RStudio. In it install MCDASupport package in latest version. You can download it from GitHub `https://github.com/psenovsky/MCDASupport` of from LMS. Installation is described in chapter 5.1. Note that newer versions of the package (if there are newer versions) may require installation of different dependencies. So examine closely error message the R produces. You will need to install missing dependencies.

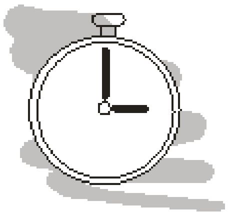
Chapter 6

Analytic Hierarchy Process (AHP)

Study guide
In this chapter we will finish problematic of multiple criteria decision making. From portfolio of problems we have been facing, only one remains, and that would be dealing with hierarchies of criteria. AHP provides us a tool to deal with this problem in effective way.

After reading this chapter you will know

- how to use AHP method for
  - weight coefficient estimation
  - check weights for randomness
  - score alternatives to find best alternative or rank all of them



Time required for study
To go through the chapter you will need approximately two hours, but we recommend to invest more time to practically try the method and experiment with the examples using for example MS Excel template available in LMS or R.

6.1 Introduction to AHP

First let's consider a question: what the problem is and why should we even consider existence of hierarchies? When talking about WSM method we formulated the requirement on criteria used for decision making to be independent. Almost immediately we found out, that it is not easy task to fulfill it. So we presumed to be sufficient for criteria to be only weakly dependent.

Problem that if such dependencies exist, regardless of whether weak or strong, then when using WSM, the results will be impacted by it, possibly providing misleading results. Please note that by assuming weak dependence we basically believe that this impact will be so low, that it will not alter the result significantly. Thus making it possible for us to use the method. Please note that distinguishing between weak and strong dependencies is not necessary easy task.

This problem is actually typical for basically all of multiple criteria decision method including ELECTRE family of methods, even thou we didn't explicitly specify this requirement.

Under these conditions, it might be beneficial to use a method, which is actually capable of working with hierarchies and somewhat alleviate this problem. Most popular method for multiple criteria decision support using criteria hierarchies is AHP method.

AHP was developed in 80-tees of last century by Thomas L. Saaty [18] to solve this kind of problem. Method formulates decision problem as a hierarchy of goals and criterias. Criteria serve as indicators for evaluating ability of the alternatives to achieve this goal.

Hierarchical organization gives us certain control on influence of clusters of criteria, which would otherwise present significant problem for WSM and similar methods[1].

AHP also provides tools to evaluate basic quality of stated preferences that are being used for weights estimation. AHP is capable to identify possible randomness in these preferences using *consistency index*, but unfortunately the quality control does not answer question of precision of the weights. So the answer AHP provides is that weights are not random, but not that the weights are correct. AHP thus does not solve problems with precision of the expert opinions, or more correctly notorious lack of it.

Because AHP method is very popular, there are hundreds or even thousands of studies for various problems, utilizing this method. Usage and limitations of the method is well described and documented, which gives us a solid base for using it. Also there are software suites and libraries for various computational environments (such as R), which makes the method suitable to be solved using computers.

One example of such software is SuperDecisions software [30] developed directly by Saaty and a team around him.

In literature the method is also sometimes being refereed to somewhat incorrectly as Saaty's method. But it is necessary to state, that it is misleading as Saaty developed multiple methods all of which are being widely used, among others ANP. So technically name Saaty's method may refer to any of these. We recommend the practitioners of the method to be specific.

ANP in comparison with AHP does not see the decision problem as criteria hierarchy, but a network of it. ANP is thus more general method capable to cover broader portfolio of problems. It could be also shown, that the AHP is actually special case of ANP. Since the space in this textbook and time for lectures are limited, we decided to focus on AHP.

Some software supports only one of these methods, but for example SuperDecisions supports both methods.

6.2 AHP Process

Process of AHP has three basic steps:

1. model decision problem as a hierarchy
2. pairwise comparison of the criteria at the various parts of hierarchy to estimate their contribution to problem solution
3. weight estimation

We can use the results either to estimate weights only or to rank the alternatives.

Let's examine simple example of hierarchy, see fig. 6.1. We decide on best Chief Executive Officer (CEO) candidate.

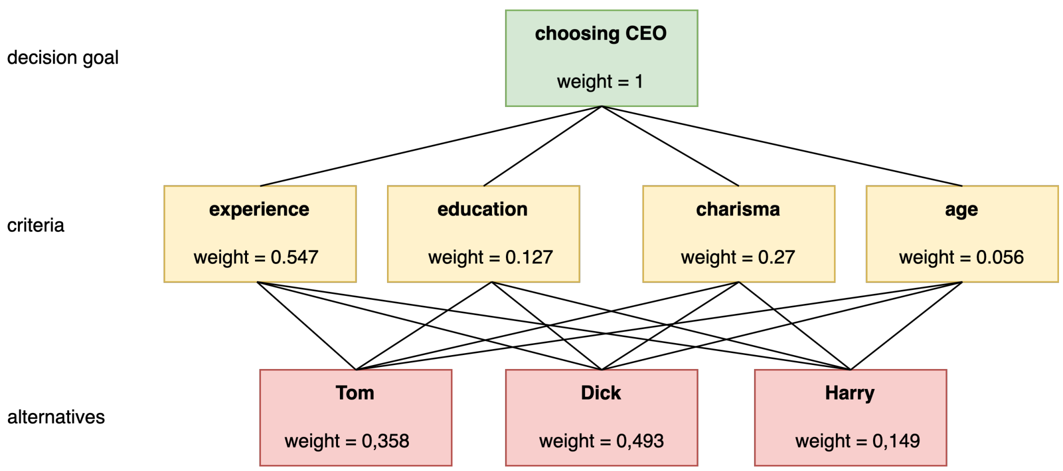


Figure 6.1: Simple hierarchy of best CEO candidate selection (adapted from [3])

[1] other methods which do not use hierarchies

Hierarchy in this case is only two levels. In first (root) level we specify decision goal, manager choice in our case. Into this goal, all criteria are consolidated. To that corresponds overall weight 1 (100 %). This weight is being distributed in level 2 to the criteria on this level. Assigned weights must comfort to condition, that $\sum w$ of the criteria is equal to weight of criterium from which these are derived.

Hierarchy on fig. 6.1 is very simple. In fact it is so simple, that its usage brings almost no benefits in comparison to WSM (and similar methods). But from point of view of hierarchy's complexity, we are not limited. The hierarchy can have three, four or even more levels of hierarchy, as needed.

That is a situation, when use of AHP will bring significant benefits.

If our goal was only to compute weight coefficients, we could limit the hierarchy only to goals and criteria and omit the alternatives and its performance in them. We need to consider the alternatives only in case we use AHP for scoring them - finding the best or rank them. Weight coefficients then correspond to overall performance of alternative across all "leaf" criteria in hierarchy.

In this case we also perform pairwise comparison. We compare alternatives performance in the criteria. Each pair is consequently being compared in all criteria, to estimate the result.

This approach is beneficial in that the performance of alternatives in criteria can stay in its natural units, even in cases, when the criterium is specified in categorical form using for example ordinal scale or even text description. Compare that to ELECTRE methods family, where we had to choose different method based on how our inputs looked like.

In case that the alternative performance in criteria is normalized and the used scale correspond to our understanding of utility we do not need to perform pairwise comparison of the alternatives. Formally we can simplify such situation into single level formed by leaf nodes of the hierarchy and use WSM to compute overall score of the alternatives.

To finish the computation of information from fig. 6.1, from nodes values it is apparent that best candidate to CEO position is Dick, followed by Tom and Harry on last place.

To estimate weight coefficients we need to perform pairwise comparison on every level and every group. For our easy hierarchy on fig. 6.1 we will need single matrix to compare the criteria and four matrices to compare alternatives performance in these criteria.

Compare that with solution of hierarchy presented later in chapter. For its solution we will need 3 matrices to compare the criteria and 8 matrices to compare alternative performance in them, see fig. 6.2.

In each matrix we compare the criteria using following scale:

- 1 - equally important
- 3 - moderately more important
- 5 - strongly more important
- 7 - very strongly (demonstrably) more important
- 9 - extremely strongly more important
- even numbers can be used to improve granularity of comparison
- decimals can be used to further improve comparison resolution

If we compare criteria a and b, with a being strongly more important then b, then $aSb = 5$. From this we can derive value of inverse comparison $bSa = 1/5$. This approach allows us to limit required preferences, which need to be collected from problem domain experts to realize the comparison.

So the preference values will conform to following conditions. If a is preferred to b, then the preference value will be $\in < 1; 9 >$. In case b is preferred to a, the the preference will be in interval $< 0.11; 1 >$, or 1/9 (0.11) to 1/1.

Furthermore, considering that comparison matrix is always square matrix: n:n, where n is number of criteria we compare. We can derive, that the diagonal must be equal to 1 as we compare the criterion with itself.

We perform pairwise comparison, but of other type then in case of Fuller's triangle, which was *binary*. Since our preference resolution is much finer then just saying that a is more important then b or vice versa, we are capable to also gain some insight in intensity of this preference.

Preference matrix P (6.1) is basis for estimation of weight coefficients.

$$P = \begin{bmatrix} 1 & p_1 & \dots & p_{1n} \\ \frac{1}{p_1} & 1 & \dots & p_{\dots n} \\ \dots & \dots & 1 & \dots \\ \frac{1}{p_{1n}} & \dots & \dots & 1 \end{bmatrix} \tag{6.1}$$

We presume, that the preferences correspond to real ratios between criteria's weights. That is strong requirement, which can be problematic, if we fail to meet it, than the results provided by the method could be misleading. This is a property of the method, we can perceive as a weakness and or limitation. Method also does not provide any mean to evaluate the quality of mapped preferences (measure of how much it corresponds to reality). On the other hand method has a measure which helps us identify situation, when the preferences has been estimated randomly.

Problem of weights coefficient estimation can be seen as optimization problem, see (6.2).

$$F = \sum_{i=1}^{k} \sum_{j=1}^{k} \left(p_{ij} - \frac{w_i}{w_j} \right)^2 \rightarrow \min \tag{6.2}$$

Where k is number of evaluated criteria.

The solution is possible using *quadratic programming* method, but the solution is usually considered as relatively computationally intensive and very complicated for computing in hand. Because of that the computation is often approximated using geometric mean (6.3). This approximation introduces some level of error into the result and is considered as undesirable for high value decisions.

$$w_i = \frac{\left(\prod_{j=1}^{k} p_{ij} \right)^{\frac{1}{k}}}{\sum_{i=1}^{k} \left(\prod_{j=1}^{k} p_{ij} \right)^{\frac{1}{k}}} \tag{6.3}$$

We can evaluate weight consistency using *consistency index* CI, computed by (6.4).

$$CI = \frac{w_{max} - k}{k - 1} \tag{6.4}$$

We can compare computed CI against *random consistency index* RCI to compute *consistence ratio* CR, see (6.5).

$$CR = \frac{CI}{RCI_k} \tag{6.5}$$

So what exactly is RCI? It is average CI for preference matrices generated randomly. Computation itself is easy if you use computational environments like R, MathLab, SciLab or any other. Start with eye matrix $k : k$, then randomly generate number in interval $\in < 0; 9 >$ as randomly generated preference value. Note that only half of matrix's fields need to be estimated in this way, the other half presents inverse preference and thus must be equal to 1/random preference. Based on this data we compute the weights.

Then we compute CI and add it to the dataset of CIs. We repeat the process, until you get robust enough dataset of CIs, for which you compute mean, which we will call Random Consistency Index (RCI).

Original Saaty's RCI values were computed on few thousand elements large datasets. In literature there are RCI computed on dataset order of magnitude larger.

Conventional wisdom says, that larger the k is the larger CIs dataset is needed to estimate robust mean. But there is also law of marginal utility in the play, so increasing number of CIs in dataset over certain threshold does not significantly improve precision. For $k < 11$ adding CIs over 500 000 will not induce change on first 5 decimal points.

Original Saaty's values for RCI are provided in tab. 6.1 and are considered good enough for most use cases.

As statistically significant value of $CR = 0.1$ is considered. If $|CR| < 0.1$ we presume, that mapped preferences we used for weights estimation are not random.

Table 6.1: AHP - random consistency index RCI (courtesy of [18])

k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
RCI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Please note that value $CR = 0.1$ should be considered more as rule of thumb then strong statistical test. When estimating weights we should try to achieve lowest possible value. For complex problems with large k-number of criteria, it is very hard to get lower then 0.1 using preferences stated by experts as expert opinions on their own are large source of random errors introduced into computation. There is also a name for this problem - *cognitive bias*. All human made decisions/opinions are influenced by it. If we understand the nature of bias, we can somewhat minimize its influence, but it is always present. We will discuss it in more detail in next chapter.

Another source of errors is created if we use group of experts as preference source instead of single person. There are many strategies we can use to integrate varying preferences. Easiest approach is to use the average (mean) value. But if there is significant variance in these opinions large CI will be probably observed for the weights.

6.3 Case study - Choosing Best Car for Johnson's Family

This case study has been published originally on Wikipedia [4], we will use slightly modified version for demonstration purposes.

Goal of the decision is to find best car for Johnson's family. The family after thinking really hard came with criteria hierarchy on fig. 6.2 with criteria relevant for utility considerations. Family also identified 6 cars, it is choosing from.

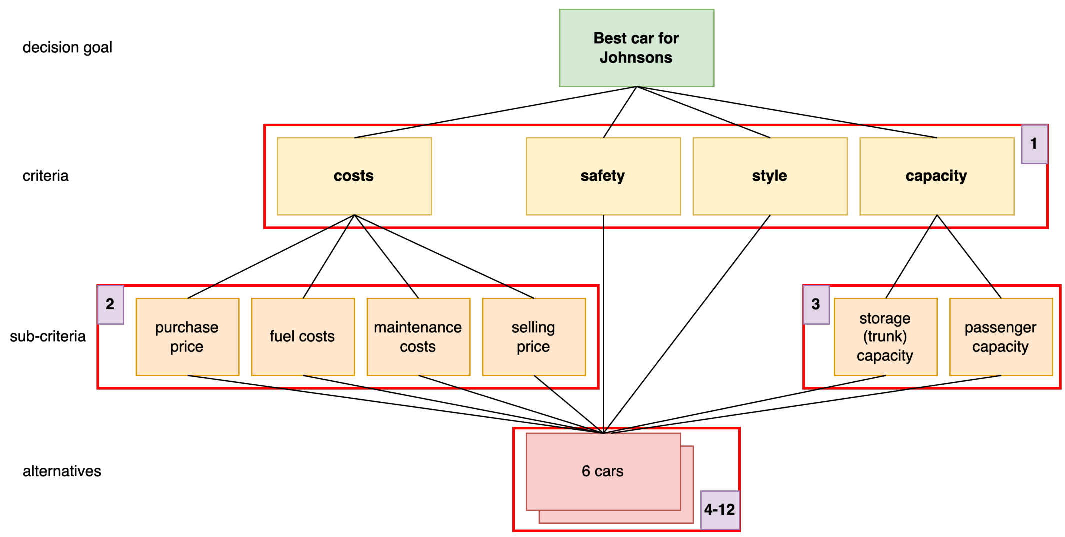


Figure 6.2: Criteria hierarchy for choosing Johnson's new car, comparison groups are visualized (courtesy of [4])

Johnson's choose from following cars:

1. Accord (sedan)
2. Accord (hybrid)
3. Pilot SUV
4. CR-V SUV
5. Element SUV
6. Odyssey minivan

First we need to perform pair-wise comparison for criteria hierarchy to establish weight system for leaf criteria. To do that weights of upper levels need to be established first.

Our hierarchy on fig. 6.2 consist of criteria level, which forms first group (and matrix) of the criteria and then we have 2 groups of sub-criteria (2 and 3). All of these will form separate comparison matrix:

1. criteria level of hierarchy (tab. 6.2)
2. costs sub-criteria (tab. 6.3)
3. capacity sub-criteria (tab. 6.4)

Table 6.2: Choosing car - criteria comparison (courtesy of [4])

	costs	**safety**	**style**	**capacity**
costs	1	3	7	3
safety	1/3	1	9	1
style	1/7	1/9	1	1/7
capacity	1/3	1	7	1

Table 6.3: Choosing car - comparing costs sub-criteria (courtesy of [4])

	purchase price	**fuel costs**	**maintenance costs**	**selling price**
purchase price	1	3	3	5
fuel costs	1/2	1	2	2
maintenance costs	1/5	1/2	1	1/2
selling price	1/3	1/2	2	1

Table 6.4: Choosing car - comparing capacity sub-criteria (courtesy of [4])

	storage capacity	**passenger capacity**
storage capacity	1	1/5
passenger capacity	5	1

Whole decision hierarchy has 8 leaf nodes. Which means we will need to create additional 8 comparison matrices, where we compare performance of the alternatives in these criteria.

We will fully explore the first of these - purchase price, for rest we only specify matrix with comparison. If you are interested in more details and explanations (rationale behind there numbers) consult original Wikipedia article [4][2].

When family considers *purchase price*, it actually uses variety of other supportive information connected to this price. First the family can invest only 25 000 USD into purchase and its extremely important that the family remains in budget for the purchase. As supportive measure price difference and price ratio of compared cars can be used.

Basic evaluation is available in tab. 6.5, weight coefficient estimation is in tab. 6.6 and finally the matrix comparing them is in tab. 6.7.

For other leaf criteria the comparison is available in tab. 6.8 - 6.14.

We will demonstrate computation in R environment using AHP package [31]. This package is not on CRAN anymore, but can still be downloaded from the CRAN archives and installed manually in same way as we installed MCDASupport package in previous chapter.

You can download the package from `https://cran.r-project.org/src/contrib/Archive/ahp/`.

Also the package is not being actively developed anymore, but what is in there is still fully sufficient for our scholarly experiments. Also the package supports only "full" regime of usage, so it requires also alternatives for evaluation. Simplified computation of weights only for the hierarchy is not supported in this package.

The usage could look like this:

[2]Johnson's new car on Wikipedia: `https://en.wikipedia.org/wiki/Analytic_hierarchy_process_%E2%80%93_car_example`

Table 6.5: Choosing car - compare price of cars, preferred variant bold (courtesy of [4])

	comparison		purchase price		better	price better		under/(over) budget	
#	A	B	A	B	price	difference	ratio	A	B
1	Accord s.	Accord h.	20 360	31 090	A	10 730	1,53	4 640	**(6 090)**
2	Accord s.	Pilot	20 360	27 595	A	7 235	1,36	4 640	**(2 595)**
3	Accord s.	CR-V	20 360	20 700	A	340	1,02	4 640	4 300
4	Accord s.	Element	20 360	18 980	B	1 380	1,07	4 640	6 020
5	Accord s.	Odyssey	20 360	25 645	A	5 285	1,26	4 640	**(645)**
6	Accord h.	Pilot	31 090	27 595	B	3 495	1,13	**(6 090)**	**(2 595)**
7	Accord h.	CR-V	31 090	20 700	B	10 390	1,5	**(6 090)**	4 300
8	Accord h.	Element	31 090	18 980	B	12 110	1,64	**(6 090)**	6 020
9	Accord h.	Odyssey	31 090	25 645	B	5 445	1,21	**(6 090)**	**(645)**
10	Pilot	CR-V	27 595	20 700	B	6 895	1,33	**(2 595)**	4 300
11	Pilot	Element	27 595	18 980	B	8 615	1,45	**(2 595)**	6 020
12	Pilot	Odyssey	27 595	25 645	B	1 950	1,08	**(2 595)**	**(645)**
13	CR-V	Element	20 700	18 980	B	1 720	1,09	4 300	6 020
14	CR-V	Odyssey	20 700	25 645	A	4 945	1,24	4 300	**(645)**
15	Element	Odyssey	18 980	25 645	A	6 665	1,35	6 020	**(645)**

Table 6.6: Choosing car - price comparison - preference estimation (courtesy of [4])

	Comparison		better	pref.	justification
#	A	B	car		
1	Accord s.	Accord h.	A	9	B over budget
2	Accord s.	Pilot	A	9	B over budget
3	Accord s.	CR-V	A	1	price almost same
4	Accord s.	Element	B	2	price of B better by 1 000 USD
5	Accord s.	Odyssey	A	5	price of A better by 5 000 USD
6	Accord h.	Pilot	A	1	both cars over budget
7	Accord h.	CR-V	B	9	A over budget
8	Accord h.	Element	B	9	A over budget
9	Accord h.	Odyssey	B	7	A over budget, B slightly over budget
10	Pilot	CR-V	B	9	A over budget
11	Pilot	Element	B	9	A over budget
12	Pilot	Odyssey	B	7	A over budget, B slightly over budget
13	CR-V	Element	B	2	price of B better by 1 000 USD
14	CR-V	Odyssey	A	5	price of A better by 5 000 USD
15	Element	Odyssey	A	6	price of A better by 6 000 USD

Table 6.7: Choosing car - comparison matrix for purchase price (courtesy of [4])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	9	9	1	1/2	5
Accord h.	1/9	1	1	1/9	1/9	1/7
Pilot	1/9	1	1	1/9	1/9	1/9
CR-V	1	9	9	1	5	1/2
Element	2	9	9	1/5	1	6
Odyssey	1/5	7	9	2	1/6	1

Listing 6.1: Usage of AHP method for choosing new car for Johnson's family

```
library(ahp)
library(data.tree)
cars <- Load("c:/path/cars.ahp")
Calculate(cars)
print(cars, filterFun = isNotLeaf)
Analyze(cars)
AnalyzeTable(cars)
```

Table 6.8: Choosing car - comparison matrix for fuel costs (courtesy of [4])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1/3	5	3	4	3
Accord h.	3	1	9	5	7	6
Pilot	1/5	1/9	1	1/4	1/3	1/4
CR-V	1/3	1/5	4	1	2	1
Element	1/4	1/7	3	1/2	1	1
Odyssey	1/3	1/6	4	1	1	1

Table 6.9: Choosing car - comparison matrix for maintenance costs (courtesy of [4])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	2	4	4	4	5
Accord h.	1/2	1	4	4	4	5
Pilot	1/4	1/4	1	1	2	1
CR-V	1/4	1/4	1	1	1	3
Element	1/4	1/4	1/2	1	1	2
Odyssey	1/5	1/5	1	1/3	1/2	1

Table 6.10: Choosing car - comparison matrix for selling price (courtesy of [4])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	3	4	1/2	2	2
Accord h.	1/3	1	2	1/5	1	1
Pilot	1/4	1/2	1	1	1/6	1/2
CR-V	2	5	1	1	4	4
Element	1/2	1	6	1/4	1	1
Odyssey	1/2	1	2	1/4	1	1

Table 6.11: Choosing car - comparison matrix for safety (courtesy of [4])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1	5	7	9	1/3
Accord h.	1	1	5	7	9	1/3
Pilot	1/5	1/5	1	2	9	1/8
CR-V	1/7	1/7	1/2	1	2	1/8
Element	1/9	1/9	1/9	1/2	1	1/9
Odyssey	3	3	8	8	9	1

Table 6.12: Choosing car - comparison matrix for style (courtesy of [4])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1	7	5	9	6
Accord h.	1	1	7	5	9	6
Pilot	1/7	1/7	1	1/6	3	1/3
CR-V	1/5	1/5	6	1	7	5
Element	1/9	1/9	1/3	1/7	1	1/5
Odyssey	1/6	1/6	3	1/5	5	1

That is not too bad. We load data file cars.ahp, which has defined decision hierarchy including preference matrices and we put it through *Calculate()* function. This function will compute weight coefficient and also scores alternatives.

Remaining lines of code only visualize the results.

Print() lists decision hierarchy to the scree. For file cars.ahp[3] the hierarchy looks like:

[3]the file is available for download in LMS (`https://lms.vsb.cz` in course *Modeling of Decision Processes*)

Table 6.13: Choosing car - comparison matrix for storage (trunk) capacity (courtesy of [4])

	Accord s.	**Accord h.**	**Pilot**	**CR-V**	**Element**	**Odyssey**
Accord s.	1	1	1/2	1/2	1/2	1/3
Accord h.	1	1	1/2	1/2	1/2	1/3
Pilot	2	2	1	1	1	1/2
CR-V	2	2	1	1	1	1/2
Element	2	2	1	1	1	1/2
Odyssey	3	3	2	2	2	1

Table 6.14: Choosing car - comparison matrix passenger capacity (courtesy of [4])

	Accord s.	**Accord h.**	**Pilot**	**CR-V**	**Element**	**Odyssey**
Accord s.	1	1	1/2	1	3	1/2
Accord h.	1	1	1/2	1	3	1/2
Pilot	2	2	1	2	6	1
CR-V	1	1	1/2	1	3	1/2
Element	1/3	1/3	1/6	1/3	1	1/6
Odyssey	2	2	1	2	6	1

AHP weight estimation

On LMS you have available simple example of using AHP to establish weight system for risk analysis of the regions using MS Excel. You can use it as a template for other tasks, where you want to establish weight system only, but are not interested comparing the alternatives using same approach.

In such case you can compute weights of the leaf node and flatten" the hierarchy to these nodes. Computation than proceeds as usual with WSM method.

```
    levelName
1  Root
2  ¦--Cost
3  ¦  ¦--Purchase Price
4  ¦  ¦--Fuel Costs
5  ¦  ¦--Maintenance Costs
6  ¦  °--Resale Value
7  ¦--Safety
8  ¦--Style
9  °--Capacity
10  ¦--Cargo Capacity
11  °--Passenger Capacity
```

Graphical representation of solution can be visualized as on fig. 6.3:

First column, *weights*, shown estimated weight coefficients. As least important criteria of trunk capacity and style have been identified with weights of 3,6 % a 4,1 %. Also note, that the style in column *inconsistency* has value 10.1 %, which can be interpreted in our context as $CR = 0.101$. Since value $CR = 0.1$ is perceived as limit, than we can't exclude possibility, that the stated preferences leading to the weight estimation are actually random. Since we are over the limit just by 0.001 and our decision problem is purely demonstrative, we can let it slide this time.

From point of view of formulation of the recommendation to Johnson's family, best car was Accord sedan followed by Odyssey minivan. On opposite side of evaluation is Pilot SUV.

As you can see usage of the method is relatively simple, provided that you have well prepared inputs for the computation. And that is the main obstruction to adoption. For our case that would be preparation of file cars.ahp.

This file is in YAML Ain't Markup Language (YAML) format, and it ain't pretty. Bellow short excerpt is presented to demonstrate basic structure of the file.

	Weight	Accord Sedan	Odyssey Minivan	Accord Hybrid	CR-V SUV	Element SUV	Pilot SUV	Inconsistency
Root	100.0%	22.0%	21.2%	18.3%	15.8%	13.3%	9.3%	7.5%
Cost	51.0%	12.8%	4.7%	9.1%	11.3%	11.2%	1.9%	1.5%
Purchase Price	24.9%	6.1%	2.4%	0.6%	6.1%	9.1%	0.6%	8.0%
Fuel Costs	12.8%	2.9%	1.1%	6.2%	1.3%	0.9%	0.4%	3.1%
Resale Value	8.2%	1.8%	0.9%	0.8%	3.4%	0.9%	0.5%	0.5%
Maintenance Costs	5.1%	1.9%	0.3%	1.5%	0.5%	0.4%	0.5%	4.0%
Safety	23.4%	5.1%	10.2%	5.1%	0.8%	0.5%	1.8%	8.0%
Capacity	21.5%	2.8%	6.0%	2.8%	3.1%	1.4%	5.5%	0.0%
Passenger Capacity	17.9%	2.4%	4.9%	2.4%	2.4%	0.8%	4.9%	0.0%
Cargo Capacity	3.6%	0.3%	1.1%	0.3%	0.6%	0.6%	0.6%	0.2%
Style	4.1%	1.5%	0.3%	1.5%	0.6%	0.1%	0.2%	10.1%

Figure 6.3: AHP - solution of choosing new car for Johnson's

Listing 6.2: YAML structure - excerpt from cars.ahp

```
1  Version: 2.0
2
3  Alternatives: &alternatives
4    Accord Sedan:
5    Purchase Price: 20360
6    MPG: 31
7    Residual Value: 0,52
8    Safety class: midsize car
9    Cargo Capacity: 14
10   Passenger Capacity: 5
11   Surb Weight: 3289
12   crash rating: 4* in side impact front
13   60K tire cost: 700
14   Brakes Cost: 1x
15   Consumer Report: +++
16 ...
17 Goal:
18   preferences:
19     pairwise:
20       - [Cost, Safety, 3]
21       - [Cost, Style, 7]
22       - [Cost, Capacity, 3]
23       - [Safety, Style, 9]
24       - [Safety, Capacity, 1]
25       - [Style, Capacity, 1/7]
26   children:
27     Cost:
28       preferences:
29         pairwise:
30           - [Purchase Price, Fuel Costs, 2]
31           - [Purchase Price, Maintenance Costs, 5]
32           - [Purchase Price, Resale Value, 3]
33           - [Fuel Costs, Maintenance Costs, 2]
34           - [Fuel Costs, Resale Value, 2]
35           - [Maintenance Costs, Resale Value, 1/2]
36 ...
```

Structure of file is very rigid. On first line, there is always identification of the version. Definition of the alternatives follows, see lines 3 - 15 in the listing. Remainder of the file is then formed by definitions of decision hierarchy and pairwise comparisons. We need to compare each pair only once, the inverse comparison is computed automatically in the package's functions, but still, it is lengthy process.

Without this feature the number of rows of the input file would have literally doubled. Even with this simplification the file has around 250 lines of text. Full listing of the car.ahp file is Available in

Annex 3 of this textbook.

Manual creation of the file is complicated. We can expect, that longer the file is, more error will be made during manual input. To streamline the process I prepared small editor, allowing its user to click and edit the file, with possibility to export it into YAML format. The program is called AHP YAML Editor [32] and is available either in LMS and or on my homepage `https://fbiweb.vsb.cz/~sen76/data/uploads/programy/AHPEditor%20v0.1.7z`. Exporting should limit number of errors, which will be present in the YAML file.

Graphical User Interface (GUI) of the program is presented on fig. 6.4 and fig. 6.5.

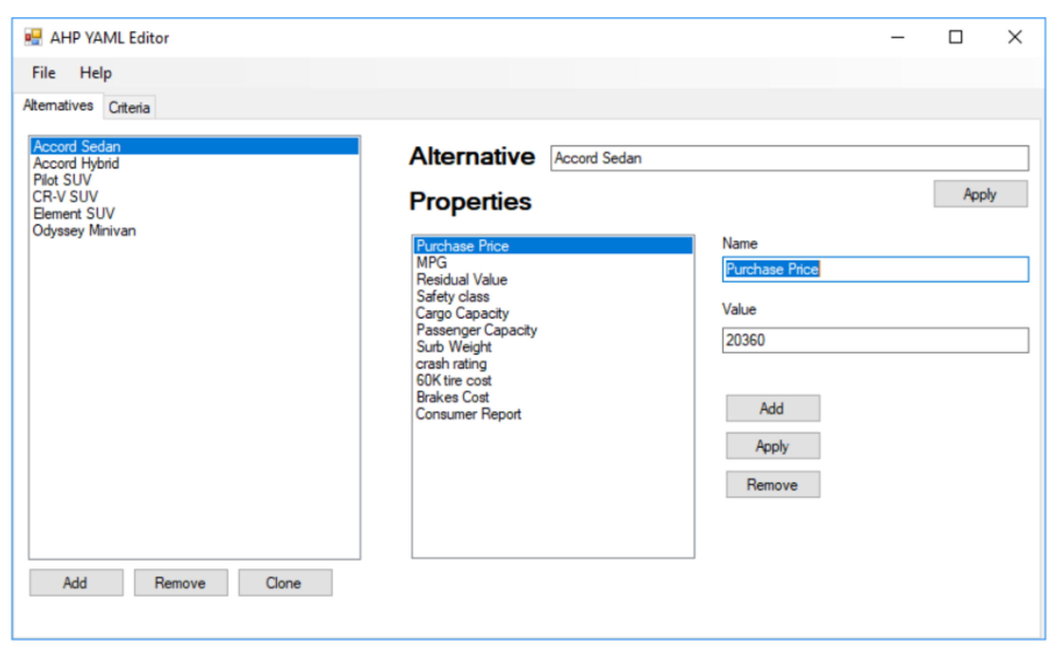


Figure 6.4: Setting up alternatives for decision problem in AHP YAML editor

Program has two perspectives - alternatives and criteria. Since AHP package works only in "full" mode, we must input data into both of these. We start with alternatives as alternatives will be also used in criteria perspective form comparison in leaf nodes. In alternative perspective we define the alternatives and possibly its properties. You can speed up the process using cloning functionality. That is effective especially when you deal with similar structure of properties and values. Just clone and edit to adjust for different name and property values of various alternatives.

You can clone the alternative by selecting it and clicking *Clone* button.

Defining properties is not mandatory. AHP package supports properties for purposed of preference derivation using mathematical functions, but AHP YAML Editor does not support that. One option you have is to define properties in editor, after finishing whole process of defining also criteria and all the comparisons, export results into YAML and manually add missing features. While not ideal, it is still better then whiting the whole file by hand. Of course, if you do not plan to take advantage of this functionality you can safely ignore it.

In criteria perspective we create the hierarchy from root down to leaf nodes.

Before starting of preference mapping I recommend to define whole decision hierarchy. Technically it is not required, but such approach is logical, because you usually do not need to change this hierarchy, at least not structurally - most changes will be probably in preference statements. It is logical to finish the structure first and then focus on preferences and computation.

For leaf nodes, do not forget to click on check box *leaf node*. It changes way how the editor works with the node. If the node is not leaf editor provides criteria down in hierarchy for comparison. But if leaf node check box is checked, it provides alternatives of comparison instead.

After preparing everything, export the file into YAML. Export is necessary as the editor uses its own internal file format to store the data. The format is JSON based, it is humanly readable, but AHP package in R does not support this format.

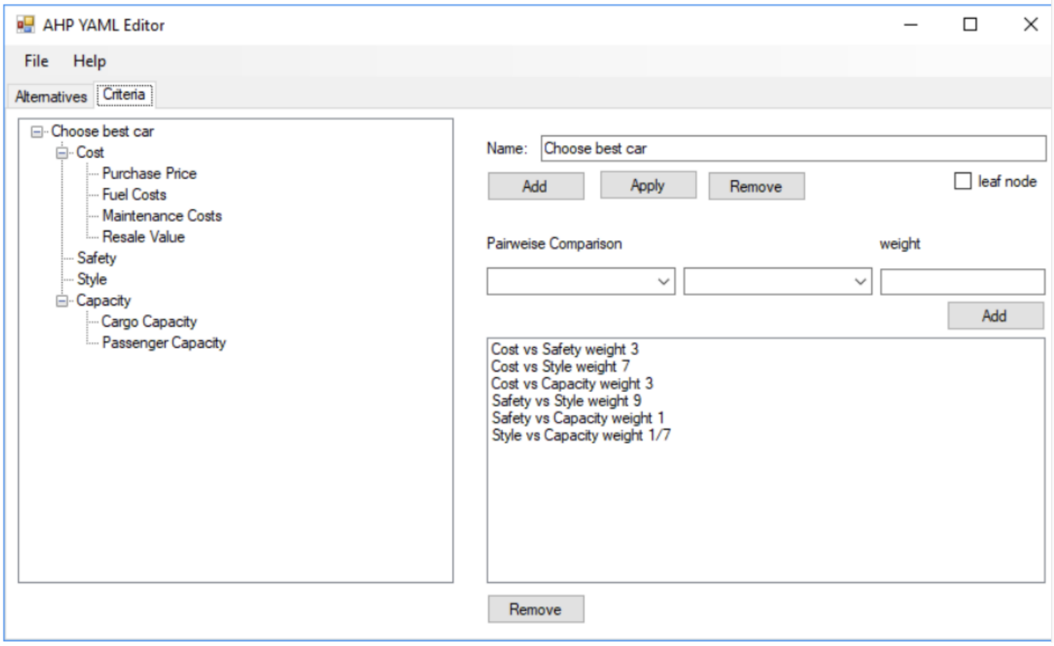


Figure 6.5: Setting up decision hierarchy for criteria in AHP YAML editor

Experiment
Try to implement the examples on your own in the editor a then compute it using R. Explore the results.

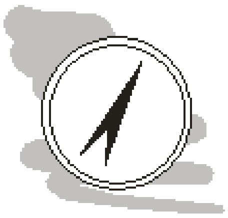
Summary
AHP method allows us to formulate the decision problem into decision hierarchy. The weights are estimated based on preference evaluation. Preferences are gathered for both criteria (for weight estimation) and alternatives (for alternative ranking). Method uses pairwise comparison, but at scale of $\in < 0; 9 >$ and allows to compute consistency index we can use to evaluate randomness of stated preferences.

Self-evaluation questions

1. What differences are between AHP and ANP methods?
2. What differences are between AHP and WSM methods?
3. Explain how AHP evaluates randomness of stated preferences.
4. Explain system of preferential evaluation in AHP.

Chapter 7

Literate programming using RMarkdown



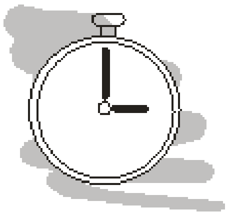
Study guide
In this chapter we will finish look into possibility to use R together with text commentary to generate good looking and easy to realize reports.

After reading this chapter you will
learn how to

- knit together R code, its output and text commentary

know basics of

- Markdown and
- RMarkdown languages



Time required for study
We will heavily lean on knowledge gained in previous chapters. So if you did not go through these or omitted trying to compute it using R and understanding how it works, prepare for long haul. You will need to revisit previous chapters and fill in gaps in your knowledge to proceed.

In previous chapters we realized our computations using R language. We did so by using script which printed textual outputs to the console and graphical outputs to separate window. That is not ideal as it requires the analyst to copy textual outputs manually into the report and export the graphical outputs from RStudio and import the result into MS Word or other text processor you have chosen to write the report.

In this approach report and analytical tool form separate entities which do not directly "communicate" together. That is a problem, because it is very common that we need to work and adjust the computational part of the analysis to incorporate new information or insights we have into the problem. But to do that we need to again do the computation and then incorporate its results into the report. That presents very large overhead and is very inefficient as result.

To deal with the problem R developers community developed a tool called Knitter, which allows its users to automatically "knit" together code, its outputs both textual and graphical and textual commentary together into single file. Knitter supports generating outputs in PDF, HTML and DOCX. It also supports other file format, but these are used most frequently.

If you are using RStudio, Knitter support is incorporated into it by default. We write the text and the code in RMarkdown language. We then instruct the Knitter to knit, which will run the code to generate it outputs and generate document in HTML or any other supported file format.

RMarkdown is a flavor of Markdown language, which is very simple language for creation of documents with minimal effort.

We will start with Markdown first, then we will look into the RMarkdown.

7.1 Markdown language

Markdown is the text format allowing us to use very simple formatting of the text including ability to incorporate figures, tables and equations, without the need to use sophisticated and "heavy" text processors such as MS Word or LibreOffice Writer.

We can write such file using any text editor, but for convenience reasons we usually prefer using specialized Markdown editors such as Obsidian [33] or Zettlr [34] and many others, which are able to render the text in formatted way and incorporate other media such as figures for easy reading experience.

Format is popular for the speed with which you can create text as these editors provide minimal distraction allowing its users to focus on text writing rather then formatting.

Rules for formatting are very simple:

- paragraphs of text are signifies by empty line
- **bold text**
- *italic*
- # heading 1, ## heading 2, ### heading 3, ...

You cal also use ordered and unordered lists:

Listing 7.1: Ordered and unordered lists in Markdown

```
Ordered list:
1. 1st item
2. 2nd item
3. 3rd item
3. 4th item (number is assigned automatically when rendering the document)

Unordered list:
- 1st item
- 2nd item
- 3rd item
```

You can also embed images into the text using following syntax:

```
![alt text](image.jpg)
```

Alt text will serve as figure caption when rendering the file. In () we provide full or relative path to the image. Our example would work only if the file image.jpg was in same folder as Markdown file.

Finally we can add links to other files:

```
[VŠB-Technical University of Ostrava homepage](https://www.vsb.cz)
```

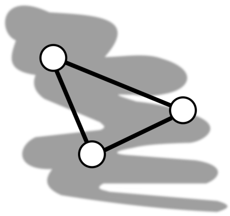
Similarly to embedding the images, the text in [] will serve as the caption for the link. If you leave it empty rendered text will use the URL instead. The link in rendered document is usually active (you can click on it and it will open file or resource).

Lastly Markdown allows us to insert blocks of code using 3 backticks. Here is small example of Markdown of using PROMETHEE I method from MCDASupport package. Note the "R" after the backticks specifying used programming language. Editors like Obsidian will use this information purposes of syntax highlighting, in similar manner as you see in listing bellow.

Listing 7.2: PROMETHEE I demonstration in Markdown block

````
```R
PM <- cbind(
  c(80, 65, 83, 40, 52, 94),
  c(90, 58, 60, 80, 72, 96),
  c(600, 200, 400, 1000, 600, 700),
  c(54, 97, 72, 75, 20, 36),
  c(8, 1, 4, 7, 3, 5),
````
````

```
  c(5, 1, 7, 10, 8, 6)
)
colnames(PM) <- c('C1', 'C2', 'C3', 'C4', 'C5', 'C6')
rownames(PM) <- c('A1', 'A2', 'A3', 'A4', 'A5', 'A6')
minmax <- 'max'
shape <- c('U-shape', 'V-shape', 'linear', 'level', 'default', 'Gaussian')
q <- c(10, 0, 450, 50, 0, 0) #indifference threshold
p <- c(0, 30, 50, 10, 0, 0) #prefference threshold
s <- c(0,0,0,0,0,5) #intermediate threshold
w <- c(0.1667, 0.1667, 0.1667, 0.1667, 0.1667, 0.1665)
result <- PROMETHEE_I(PM, shape, w, minmax, q, p, s)
```
```



**Need a second brain?**
Linking capabilities of Markdown editors are actually so powerful, that it become one of central tools for building sophisticated workflows and knowledge management system often called *second brain*.

Second brain was a term coined by Tiago Forte in his book *Building a Second Brain* [35] for effective systems for information and task management. Markdown tools are fully capable to fulfill requirements on such tools.

If you are interested in this problematic you can start on YouTube channel of mr. Forte (https://www.youtube.com/@TiagoForte) for various productivity related interviews, tutorials, tools demonstration.

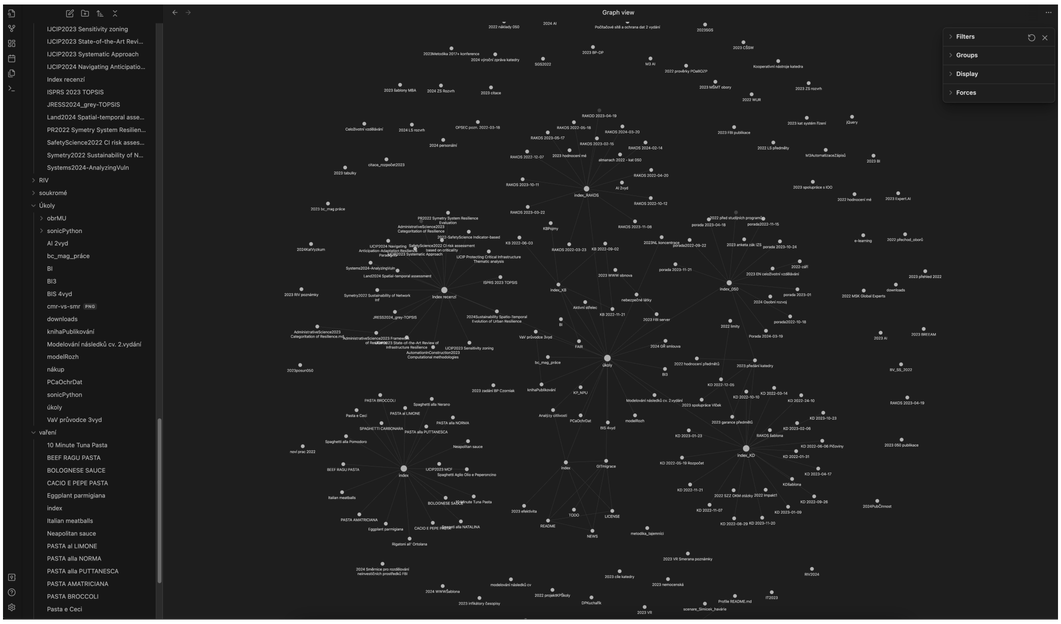


Figure 7.1: Graph of interconnections between documents in my vault in Obsidian

RMarkdown uses all the features of Markdown and adds some additional feature to allow effective code development and using of its analytical outputs.

## 7.2 RMarkdown

As we saw in previous section of the chapter, Markdown has the ability to add block of code into the document, but doesn't posses the ability to run this code or process its output. This ability is supported only by RStudio Integrated Development Environment (IDE) and Knittr tool. Both of these use R engine to process the code and integrate its outputs into the document after it is processed by RStudio or Knittr.
```

Computed values also can be directly incorporated into the paragraph text.

RMarkdown has some special properties in document header, you need to set before you start to create the text and code inside your RMarkdown document:

Listing 7.3: RMarkdown header

```
---
title: your title goes here
author: Your Name
date: 2024-05-30
output: html_document
---
```

Most parameters are self-explanatory (by the way this is not full set of available properties, you can also set for example keywords and many more other parameters for the document), but the *output* merits some explanation, because you have a lots of options here. The html_document is the default option, which is indicating to Knittr, that HTML file should be produced. This file will embed all the text, code, its outputs (both graphical and textual) into single file, in this case HTML file.

Apart of speed of working with this file using common web browser it has benefit, that it allows using interactive graphics, which is not available in any other type of supported outputs.

Other supported outputs are: PDF, RTF, md and many others. For example if, you wanted to produce pdf, you would write: output: pdf_document. If you wanted to produce both HMTL and PDF, you would write: output: pdf_document, html_document.

If you need for your outputs to be editable, use RTF or md, but be aware, that changes you realize into such document using MS Word (RTF) or Obsidian will not be incorporated in original RMarkdown document, should you need to rerun the code with modified or entirely different data.

After the header section you can write your document as normal Markdown document. There is slight difference for the code blocks. RStudio supports only R and Python block of code, instead of variety of languages for whose syntax highlighting is available in Markdown editors. And parameters of the code block are also slightly different. Let's go back to PROMETHEE I example, but this time around in RMarkdown.

Listing 7.4: PROMETHEE I demonstration in RMarkdown block

```
‘‘‘{r PROMETHEE I exmaple}
PM <- cbind(
c(80, 65, 83, 40, 52, 94),
c(90, 58, 60, 80, 72, 96),
c(600, 200, 400, 1000, 600, 700),
c(54, 97, 72, 75, 20, 36),
c(8, 1, 4, 7, 3, 5),
c(5, 1, 7, 10, 8, 6)
)
colnames(PM) <- c(’C1’, ’C2’, ’C3’, ’C4’, ’C5’, ’C6’)
rownames(PM) <- c(’A1’, ’A2’, ’A3’, ’A4’, ’A5’, ’A6’)
minmax <- ’max’
shape <- c(’U-shape’, ’V-shape’, ’linear’, ’level’, ’default’, ’Gaussian’)
q <- c(10, 0, 450, 50, 0, 0) #indifference threshold
p <- c(0, 30, 50, 10, 0, 0) #prefference threshold
s <- c(0,0,0,0,0,5) #intermediate threshold
w <- c(0.1667, 0.1667, 0.1667, 0.1667, 0.1667, 0.1665)
result <- PROMETHEE_I(PM, shape, w, minmax, q, p, s)
‘‘‘
```

In code chunk we see language specification "r" followed by code segment name "PROMETHEE I exmaple". Segment name is useful for long documents with many code chunks. When you knit the document together, the Knittr will process one chunk after another and write the name of processed chunk into the console. This way you see where in processing the system is and if there is a problem, it will help you identify where the problem is.

You can also provide some modificators specifying, how the outputs should look like

Listing 7.5: Code chunk modificators

```
‘‘‘{r, label = ’my-chunk’, echo=FALSE, fig.height=4, dev=’jpeg’}
# do not produce textual output to "console"
# specify size and format of outputing graphics

```

````
```{r, setup, include=FALSE}
# the code will not be wisible in the final (rendered) document
# useful for setting things up

... code ommited

```
````

Running the code is easy. You can click on |> sign (play button) in upper right corner of the chunk to run the chunk of code, or upper right corner of the window to run all of the code in the document, see fig. 7.2.

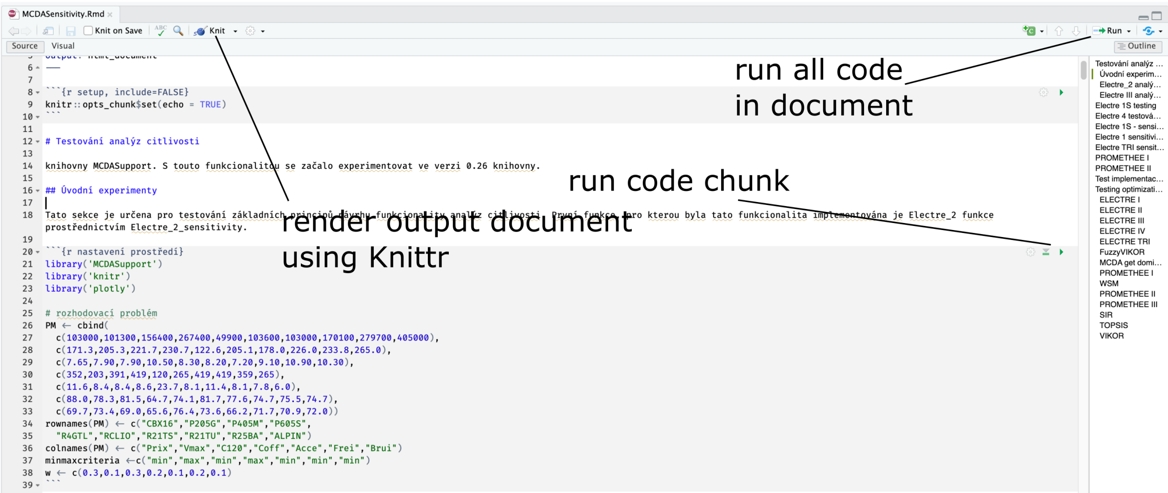


Figure 7.2: Run and render document in RMarkdown

Especially during development process it is useful to see immediately effects of the changes in the code. You can render all of the code, or a chunk, but sometimes it is sufficient to run just few lines of the code you are working on. You do that by selecting the lines (like in text editor) and pressing CTRL + Enter (or CMD + Enter on Mac).

Beware that the chunk of code may depend on results of previous chunks of code to work properly.

Running the code will produce its output (if the code produces some output on screen and not only in memory) directly in the document, see fig. 7.3, in this case we are producing ordered list of alternatives using PROMETHEE II method.

Figure 7.3: Run and render document in RMarkdown

If the code chunk produces several results only one of them will be visible directly, remaining will be available as thumbnails, you can click through to inspect them.

If you knit the code the behavior will be different and the results will be rendered all one after another.

Data frames and matrixes are presented in the form of formatted tables. You can influence how exactly using kable() function. Its first parameter is always a data frame or matrix to render, other parameters like caption and many others specify how the output should look like. Please consult the documentation for the function for the details.

To support you on your learning journey, we prepared two demonstration of RMarkdown usage, you can download from LMS. First (WSM.rmd) demonstrates usage of WSM method and also explains in detail usage of some features of Markdown. The second one (ELECTRE.rmd) just focuses on computation of ELECTRE I and II. Both documents are based on e-ink e-book reader selection problem, we were using for demonstration purposes in previous chapters.

You can use these files as a template to solving your semester project.

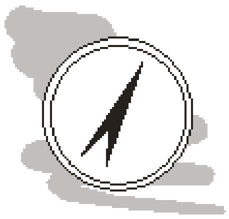
Experiment with templates
We are at the end of chapter. Usually you would find a summary here, but this time it is different. We approached the problem practically, so it is required that you also try things practically. Take the provided RMarkdown files and start to experiment with them.

Try to understand way it operates, run the code both in the editor and by knitting the document. Then try to adjust the code and observe how the output changes. Perhaps use different method from MCDASupport package - there are lot of them we did not discuss in this textbook.

The package has detailed documentation including tested examples you can use as your starting point.

Chapter 8

Other methods usable in general decision situation



Study guide
In this chapter you will learn about other supporting methods, which can help in almost any decision situation.

After reading this chapter you will understand how to

- do brainstorming
- use mind maps
- use Delphi method

know

- what problems are connected to data gathered by questionnaires and how to minimize them

Time required to study chapter
For getting through this chapter you will need at least one hours, perhaps more if you try to solve the tasks practically with software support.

8.1 Delphi method

Delphi method has been developed by RAND in fifties of last century as side product of research financed by US Air Force - Project Delphi. Project's goal was to develop methodology to identify possible targets of Soviet nuclear strikes on US soil. Basic problem was incomplete information. If we look at the problem logically we can form some assumptions. If nuclear war took place SSSR would intend to do as much damage to USA as possible. USA, since it has in depth knowledge on its infrastructure posses the knowledge of which targets would induce such damages. But this knowledge is not public. Target selection by SSSR is limited from this point of view on what is knows about infrastructure of USA. That means that infrastructure with most damaging potential is not necessary most probable target of attack.

So the problem can be reduced to question on what exactly does SSSR know about the infrastructure of USA. The answer to that question could be partially based on hard data. Since we are talking here about nuclear weapons, then we can presume the espionage and intelligence gathering could be good source of information, but with low probability to get actual target list, as such list would be highly privileged, and accessing it would be very hard.

Other then that there is always possibility to use expert opinion on capabilities of the adversary, his technical capabilities and derive possible answer from that.

In this environment RAND developed general method to bring answers in situation with limited or no hard data to support the decision. Since it is based on expert opinions the name Delphi method has been chosen after famous Oracle of Delphi from ancient Greece. Method tries to formalize intuitive understanding of decision situation by the experts. For its ability to work with minimal formal inputs and to always provide some kind of answers the method become very popular.

On the other hand, since the method works with minimum hard data and with expert opinions it is also less precise then methods based on mathematical models. Results are also prone to various errors and influences introduced during preparation phase by the decision maker (i. e. wrong questions), during the information gathering phase by decision maker (improperly leading process of gathering information) and experts (cognitive bias - wrong answers) and in evaluation phase again by decision maker (wrong interpretation of information).

Since the method has been so popular, it evolved in time and today there are many of variants of the method to choose from. We are going to discuss only basics of the method. For advanced information on the method we can recommend *The Delphi Method: Techniques and Applications* [36] by Lindstow and Turoff[1].

Since the method focuses on gathering information from experts. We need a way collect information from them, set some kind of process:

1. problem specification
2. expert selection
3. creating questionnaire (to collect the information from experts)
4. create documentation the the questionnaire
5. send the questionnaire to experts
6. evaluate answers
7. in case of identified problems, repeat from step 3. In additional iteration we often send the questionnaire together with its evaluation from previous iterations
8. we can perform as many iterations as needed - we repeat until we get the results we need (quality wise! not the answers we want!)
9. summarization of the results

While there is relatively high number of steps in the method's process, it doesn't appear, that the method is complex. Unfortunately practical experience with the method prove, that its usage is very resource intensive in both financial requirements and time required to realize it. As bonus to that, the results provided by method can be easily misleading if we do not ensure impartiality of experts, by improperly prepared questions or by improper choice of experts.

On the positive note there are also numerous benefits of method usage. It performs even if we have only very limited information, or in situation that we have extremely (prohibitively) lot of information and are not capable to process them using for example common statistical methods. Method is good at eliminating extreme opinions of experts, provided that the expert's pool is large enough to form broader opinion consensus.

Elimination of extreme opinions can under certain circumstances be considered as a limitation of the method, because we can argue, that such opinion does not necessary correlate with correctness of it (there could be correct extreme opinions).

If we were to choose one most critical factor regarding our chance on succeeding in using the method, it would be *choosing of experts* step. That is why we have relatively strict requirements on them:

- they must be experts on certain aspect of the problem or generalist with knowledge of the problem as whole (expert pool must have both types of experts)
- experts need to be independent
- experts need to be capable and willing to answer our questions

Requirements on expertise of the experts is logical and intuitive. In simple term we try to avoid the situation when we for all the trees do not see the forest (too much focus on detail) and opposite problem when we see the forest but missing the trees (our view lack necessary details). As a rule of thumb

[1]the book is provided on-line free of charge by its authors from https://web.njit.edu/~turoff/pubs/delphibook/index.html

we need to model/gather information as generally as possible and as detailed as necessary. Choosing right "resolution"/detail level for the problem may not be apparent, especially in the beginning phases of the problem solution.

More complex is the question of *experts independence*. If we say independence, then we may refer to many very different things. Dependence can be for example financial. If it exist the expert is motivated to follow line of thought of the institution which is paying him/her. If the expert is being paid by us to answer our questions, he may be inclined to say what (s-)he think that we expect, or avoid pieces of information, which may be perceived by us as controversial or opposite to our believe system.

Dependencies can be much more sophisticated. The dependency in general can be on certain result or on usage of same information, which leads to same results. Neither of these is easily identifiable.

Good example of dependence on results are Willigness to Pay (WTP) and Willigness to Accept (WTA) studies when respondents may be trying to steer our decision outcome by certain answers, see separate Study guide in this chapter on these methods (marginal utility measurement).

Second type of problems is closely connected to imperfect information available. We know that for decision making we have rarely available all information, we would like to have. If there is acute lack of information available the chance of interpreting available information wrongly significantly increases. If multiple experts all use exactly same data source to form their opinions, then while they are technically independent, in fact they may be even unaware of each other's existence, the will probably come to same or similar conclusion. User of these opinions may be swayed by "support" of such opinion by multiple experts. That support may not be grounded in reality. What if underlying information is false? Conclusions based on it will also be false regardless of how many experts say it.

Since the experts are technically independent, they have no way of identifying the problem. Only way of how to do it is by the decision maker, who organizes the information gathering to ask right questions to identify such problems, like for example examining on detail on what the expert's opinion has been based.

All answers collected from the experts need to be processed. The way we do it will significantly differ depending od type of the answer. Basic typology of the answers can be following:

- numeric
- ordinal (choice from several possibilities, number can be attached, but has no numeric meaning)
- free form answer

There are variety of problems that are common to all types of answers. These can grouped into two basic groups: 1) missing values and 2) dealing with outliers.

By missing values we mean experts answers which are either empty (expert for example forgot to write answer) or refused to answer. Reasons for refusal can be multiple from identifying his/hers own bias, or lack of knowledge to ethical reasons. Since in Delphi method the surveys are not anonymized, we can (and should) find exact reason of missing data. In case of simple forgetting, we can even allow the expert to amend the survey, thus solving the problem. But if there are other reasons - refusal to answer, we will actually need to deal with missing value.

Simplest way of how to deal with it is to omit such answer. Not always such approach is possible. For example the survey may be constructed in such way, that it gives proper answer only if it has been filled in whole, or some subset of answers is required. Missing value in such case may invalidate these answers too. By removing the answers, or whole surveys we also diminish to certain degree robustness of the dataset we are going to use to derive information from. If the number of respondents is low, then by removal of some answers may lead to devaluation of the explanatory value of the whole statistical population.

For numeric answers various imputation method exist, which may be feasible to use. Such methods always come hand in hand with some level of added noise to derived information. Analytic needs to decide, how to approach the problem considering all pros and cons. Further discussion of imputation problematic goes beyond scope of this course, as it requires also basics of statistics in addition to understanding imputation methods.

Another problem are so called *outliers*. By outliers we mean values which can be considered as extreme in comparison to other values for the answer. Say you let experts predict GDP grow a year from now. Most of the answers will fall for example into interval $\in < -1; 3 > \%$. What if one of experts predicts 15 %? Such value can be considered as outlying. To understand why it can be problem we will need to address a question what do we we actually do with these values? How do we process them?

For numeric value we may be interested some statistical properties of the dataset for the answer, such as mean, median or distribution of the answers. Consider two dataset of GDP predictions $x_1 = -1, 0, 1, 2, 3$ and for x_2 we add extreme prediction 15 %, so $x_2 = -1, 0, 1, 2, 3, 15$. If we compute mean for the variables we will find that $\overline{x_1} = 1$ but $\overline{x_2} = 3.3$, which is significantly more. The single value did significantly influence the result. Consensus of economic experts in this case was prediction of weak economical growth, x_2 dataset with extreme value predicts very healthy economic grow, if we use the average as a measure.

Please note that median value is not so sensitive to these extremes. $median(x_1) = 1$, while $median(x_2) = 1.5$. There is another classic example, but one that we see every day around us: difference between average pay and median of pay or in mortality due to covid infection - average for whole dataset, compared averages for age stratified dataset. Similarly, in field of security, average damages vs peak damages or expected yearly loss. We could continue on.

With extreme values, removal is an option, with same impact on robustness of the dataset as in case of missing values. First step is usually identification of such values. Good graphical approach is *Box Plot*, see fig. 8.1, we could also use histograms to see value distribution and "eye ball" its type. With this knowledge we can identify the extremes and start thinking on what to do with them.

Q1 and Q3 are first and third quartile, Inter Quartile Range (IQR) and σ^2 is variance of the dataset. Values lower $Q1 - 1,5 \cdot IQR$ or higher than $Q3 + 1,5 \cdot IQR$ are usually considered extreme.

Some inspiration for the extremes identification can be found in existing literature on Delphi method. For example relatively popular is the upper and lower 25th percentile as limit for identification. As this value can be too low or high for the purpose, we need to decide on these limits on case per case basis.

When evaluation choices from predetermined variants (*ordinal choice*), we can't compute averages etc. Or more specifically we can compute them, but the results are meaningless. When evaluating such type of answers we should focus on number of occurrences of the opinion. Is some choice preferred significantly more than the other? Using percents for the purpose simplifies the process. Results then are relatively easy to understand. We can also use pie or donut charts to visualize answer distribution. Bar charts are also usable for the purpose, thou they are less popular.

Free text answers (open questions) are most problematic to process. There are no statistical tools, charts, etc. available to kick start our analysis. We usually need to process the answers by hand and derive additional information from the answers. This information can be processable by statistical methods, or not, depending on the information provided by experts.

Evaluation often tries to find common properties in the answers. Does some idea keep appearing in the answers of the experts? This can be interpreted as a consensus among the experts (something the experts agree on). Question of consensus is one of more problematic areas of the method, we will address it also separately later in chapter.

We can also be interested in some different/interesting answers. Expert may communicate some piece of information or angle, we originally did not think about. Maybe we can identify the need to review the survey and add or change the questions. In other worlds we may have identified a gap in our understanding of the problem.

Since the free text answer is very flexible, open, the surveys always do have some questions available in this form. Minimally as a place to facilitate the communication between the expert and analyst.

Disadvantage of the open questions is the time and effort required on side of both analyst and respondent to deal with them. Will the respondent have time to answer it properly or will (s-)he be forced to cut the corners? While these questions are important and to certain degree always present in the questionnaires, they should form only minority of the questions.

The basic message we are trying to is that you need to truly understand the nature of the data you are processing. Are you using proper measures? What do the results tell you? How can we interpret them?

Last question we are going to discuss around the Delphi method is problem of experts consensus. We actually saw symptoms of consensus or lack of it in the data in paragraphs above. If there is no clear consensus, blindly using average, mean or any other measure will probably be not helpful as experts either understand the question differently, or there is not one "right" decision.

To effectively process data, some kind of consensus must be reached. To increase chances for consensus one can prepare:

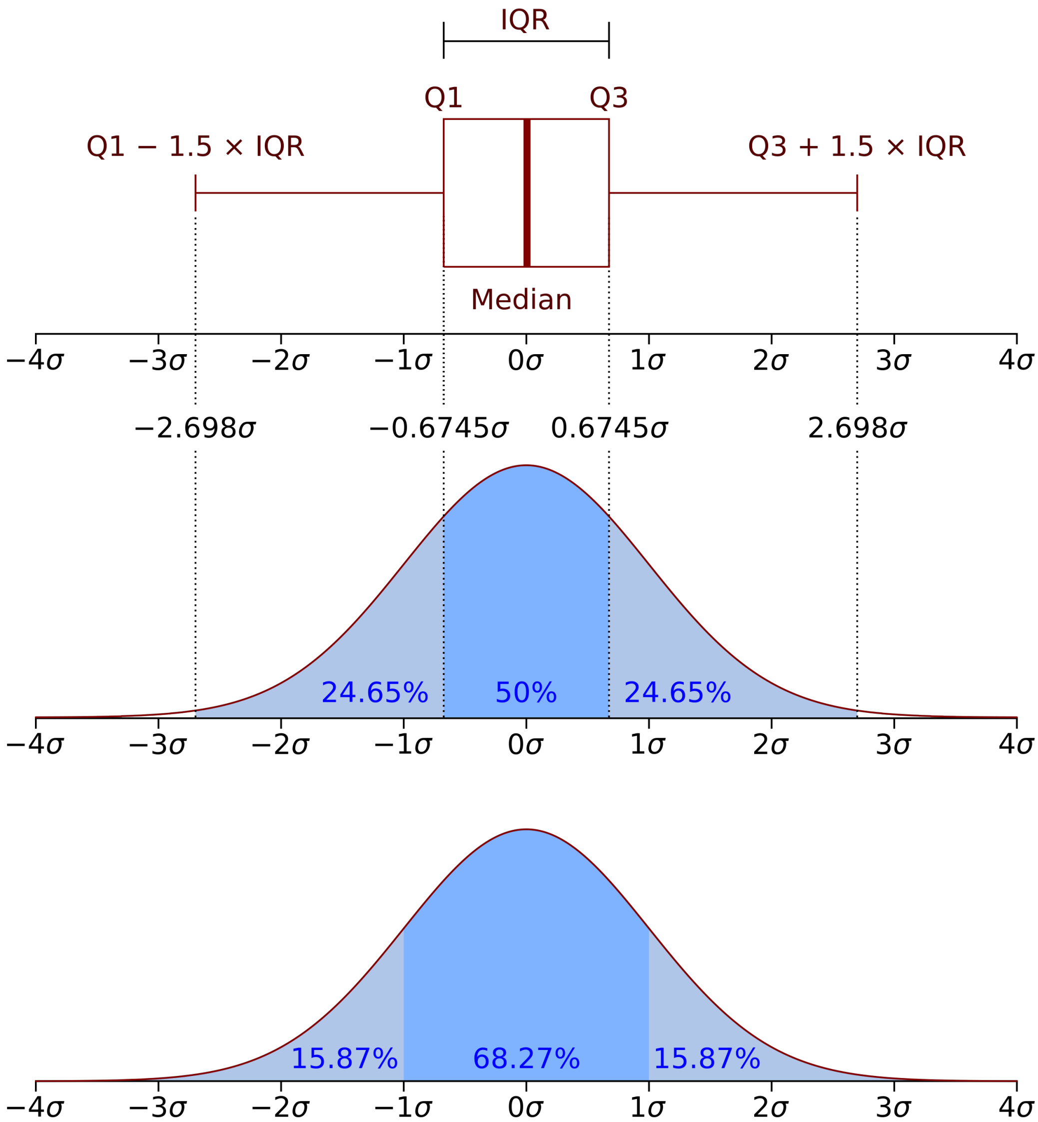


Figure 8.1: Boxplot and a probability density function (pdf) of a Normal N(0,1 σ^2) Population. (courtesy of [5])

- the survey in such way that it is clear what kind of information is needed
- in depth documentation to the survey, so that the expert has a reference point to lean on
- indoctrination seminary where the experts are taught what is expected of them in survey (rationale of the survey)
- multiple rounds of survey. Each round includes all information gathered in previous round. Previous round serves as limiter to answers.
- if nothing else helps prepare seminary with intend to find a consensus - experts will discuss the problem and decide on answer.

As you can see, reaching consensus is not an easy task and is one of reasons why Delphi method is so resource intensive.

8.2 Cognitive bias

One of reasons why Delphi method is so hard, is a keen ability of the human brain to come almost instantly to conclusions, with almost no requirements on input data. Modern science sees this ability as a result of adjustment of our ancestors to survive in hostile environment. Closely connected to it is *fight or flight mechanic*. This refers to decision whether to stand up and fight or try to flee in presence of possible danger. This decision needed to be instant to be maximally effective and it doesn't need to be that precise as most signals of danger have been false alarms.

In case of false alarm, the decision does not matter as a danger does not exist. But in case of real danger immediate reaction has potential to save life further strengthening the mechanism. Today this ability is not so useful for normal life but it is part of us and we use it everyday.

From point of view of thinking there are two basic types: 1) fast (immediate) decision making and 2) slower analysis based decision making. Daniel Kahneman described this in his bestseller *Thinking, Fast and Slow* [20], where he summarizes 40 years of research with his college Amos Tversky in the field of what we today call *cognitive bias*.

Main idea of the book is that we all are primed to behave in certain way. Our split second decision may seem random, but they are determined a lot of things which basically force the decision. That all is part of "fast" thinking. In that sense we all are biased. Since results of fast thinking are wrong lot of time, we need to actively pursuit "slow" type of thinking, which is analytical, makes lot of effort but also has much better results.

There are many types of cognitive bias. From the better described ones we can for example name *central bias*, which explains problems with ordinal choices. Central bias states that people tend to eliminate extreme values. If we have five point scale $< 1; 5 >$, and we expect experts to equally distribute their opinions in this scale, it will not happen as 1 and 5 options will have significantly lower preference rate. If we were not aware of this our logical conclusion would be that a true value is somewhere in $\in < 2; 4 >$ instead of correct conclusion that the value may be in whole interval and that in fact there might be no "true" value (random selection).

By the way t his is why we usually recommend 5 - 12 point ordinal scales for qualitative risk analysis methods. At 5 points we get 3 "actionable" points, which is minimal threshold for distinguishing between the risk levels. Upper limit of 12 is based on cognitive limit of most humans, as we are not good to distinguish more then 12 points ... of anything. In that regard is doesn't make any sense to implement stales with more then 12 levels if we are using expert opinions as an input.

Hyperbolic discounting also known as *present bias* is another form of bias well known from behavioral economy. This form of bias describes situation, when people prefer (choose) options leading to smaller but immediate rewards, in comparison witch choices leading to much larger rewards/benefits, but which are realized at later time.

From "homo economicus" point of view, where rational choice should be based solely on size of the rewards such behavior seem to be illogical, neverless such behavior is observable very often in real world on all levels of decision making: prom personal level to state policies formulation. This bias can be seen as largest obstacle to implement any policy requiring more than single election period such as pension or school system reforms or implementing of climate change prevention policies.

Decision support methods were designed to support this slow, analytic thinking and minimize effect fast thinking. Kahneman was awarded for his research by Nobel price for economy in 2002. There are also broad applications of the cognitive bias in other research fields. In economy Richard Thaler developed whole sub-field of economy called *behavioral economy* to explain certain behavior of markets and societies, which were previously unexplainable by traditional economy and perceived as irrational and thus to be considered being an anomaly and ignored. Thaler proved that not only this is not a case, but by adjusting for example policies we can use these to influence the society.

Since influencing can have negative connotation related to all oppressive regimes in their everlasting pursuit of perfect societies supporting the regime, he uses different name for these influences: *nudge*. Nudge is small, unobtrusive measure which motivates the people to make a good decision. Nudge is also a name of a book [37], where Thaler describes his research and onset of behavioral economy.

For his research Thaler was awarded in 2017 also by Nobel price for economy.

As you can see, the problematic of cognitive bias is very broad topic. Present state of the research on topic in form of cognitive bias typology map is available on fig. 8.2.

Before we move to another topic, let's explore some more how the cognitive bias influences Delphi method and survey construction and evaluation in general. There are two notable biases:

- hypothetical and
- strategic

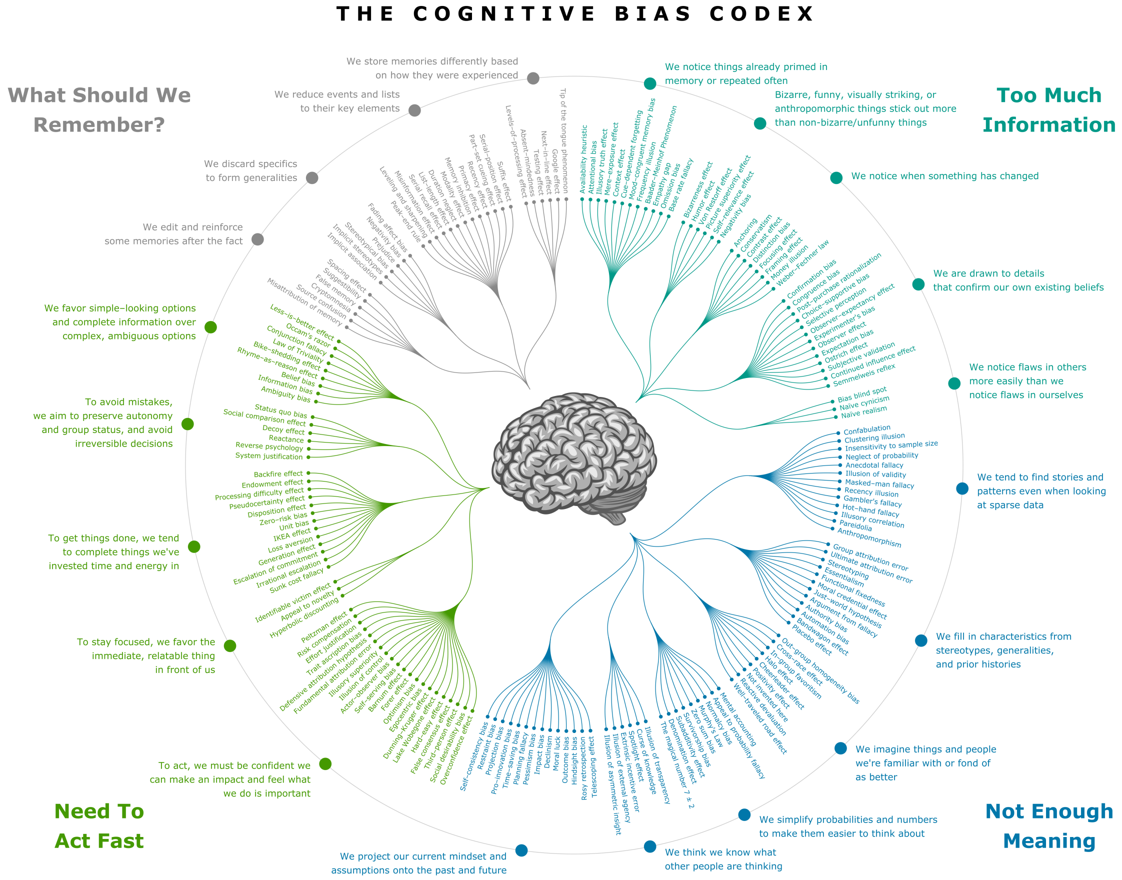


Figure 8.2: "Codex" of more then 180 forms of cognitive bias (courtesy of [6])

We in fact touched the topic briefly when talking about the answer evaluation, thou we had no name for effect, until now. *Hypothetical bias* happens when the respondent sees the question as purely hypothetical, cannot happen in real life situation. In such case the respondents do have a tendency to underestimate the answer or not put much effort into it. This may lead to heavily flawed answers.

We can defend against this bias by specific formulation of our questions. Especially important is proper framing of the question and supporting it by examples which happened recently. Goal is to introduce expert to the situation and possible gravity of the answer. This should motivate him or her to put necessary effort to the answer.

But be careful because framing and recent occurrence of the event or issue may work too well. We call this situation *anchoring bias*.

Supporting documentation with in-depth rationale behind the questions together with indoctrination seminaries can also help. The seminary is recommended also in case the questions are complex and we anticipate problems. Under such conditions investment into seminary would be a rational decision as it has potential to prevent these anticipated problems or at least minimize them. Early investment may limit number of Delphi method iterations analyst will have to do, thus decreasing overall time and money required.

The respondent (expert) will need to study the documentation and participate in seminary. That may limit number of experts available, especially if we do not pay them.

We can limit this negative effect by using online questionnaires and organizing online meetings or seminaries using videoconferencing tools like MS Teams, Zoom or any other platform of choice.

Remote meetings are less effective then in-person meetings, but are easier to organize. Both from perspective of speed (date and time participant need to agree on) and geographical distances which needs to be traveled to in-person meeting. Because of the easiness, we can organize multiple more focused meetings to further limit lesser efficiency of of such meeting.

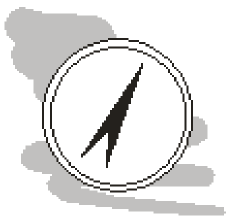
Strategic bias is almost exact opposite to hypothetical bias. In strategic bias the respondent not only believes, that the situation presented in survey is real, but (s-)he also believes that something can be gained by certain type of answers. As to what motivates certain types of answers. It may be the effort to make him/her-self more attractive to potential employer or benefactor. In this case the expert may be motivated to give opinions which conform to image of the analyst organization, or simply what (s-)he believes the analyst wants to hear.

Another example can be studies conducted on endangered regions to map resident's sentiment toward possible protective measures against floods. If respondents believe that certain types of answers will lead to change in policies and public investment in protective measures.

Respondent may change the response in such way to maximize public investment which as side effect will also increase the value of properties in the region/area being analyzed the respondent owns.

Since side effect of such investment would be increase of utility (value) of all property present on former flood plain, respondents would be heavily motivated to give answers which improve chance of such investment.

Preparing survey is very hard. In addition to the wording of the questions themselves, it is also necessary to address whether there will be a need to create supporting, training materials, organize training for respondents, etc. The form in which the survey will be conducted is also an important decision. Will the forms be sent out in paper form or electronically? One can also opt for face-to-face consultations, either by telephone or face-to-face. Will it be one-on-one or organized more as discussion with goal to find consensus.



Methods based on marginal utility measurement
In the text we have been discussing Delphi method and survey realization. There is variety of other methods based on similar principles, we can use for decision support. Methods like Contingent Value Method (CVM), Travel Cost Method (TCM) and others are based on measurement of marginal utility and use surveys to collect required data. Difference to Delphi method is that we are not asking experts, but general public in certain area of interest. It may be city, region or even relatively small area defined by polygon of expected impacts of some kind of accident.

These methods are also based on measurement of WTP or WTA. WTP measures how much are the residents willing to pay to avoid or minimize the risk. We can safely presume, that higher level of endangerment will be, the more the residents will be willing to pay. WTA is based on similar principle, but it measures how much needs to paid to residents so that they would be willing to accept risk existence. Such payment may have different form, for example rent discount, tax credit, subsidies to realize protective measures, etc.

CVM is being used for measurement of so called non-tradable goods. It can help us when dealing with questions of environment, or in public goods in general, where these is no clear monetary value for such goods (value of existence of certain endangered species, ...). TCM can be used to establish "price" for recreational areas, nature reserves, etc. We estimate the price based on willingness of the tourists to pay for visit of analyzed area. The more the visitor is willing to pay, the higher value area has.

Unfortunately we have no room to discuss these methods in detail, there is lot of studies, which describe use cases for these methods and can be recommended as further study material, see for example [38, 39].

8.3 Brainstorming and mind maps

Brainstorming of one of most used methods for analyzing (not only) decision situation. This method works best for smaller teams trying to find all aspects of the given situation. As such it is especially useful in beginning stages of decision making.

Questions

1. How Delphi method works?
2. What kind of answers can Delphi method support and how do we process it (are there differences in processing between the types of the answers)?
3. What is cognitive bias?
4. What is difference between hypothetical and strategic bias?

The method itself is not usable to identifying optimal solution of the problem, but rather to describe it or identify all important aspects of it. The solution itself must be reached using different analytical method.

When brainstorming, there is always one person in charge, who set the stage for brainstorming session, defines the problem and writes down opinions and ideas of other team members. On black board or in the computer then gradually grows network of subject and ideas with main topic in the center of it. Brainstorming team should be broad enough to cover whole problem domain of the topic on question.

During the session all ideas are being written down without prior evaluation. If we immediately performed evaluation, it would have negative impact on level of participation (number of generated ideas). Remember, that it is easy to filter out bad ideas afterward. Afterward is also safe from point of view number of generated ideas.

Some of more exotic ideas, which look weird or even stupid at first, may just be misunderstood or may function as a source of inspiration that can be worked on.

All participant have same rights, nobody is preferred. Leader and speed (s)he writes down the ideas is limiting factor. Because of that the leader must also function as moderator and specify who will say his idea.

Since there is connection between number of participants of brainstorming session and number of ideas they generate, it is not possible to increase number of participants without limits. For complex problems multiple brainstorming sessions with different set of participants may be necessary, leading to multiple networks. Separate idea integration brainstorming session may be good option.

There were some studies on correlation between number of participants and number of ideas generated, see fig. 8.3 and 8.4.

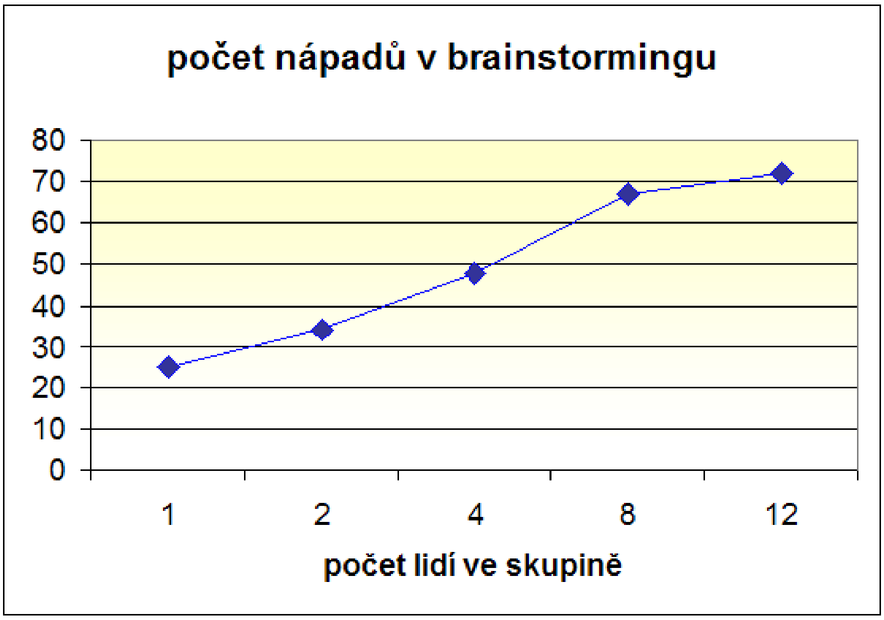


Figure 8.3: Dependence of number of ideas generated on size of group in brainstorming (courtesy of Osuský and Fajmontová [7])

We can generally say, that to certain point number of ideas grows with increasing number of participants. We can say that they inspire each other. Beyond that point the time each participant

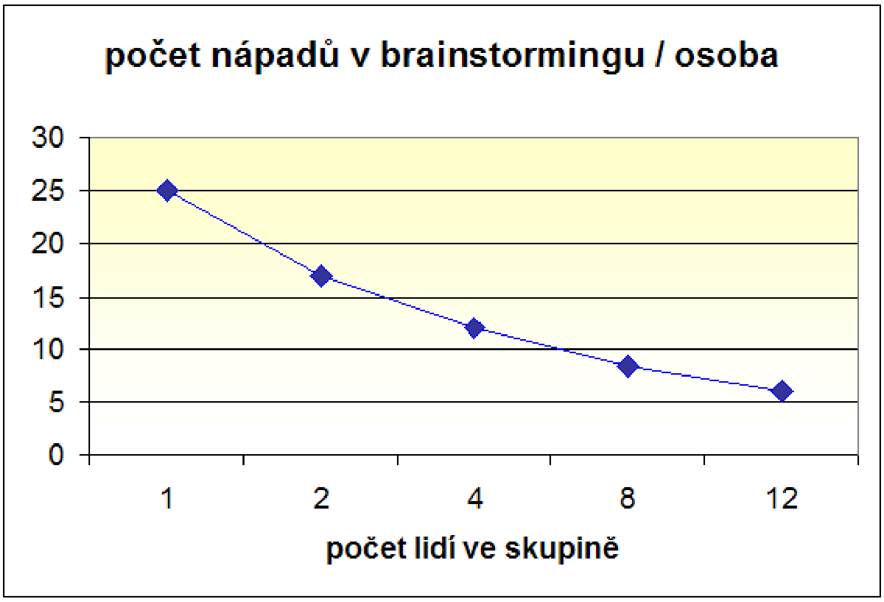


Figure 8.4: Dependence of number of ideas generated per participant on size of group performing brainstorming (courtesy of Osuský and Fajmontová [7])

has to voice his/hers opinions diminishes significantly and the yield of ideas stagnates of event shrinks. With limited time only part of knowledge of the present participants is mapped, which would limit usefulness of the method as starting point of decision making.

We can conclude that there is no single number we can use as optimal size of the group, or number of ideas we absolutely need to generate in brainstorming session. It is responsibility of the leader to decide on all organizational aspects of the session. Its the experience and ability to detect subtle nuances of "energy flow" during the session to conclude whether the session was successful, its length, or whether to organize subsequent sessions with some subgroup. Did all participants have possibility to express fully their opinions? (And did they do so?)

We can also use computers as support tool for brainstorming. Modern conference room have either large displays or allow to project screen using data projector. Writing down ideas in the computer introduces flexibility to the session, which was not available before. Is some part of the diagram getting out of hand size wise? Reorganize the diagram by dragging and dropping it parts, or even create sub-diagram to separate more complex parts of it. Use links, icons, free form notes ... None of it is available when using blackboard.

While we can use general purpose graphical program for that, it is more efficient to use *mind mapping software*. Mind maps (as a technique) have originally developed by Tony Buzan [40] as a tool to better understand problems, make notes, learn new things, etc. His original idea was to use mind maps more as self-help tool. But with proper software packages, we can successfully use the approach to support our brainstorming sessions.

While it is possible to draw mind maps in hand, see fig. 8.5 for artistic take on it, software is usually preferred.

There are many software packages available, some are paid like MindManager [41], some are also available free of charge, such as FreeMind [42]. In comparison with hand made mind maps, the software generates more visually unified maps, maybe even a little bit boring. But we are able to generate them fast, which is huge benefit of using software.

There are significant differences between creating mind map and brainstorming session. Usually only one person works with mind map at time - the person sitting before the computer. In comparison brainstorming is group effort.

Software also allows to create interactive mind maps, link them together, or link them with other resources - be it documents, images or videos.

As you can see on fig. 8.6, we are starting with single noun describing our problem. From it we derive various aspects of the problem and describe it again by noun. The number of the nodes in the map is virtually unlimited. Even large mind map is easy to work with, because the nodes can be

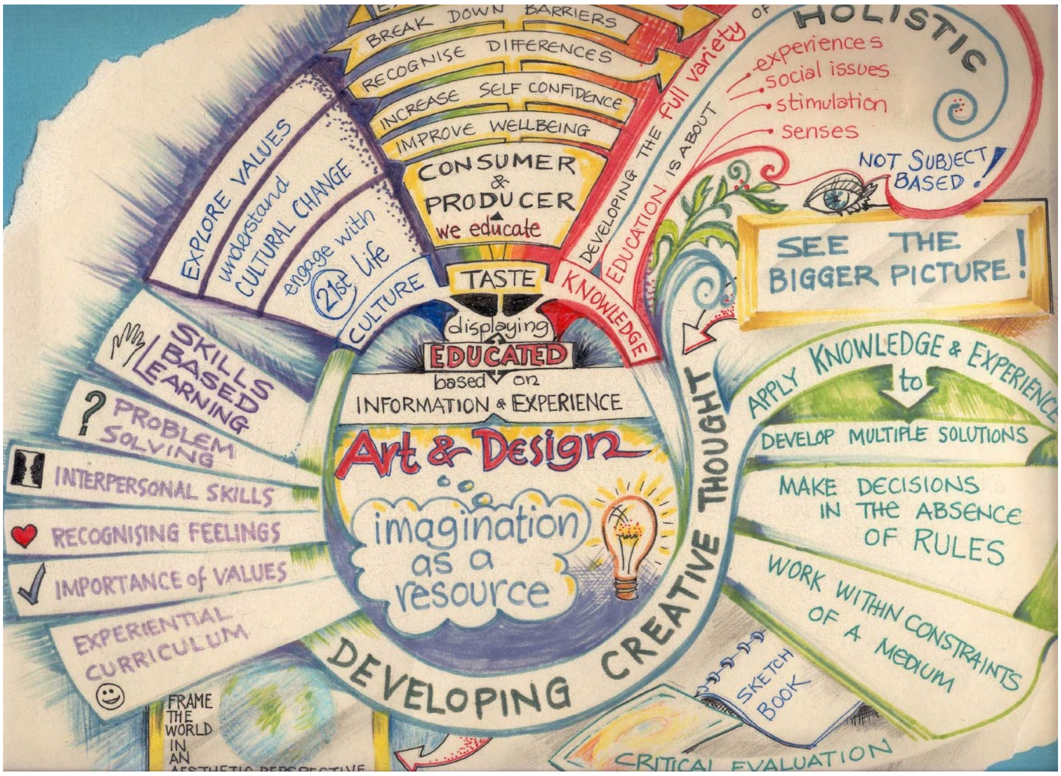


Figure 8.5: Mind map of Art and Design (Courtesy of [8])

Mind map example
Let's try to construct short mind map around the theme – modelling of decision processes, see fig. 8.6.

collapsed or folded to hide large parts of the map, when you do not need to see it.

We can link different mind maps together to form huge, self describing structures. Because we can also link leaf nodes to the documents on local computer or on network (LAN or Internet), the mind map is not limited only to interconnected brainstorming nodes. Better orientation is also possible by using icons visually describing the node.

FreeMind as a tool written in programming language JAVA is capable to function as JAVA applet and allows for diagrams to be viewed inside web browser over the Internet. Of course, that on the other hand also means, that you will need the Java Runtime Environment (JRE) installed. Fortunately Oracle provides JRE free of charge for many platforms at its dedicated web page [43].

There are available many more free or proprietary software for this tasks.

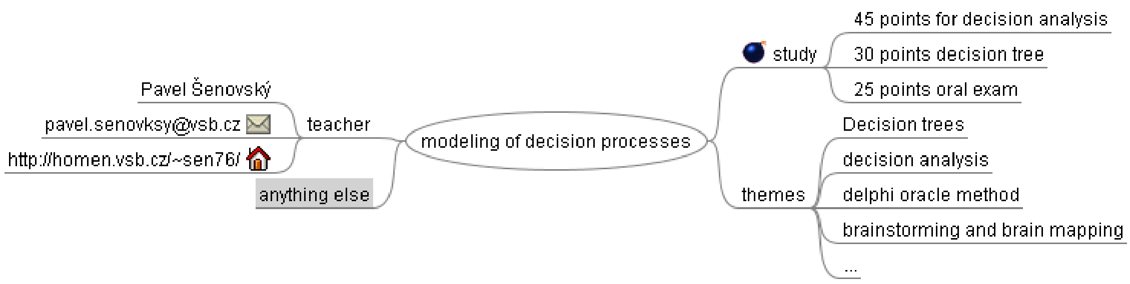


Figure 8.6: Mind map example

Create mind map
Download FreeMind (or any other freely available tool for the task) and create mind map of emergency: accident of tank transporting methane under pressure.

Questions

1. What is mind map good for?
2. When in decision process should brainstorming session take place?
3. What is the role of team leader during brainstorming session?
4. What is the connection between number of participants and number of ideas generated during brainstorming session?

Summary
Delphi method is one of methods we can use in case no or not enough hard data are available to support our decision. The method is based on problem assessment by experts. Analytic prepares the questionnaire to guide the experts through the problem. The questionnaire is sent to selected experts in problem's domain, who will answer the questions. Analytic then collects and processes answers. In case no consensus is reached by the experts, another iteration of the method is needed. In this case the existing answers are provided to guide experts in their thought process. Since the method is expert based, it takes a lot of time, and has large requirements on quality of supporting documentation and abilities of analytic to process the answers and interpret them correctly.

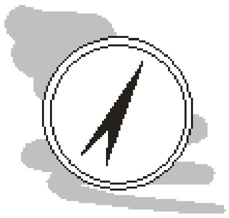
Brainstorming is well known method usable for variety of problems, especially in beginning phase of decision making. We can use it to better specify or understand the problem, its various aspects and summary the information in one place. Brainstorming is a team method, during which leader writes down ideas or opinions of brainstorming session participants on the topic.

Mind maps are usable for organization of notes, ideas, information, etc. Each map has a main topic of the map in its center. Map then works on this topic and develops it to the sides in form of trees of ideas/notes describing given aspect of the topic. We can also use mind maps as a support tool for brainstorming. Mind map is used as replacement to blackboard.

To improve speed of mind map creation, the maps are usually visually standardized, when using software to create them. Most maps today are created using such software.

Chapter 9

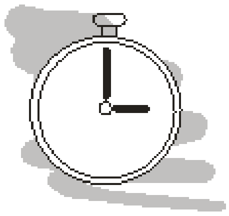
Surveys



Chapter outline
In this chapter we will discuss the surveys preparation and its evaluation using statistical methods. Surveys are one of most often used methods for information gathering on almost any topic across research fields. That also means, that you should know a thing or two about them too.

After finishing this chapter you should know

- what are the surveys and how to make them (at least the basics of it)
- what kind of problems can we meet when evaluation results of such surveys
- what methods do we have available for the evaluation



Time required for study
To study topic presented in this chapter you will need approximately 3 hours. Thou if you are not confident in basics of statistic you may have problems understanding presented concepts of the surveys. That may lead you to invest additional time to refresh basics of statistics.

Only basics of statistic are required so any textbook on the topic will suffice.

Surveys are being used extensively almost every day, for variety of problems. We can see its results in the news, Statistical offices of the states use them and they are also often being utilized as an input for theses, dissertations, research and many other things. So, since we realize these surveys so often, it is an easy thing to do, right? Well, not really, while creation and evaluation of the survey is not a rocket science, it is easy to do mistakes which can lead to errors in interpretation of the results, provide misleading or strain wrong answer.

Also if organization and processing of surveys was so easy, there would be no need for specialized firms to realize them. Since there are such firms and they strive today more then ever, we can presume that surveys can be (and are) complicated. So let's start from beginning - by creation of the survey.

9.1 Introduction to survey creation

Basic obstacle in surveys, both creation and evaluation is that for uninitiated lot of problems is not obvious - intuitively derivable, apparent on first sight. To understand them we will need to define some basic terms first.

First term is *population*. In statistics the term has specific meaning which is different then the population definition in for example course *Civil protection*. In civil protection we see population as residents and persons working (or being) at given time at given place, be it state (state population), continent or region. In context of surveys we define term differently as part general population, we are interested in from point of view of the properties being explored by the survey. So we are never speaking about everybody (entire population). We are always working with some subset of it.

The survey itself is also usually a "*sample survey*" - that is, data are not collected on the population of interest as a whole, but only on a sample of it, which we refer to as a *representative sample*. It is not that the full sample of the population cannot be measured (using a questionnaire), but it is done only in limited cases, e.g. when the population of interest is small to make the survey financially manageable (e. g. a survey of regional directors of the Fire Service). A full survey of the population of a country is carried out, for example, by the Český statistický úřad (Czech Statistical Office) (CSU) as part of the Population census [44][1]. However, this is a process so demanding, even for state, that it can only be carried out once every 10 years.

For purposes of this textbook, we will see representative sample as such sample of the population, whose characteristics, we are interested in, are statistically similar with population as whole. Meaning, that we can use results of the survey we can describe some characteristic of the whole population.

We can describe the situation in graphical form, see fig. 9.1.

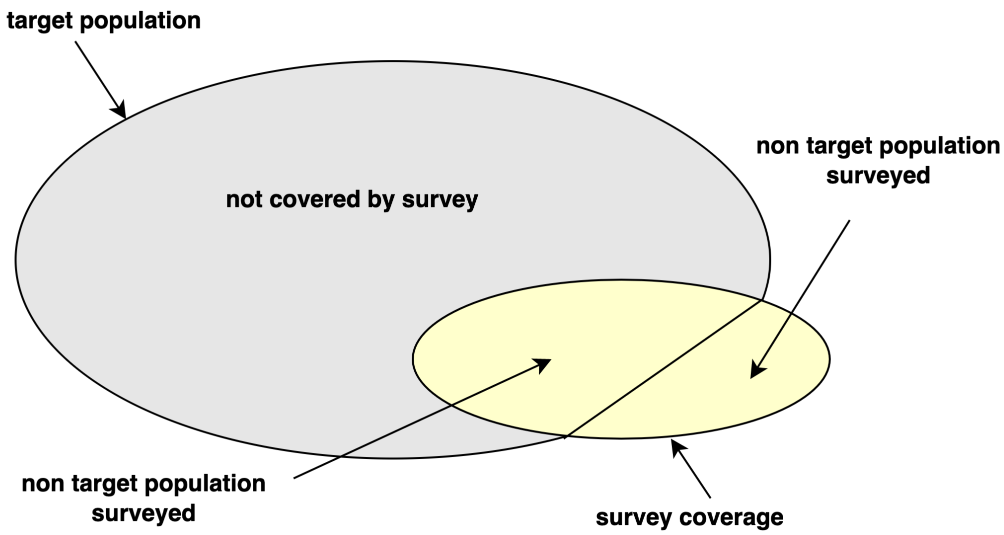


Figure 9.1: Sampling frame for the survey (adapted from [9])

From fig 9.1 we can deduce some problems. The survey must be prepared in such way to target target population **only**. So for example if we conduct survey on awareness of public safety in selected part of the city we are interested only in respondents who either live there or work there (generally spend significant time in there). But if we randomly ask people on the street, we can also ask people who do not belong to this target population. For example they can be tourists, or they randomly go through the area for shopping. If we were not able to identify such respondent, using such answers would influence results of the survey as whole. If there was a lot of such respondents, it could even invalidate the results.

Opposite scenario, when our sample size population is not large enough to be reasonably representative of true population, is also possible. One of first question, we will need to answer would be, how large the sample size should be to be representative. This question needs to be answered ideally in advance. We can also compute it afterward, but there will be no guarantee, that the collected sample size is large enough. Question then would be, what to do if we compute that our sample size is too small? Are we going to throw out results, accept larger then anticipated error, try to collect information from more respondents?

Preparation is a key in this case. Computing required sample size will allow us to plan our survey campaign including required resources, and will also tell us when we can stop.

The survey itself is realized in following steps:

1. develop survey
2. data collection (realize the survey)
3. data edit
4. data correction (dealing with missing values)
5. data evaluation

[1] Census of People, houses and housing

6. publish results

Survey design

The survey always tries to achieve some kind of goal. So the first thing we need to do is to answer the question *what we need to find and why*. Answer to this question will serve as basis for *variable* selection, whose value we want to find using survey. We formulate our questions to find them.

Some survey prefer to also formulate working hypothesis before realizing the survey. Survey then has a potential to confirm or disprove it. Reason for this is that without such hypothesis, we will see pattern in data anyway. Intuitive approach would be to describe the pattern and formulate hypothesis explaining it.

Such observed pattern may be result of the proposed explanation or may be just a random pattern emerging from data used and is not representative (does not apply) to explanation of analyzed phenomenon. Think of it similarly to the patterns we are able to see in the clouds. It is just an explanation our brain is providing us, just the noise.

But if we formulate the hypothesis before starting of the survey and then see expected pattern, supporting our working hypothesis, then we are maybe on to something, because probability of such event happening randomly is much lower.

Another way to think about the problem is, that if we formulated hypothesis based on survey, then we will need another survey to confirm (validate) such hypothesis.

Not all surveys require such hypothesis. The goal of survey might be to find previously unknown distribution of some variable. For example we may be interested in some preferences mapping in population, or how many people have certain knowledge, or equipment, etc. In such a case, however, even greater demands are placed on the analytical part of the survey, to ensure that the data collected are appropriately "mined out".

There are variety methods to use for such purpose with entire sub-field of informatics called *data mining*. Discussing data mining goes beyond scope of this course and textbook, but you can get acquainted with some of these methods in courses like Statistics, Informatics in Security, Artificial Intelligence and others.

There are two basic types of variables:

- target and
- supplementary

Target variables are directly related to survey's goal. Quantifying (finding for non numerical) these is basic requirement to consider for the survey to be successful. It is a must, without these we will not be able to meet stated goal of the survey.

Supplementary variables are not directly related to survey's goal, but these variables can help to stratify the dataset compiled from survey, or to identify respondents who are not part of target population. Logical next step would be to remove such respondents from the dataset.

Such stratification is also useful for proving that the measured sample of the population can be considered representative sample of true population.

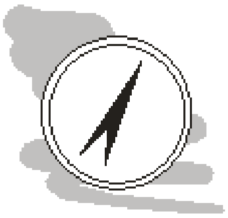
Typical examples of supplementary variables are age, sex, marital status, relation to place (resident, going through, ...), employment, specialization, etc.

The questions need to be formulated in such way, that they are short and easy to interpret for the respondent (amateur). This requirement is especially important if the respondent fills in the survey unassisted. Typically that is the case for on-line surveys, but sometimes it is also case for in attendance surveys. Clerk only serves to give out and collect survey forms.

The respondent does not have anybody to ask if (s-)he has some question. Respondent will deal with the question somehow in the end, but there is no guarantee, that question's interpretation will be same across all the respondents. Which is undesirable property, especially if we are not aware of it.

To properly fill in the survey even such minor things like the order of the question may be significant. The survey as whole must function in wholesome/consistent manner. That means that we need to group the question based on its topic and order them to form certain "story". We need to remember, that previous questions will influence somewhat the respondent. That may be desirable, if we constructed the survey to utilize it, but it may become major source of errors, if we didn't.

When designing survey we need to also understand that our respondents are usually unpaid volunteers, who value their own time. That means that with increased number of question and needed



Cognitive bias
When formulating the questions for survey, we need to also consider the possibility, that the respondent will not be impartial to the question, or more specifically answer to this question. We are already aware of the problem as we discussed it in *Delphi method* section of previous chapter. So if the term *cognitive bias* seem unfamiliar to you, now its time to revisit the topic.

to fill them, willingness to participate will decrease. We can also expect the respondents to be less inclined to answer questions perceived as personal or sensitive. Examples of such could be age, health or income. That is a case event if the survey is anonymous.

We can somewhat alleviate negative perception of such question by putting them in the end of the survey and marking them as voluntary. Adding option "I prefer not to answer" may also help. It will also allow us to easily filter out such answers when evaluating them. If we do not present escape option to the respondent, (s-)he may be more inclined to choose random or intentionally wrong answer. Which we would not be able to easily identify, thus introducing further error into the dataset.

Data collection
In this phase we have prepared the survey and start with its deployment. The preparation may be short, for example we may need to print the survey and publish it on Internet. But in case we are planning to perform assisted survey, the preparation will be more complicated. We will need to select the future assistants and educate them on rationale of the questions asked and the way they should deal with expected problems.

Apart from explaining the meaning of the questions (and answers), the training can also focus on the way of filling in the questions. For example some answers may narrow down number of questions - in other words some questions in context of our knowledge about the participant gained answering the questions may make some questions meaningless. For example if we learn in one question that the respondent commutes to work using his/hers car, that it would be pointless to ask on quality of city's mass transit system.

Finding possible walkthroughs through the survey has also one positive feat in that it limits number of question the respondent needs to answer. But if the walktrough is set in a wrong way it may lead to state when we will force meaningless questions on our respondent and possibly invalidate data gained that way.

To better understand various paths through the survey, it may be good idea to visualize it. If the survey is simple, such visualization will bring only small or no benefits. But for complex surveys with interconnected question visualization can be invaluable tool for both design and validation of survey implementation if we are using software or on-line form.

We can use flow chart [45] or activity diagram from Universal Modeling Language (UML) [46]. Simplified example of possible survey traversal are available on fig. 9.2. Resulting diagrams from flow chart and activity diagram are similar, but not the same.

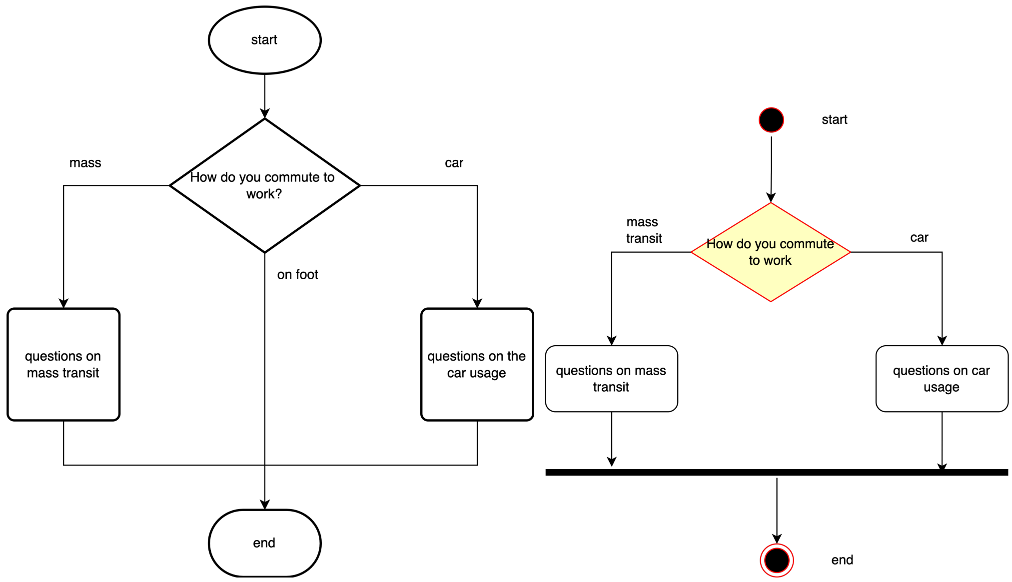
As you can see on fig. 9.2 ve can use UML's activity diagram as a logical extension of the flow charts. UML is also much more modern standard supporting variety of other diagrams for various aspects of system analysis.

Data editing
By *data editing* we can mean a lot of things. We can mean a process in which we transcript data from paper to electronic forms for further processing. Editing may also mean correction of detected errors.

There are three basic types of errors in surveys:

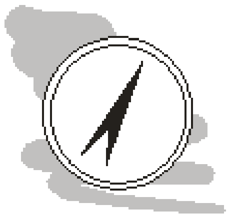
1. range errors
2. consistency errors
3. errors in traversal of the survey

We can at least somewhat check validity of the variables against in advance set limits for the variable. If the variable's value is outside these limits, we talk about the *range error*. For example age over 100 is improbable for respondent, but possible, age over 130 is unfortunately not possible at this time. If the responded was expected to choose from preset list of options, we can check, if (s-)he really did so and didn't create another type of answer on his/her own.



(a) Flow chart (b) Activity diagram

Figure 9.2: Flow chart vs UML's activity diagram



UML
You can learn basics of UML language in course *Artificial intelligence in security*, you can choose during your study.

By *errors in consistence* we mean the situation when the survey has been filled in, but the answers contradict each other. For example person with age: 8 and marital status: married is in Czech Republic basically unheard of. One or both answers then are probably wrong. These contradictions need to be detected and dealt with.

In fact, we have only limited number of options available, if something like that happens. If the survey was not anonymous and we have contact information, we can do follow up on survey and amend it. If the survey was anonymous, than we can only decide to throw out contradicting answers or all answers by the respondent.

Errors in traversal happen when the survey is wrongly implemented, or the assistant makes a mistake in guiding the respondent. This result in situation, when some questions which should have been answered were not asked, while others we were not interested in were answered.

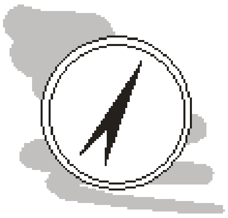
We deal with these errors in similar way as with consistence errors - so either amend it if possible or throw out part or all answers for respondent.

For this type of error it is also important to identify reason for error. Was the question sequence wrongly encoded? If the survey continues we might want to correct on-line form and rules for questioning themselves. For assisted surveys human error is possible. If we see that the assistant has higher then usual level of errors in surveys it may be good idea to train him/her further or to fire him/her if the errors are presumably result of negligence for example.

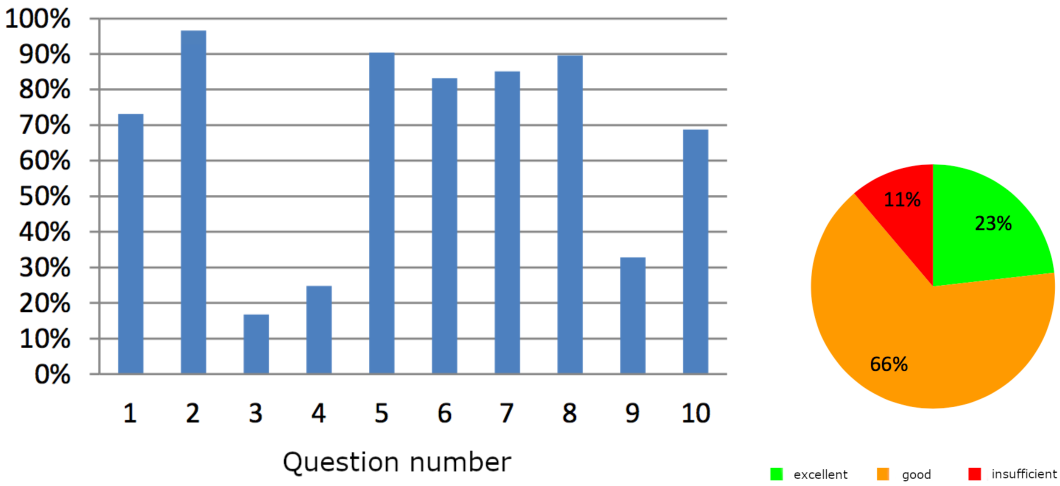
9.2 Basics of survey evaluation

The moment we get the task to evaluate the survey, most of the people images something similar to graphs on fig. 9.3 [10].

There is nothing inherently wrong on using the charts, on the contrary - we recommend it wholeheartedly. Graphs on fig. 9.3 are taken from thesis of Ing. Dratva [10]. The survey has been realized to estimate level of knowledge in area of civil protection and crisis management among the secondary



Please note - examples used
The examples used in this section are only illustrative, they are intended for demonstration purposes of the evaluation process only.



(a) Bar chart (b) Pie chart

Figure 9.3: Bar and pie charts (courtesy of [10])

school students and among normal population.

Bar chart represent percentage of correct answers in the questions. So what is the problem, if there is a problem? Well, the graph itself should be at least somewhat self-sufficient. By self-sufficiency we mean, that if we look at the chart we should be able interpret results it presents without need to study accompanying text extensively. It should communicate clear message.

We have only limited amount of tools available for the purpose:

- graph label (i.e. *Fig. XY: Results of civil protection and crisis management evaluation among secondary school students in 2014*)
- main heading of the graph (i.e. *Knowledge test: civil protection and crisis management - secondary schools*)
- labels graph's axes (i.e. X axis: *question number*, Y axis: *correct answers [%]*)
- legend

In context of what we know about the survey carried out we could replace bar chart from fig. 9.3 by different type of bar chart, see fig. 9.4. We can use this type of graph also in case we need to split the result into more target classes then 2, as in our illustrative example.

The graph should not be only way we present the information as it doesn't contain all the information for reader (report user) to form adequately his/hers opinion. Data should be presented also in tabular form. Survey results in graphical and tabular form are usually the most interesting part of the report and also most useful. Graphical form is often more connected to the intend of the survey and its goal. But it is not useful to think, that we know in advance full value of collected data. Perhaps it can be used in the way we did not think of. Or they may present additional values when integrated with other datasets.

So the reader might want to use the data in different way. If done properly, part of original data can be reconstructed from the graph. But it is lot of work and there is large potential for errors. If the data are also available in tabular form, such reconstruction is not necessary.

Therefore we recommend using both forms, thou data tables may be stored separately from graphs in report annexes or even entirely separate from report itself using document repository. In this case report must contain information on how to reach the underlying data.

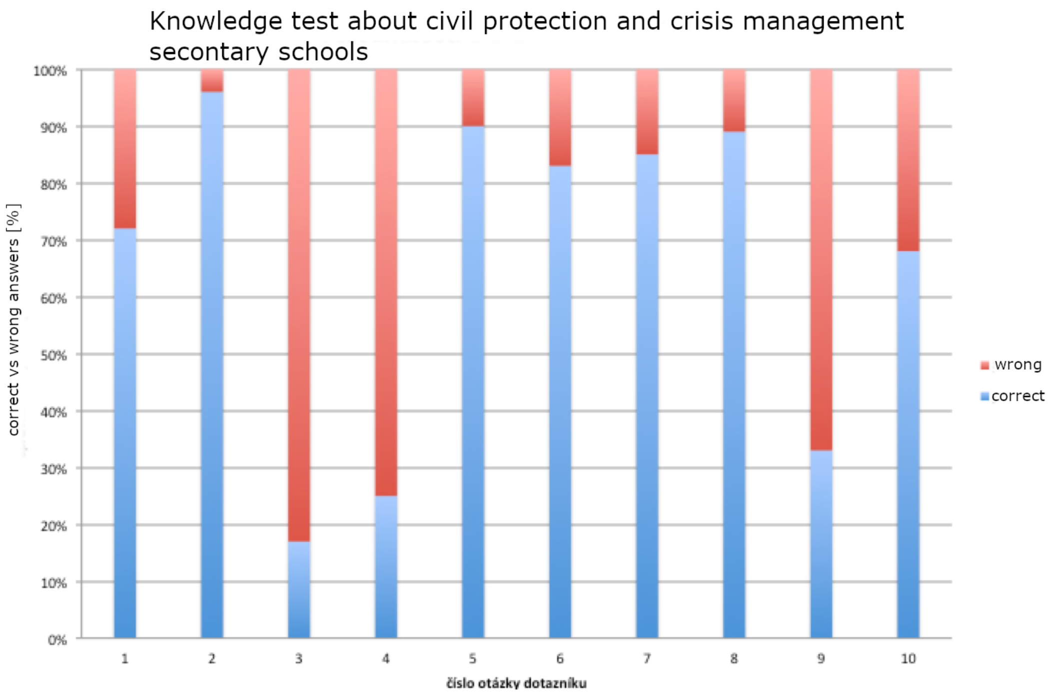


Figure 9.4: Results of Knowledge evaluation in civil protection and crisis management among secondary school students in 2014 (data: Dratva [10])

We can demonstrate the problem on question 1 from fig. 9.4, try to estimate the percentage correct answers. Was it 72 % or maybe 73 % of respondents? Without underlying data we will never know for sure.

If we process the results separately for secondary schools and adult population, similarly to the fig. 9.3 or 9.4) we can estimate level of knowledge prevalent in area of civil protection and crisis management in these target populations. Requirement for such stratification is existence of variable in dataset specifying whether the respondent is student or adult. That was a case of the study we are using for demonstration purposes.

What if we need to estimate properties of the population as whole? Such population has both students and adults. Can we just use the whole dataset without stratification prepare graphs and be done with it? Well we can do that, see tab. 9.1, but what does it tell us exactly?

Table 9.1: Students vs adults (data: Dratva [10])

evaluation	students [-]	students [%]	adults [-]	adults [%]	total [-]	total [%]
excellent	29	23	8	31	37	24,6
good	82	66	13	52	95	63
sufficient	14	11	4	17	18	12

The *total* value is very problematic in tab. 9.1, because statistically it does not make any sense. To derive correct conclusions from data and integrate these values to form meaningful total, we also need to consider:

1. size of population sample - in our case 125 students and 25 adults
2. size of real population (we are interested only in the target population, to be specific) - students on secondary schools in ČR in school year 2013/2014 *313 130* [47], number of adults in ČR is *8 665 578* according ČSÚ [48]

So according to this information we covered by survey 0.04 % of secondary schools students and 0.0003 % of adult population. In other words secondary school students form 3.5 %, while adults form 96.5 % of target population. Presuming that the respondents of our survey form representative sample of its part of population, we can adjust the sample data using eq. (9.1).

$$V = \frac{\sum_{i=1}^{N} w_i v_i}{\sum_{i=1}^{N} w_i} \tag{9.1}$$

where V - is the value of the variable for whole population, N - number of samples, w - weight of sample in target population.

We can demonstrate the computation in our case to recompute size of population with excellent knowledge in civil protection and crisis management:

$$V = \frac{3,5 \cdot 23 + 96,5 \cdot 31}{100} = 30,6 \tag{9.2}$$

Logically the overall result must be very close to the results for the adults as they represent majority of the population.

In the text we several times presumed, that the sample of the population is representative, but we did not prove it. We even didn't explore how large the sample size should be, for it to be representative. Related question would be, if I have a sample of certain size, what level of accuracy regarding derived information from it can I expect? These questions are very important, so let's try to find answers for these, shall we?

First we need to understand, that whatever we "measure" in the survey will be only an estimate of true value in target population be it average, mean or anything else. Difference between the estimate and true value is given by fact that we deal with much smaller sample of population then the target population is. So statistically the sample population, if representative, is similar, but not same as target population.

As a guideline we can use for example confidence interval estimation of the error of the mean, see eq. (9.3).

$$E = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \tag{9.3}$$

where E - error, z - critical value of normal distribution for chosen confidence level α, σ - standard deviation, n - number of values measured.

We can derive easily sample size n from this equation for chosen confidence level, see eq. (9.4). In other words, if our sample size will be $> n$, then we can expect the results in be reliable at chosen confidence level α.

$$n = \left(\frac{z_{\frac{\alpha}{2}} \sigma}{E}\right)^2 \tag{9.4}$$

Equation (9.4) is usable for numeric values with known or estimated standard deviation.

If our data are of ordinal type, we were using such data in our illustrative example, we need to approach the analysis in slightly different way, see eq. (9.5).

$$E = z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \tag{9.5}$$

where p is the estimate of the variable, other mathematical symbols are same as in eq. (9.3). From that we can easily derive necessary sample size n, see eq. (9.6).

$$n = \frac{z_{\frac{\alpha}{2}}^2 p(1-p)}{E^2} \tag{9.6}$$

Let's apply the approach on estimate of number of adults with excellent knowledge in civil protection and crisis management. We will work with the data used in our illustrative example. Usual requirement would be for estimate to be valid at confidence level 95 % and expected error, let's say 4 %. The 95 % level is standard, the 4 % error is more our choice. For variable estimate p we will use value 31 % as measured in original survey.

$$n = \frac{1,96^2 \cdot 0,31(1-0,31)}{0,04^2} \doteq 514 \tag{9.7}$$

We computed, that to evaluate the property present in 31 % of population, in confidence interval of 95 % a expected error 4 %, we need to use sample size of 514 respondents, presuming that all of them will respond and we will not be required to exclude some of these due to uncorrectable errors in their answers.

Since our sample size was 25 adults and 125 students ... the results suddenly do not look so good. Sample size of the survey is simply too low. We have unrepresentative results and try to derive some knowledge from it and form some recommendation based on the survey. Validity of such recommendation would be very questionable at best. To prevent such questioning, we plan the survey to improve our chance on success!

Yes, its lot of work, but we will get results we can trust and we can safely build our recommendations on them.

The problems may also be introduced with incompetent data processing, visualization and interpretation.

Students often think that visualizing gathered data using bar chart and provide some trivial explanatory text to it is good enough for any kind of work. Well, it is not, but it is good start. We need to use gathered data, derive some conclusions from them. It may be possible to further process the data, maybe enhance it using other data sources.

We also need to compute some statistical measures for the data to understand how the data are distributed. These information improve interpretation context for the data.

Remember that even simple statistical measure will provide misleading information if rationale behind it is not understood properly.

Good example of such problem would be using statistical measures intended for continuous variables, but apply it on variable with ordinal scale. Such scale is often labeled numerically, but the scale does not represent numerical (continuous) variable. Technically the computation using this in-proper mathematical apparatus will yield the result, but it will be wrong.

So error and confidence level for sample size
Considering our illustrative example, sample is clearly not representative for chosen error and confidence interval. Try to experiment with the computation and find confidence levels and error for which the sample size would provide valid answers.

Statistics
If the previous paragraphs in this chapter sound like gibberish to you and you cannot understand of how we come to our conclusions, now its time to return to your study materials on *Statistics*.

Summary

Survey organization and evaluating its results is not an easy task. In preparation phase it is necessary to focus especially on setting appropriate goal for the survey and prepare the question to allow us collect the data to reach this goal. Questions should be short, to the point and clearly interpretable.

When we release the survey, we need to realize, that we will always measure only sample of target population, that is why we need it to be large enough for the results to be considered representative for this target population. (The results to be statistically significant.) When choosing the sample size, we need to consider target confidence interval and error and also the prevalence of searched for measure in the population.

We often visualize the results using graphs, such as bar or pie charts. Survey results should be also available for reader in tabular form, either directly in text accompanying the graphs or in annex. If the resulting dataset is stratified using some variable (i.e. education level, sex, age, etc.) we need to consider true size of these sub-groups in target population and transform measured values to com-form it.

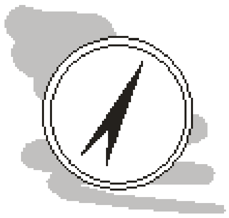
Numeric values need to be further processed, meaning we need to compute standard deviation and use it to interpret the data.

Self-test questions

1. What do we mean by target population?
2. Define term representative sample.
3. Explain the process of adapting in survey measured values to size of target population.
4. Explain what impact will have choice of confidence interval and error on sample size.
5. What is survey traversal error and how to solve it?

Chapter 10

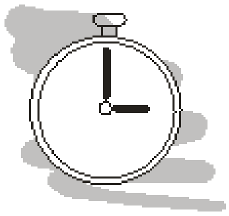
Network models



Study guide
In this chapter we will acquaint ourselves with application of the network models for description of the activities as part of the project realization as a basic tool of manager for identification of problematic activities inside of project – allows him to redevelop them.

After reading this chapter you will
Know

- what the network graph is
- what the progress chart is
- how is used CPM method



Time required to study chapter
For getting through the theory in this chapter you will need at least an hour, but if you try network models practically, reserve few hours for your experiments.

10.1 CPM method in project management

Network models are commonly used models for analysis of so called *Critical Path Method (CPM)*. Network models are also used for visualization of interconnected project activities, for analysis of distribution networks etc.

When talking about distribution network, the configuration of network is derived from physical configuration of modeled network. When modeling project, the configuration of model is derived from activities necessary to finish the project, time required to finish them and connections between them.

Before we dwell into network model construction, let's define several terms first, we will rely on them on next few pages. First of them is management. The term *management* is derived from French word manége - as circular arena for horses training or French word managere – in English management. There are many definitions of management; one of definitions used more often is that management is completing the goals using others. Another definition of management is that management is mobilization of all company's resources to accomplish planed goals.

Management as science branch is considered to be art, that's why it is not completely possible to for "hard" mathematic methods to hold all the answers we need. Such methods are usable as important support tools. Its usage will not enable the manager to magically succeed whatever he is doing, but if used properly it can increase his chances for success.

Another term we will discuss in this chapter is *project*. Project means chain of activities, which lead to the goal of the project. For each project we have clearly stated start and finish dates, we also have only limited resources to achieve the project goals.

The activities of the project have to take place in certain order. It is possible to express such order in tabular form. Let's demonstrate the approach on example of simplified building construction (see table 10.1).

Table 10.1: Activities of the project

	activities	followers	Predecessors	Length [days]
A	**Foundation excavation**	B	-	2
B	**Concreting of excavations**	C	A	10
C	**Fabric of the building**	D	B	17
D	**Roof**	E,F,G,H	C	5
E	**Electro installation**	I	D	5
F	**Piping (water, gas)**	I	D	5
G	**heating**	I	D	5
H	**windows**	I	D	14
I	**Facade rendering, flooring**	J	E,F,G,H	21
J	**Building acceptation**	-	I	1

The project represented by activities as on tab. 10.1 and figures 10.1 and 10.2 is very linear. The reason for it that we simplified the example little bit too much - it is scholarly example to demonstrate the process after all. In real world even a simple project can easily have much more activities and connection between them. Resulting graph of such project would be much more complex.

We can use sum project for purposes of demonstration of the approach and scale it up to cover more realistic real-world projects. Such projects can have hundreds or even thousands of interconnected activities.

Rule of thumb for tool usage it the more complex the project is, the more the manager can benefit from using project management support tools.

Let's go back to our simple example project and focus some more on dependencies between the activities.

We can describe the activities sequence by specification of followers (activities which have to follow after finishing present activity) and predecessors (activities which have to be finished before present activity can start).

The both approaches are completely equivalent by its meaning; they differ by the direction of construction. When considering followers we are going from start to finish, while when considering predecessors we do opposite – going from finish back to start.

Some software tools for project management such as Microsoft Project [49], open source Libre Project [50] or Open Workbench [51] allow the user to choose follower or predecessors approach as needed or as preferred by user and even to change then between each other – followers and predecessors will convert accordingly.

There interdependencies form basic structure for the project's network. This type of dependencies can be also considered as structural or direct.

Another type of dependencies can be covered by introducing resource management into the project. Resources form certain constrains on our ability to proceed with the work on activities. Resources can be:

- human resources (workforce) or
- material resources.

Each activity will have its own resource requirements and we will need to fulfill it to finish the activity. The resources can be shared across the activities. In such case using it on one activity may make it unavailable to another activity. Logically, we will need to push forward finishing date for such activity and perhaps even push forward overall finishing date for the project as whole.

List of the activities in tabular form is not entirely usable for operative control of activities progress, resources planning etc. That's why the graphical interpretation of this information is actually chosen, in the form of *progress charts*, sometimes also called *Gantt chart* and *network graphs*.

As you can see on figure 10.1, the red path here is so called *critical path*. By critical path we mean sequence of the activities, for which any increase in the duration over the planed time would automatically lead to increase in the time needed for the project as whole. Because of this property it is necessary for manager to especially thoughtfully manage these activities.

Software support

Creating the diagrams by hand is tedious task, which takes time and you can easily make a mistake. That's why there are many software tools to support us in project management.

The leader in the branch of project management is Microsoft with its MS Project Suite [49]. It is proprietary software and is usually considered as standard for such type of the software. Proprietary nature allows MS Project to cooperate with other softwares in MS Office family and with Exchange to take project management one step closer to the project members by planning their work task and propagating them into their own calendars in MS Outlook.

The are also available various other softwares to support us which are open source. Example of such could ProjectLibre (formely known as OpenProj, [50]). ProjectLibre is available on practically any platform (MS Windows, Linux or Mac OS). Alternative could be Open WorkBench [51], but only MS Windows operating system is supported and it has far less features then ProjectLibre.

You can find practical, hands on example in ProjectLibre in next section of this chapter.

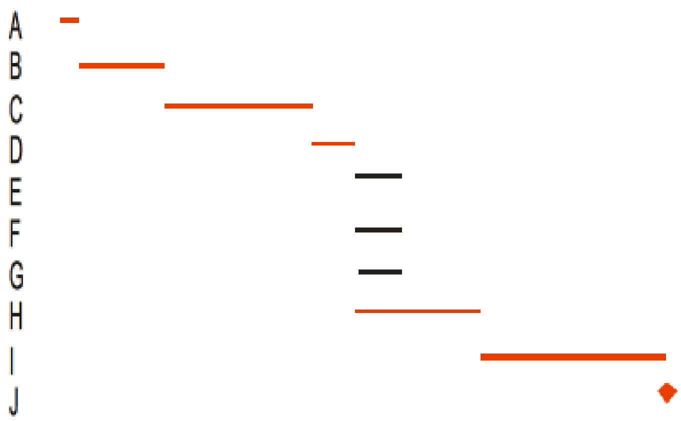


Figure 10.1: Progress chart of the project

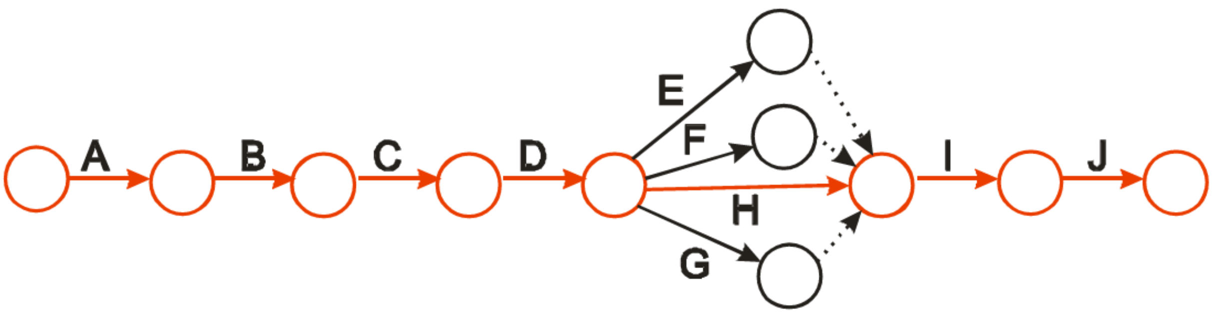


Figure 10.2: Network graph of the project

On network graph the activities are represented by the edges. Edges E, F, G, H may occur at the same time, so they have to be finished before activity I takes place. To demonstrate it we add so called virtual activities (look for doted arrows on figure 10.2).

These virtual actions do not represent real activities; we add them to graph only to improve its readability.

We can also note critical path in the graph. On figure 10.1, the critical path is noted in red color.

Maybe you ask why we use two types of graphs for single task (identification of critical path). The answer is that we use them both because identification of critical path is only one task the graphs are meant to solve. Primary goal of the network graph is to visualize interconnection between the activities of the project. Primary goal of the progress chart is to visualize activities from point of view of the time to allow its optimization. For this task all non-critical actions may be used, since we can increase the time needed for such non-critical task by sparing some resources (as people hours) and use them somewhere else, where they are perhaps needed more.

It's obvious that the time increase for noncritical activities can't be infinite, because by increasing length of the activity it may become critical under these changed circumstances.

Advantages of progress chart usage:

- identification of the critical path
- optimization of resources usage for non-critical activities
- overview of realized activities as a good source of information for effective management

Critical Path Method
Compute length of the critical part for example we worked with in this chapter.

Critical Path Method - solution
Possible paths
1 – A,B,C,D,E,I,J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61
2 – A,B,C,D,F,I,J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61
3 – A,B,C,D,H,I,J = 2 + 10 + 17 + 5 + 14 + 21 + 1 = 70
4 – A,B,C,D,G,I,J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61

Critical path is the path 3 (A, B, C, D, H, I, J), project will last for 70 days.

Questions

1. What is CPM method?
2. Explain interconnection between the project activities from point of view of the follower and predecessors.
3. Why do we use both progress chart and network graph, wouldn't it suffice to use one of these?
4. Why do we try to find critical path?

10.2 ProjectLibre – software support to project management

Study guide
In this chapter we will be using ProjectLibre for project management, but most of what is written for it should apply also to any other project management software. Though GUI will be slightly different. So if you have installed different tool, you do not have to install ProjectLibre, if you do not want to.

First we to download the ProjectLibre from its homepage on SourceForge [50]. To beginners I recommend downloading binaries of Project Libre intended directly for your operating system. But, if you are inclined so, you can also download source code for the project and compile it yourself. Source code is not required if you use binaries.

To run the software you will need either JRE or Java Development Kit (JDK) installed. Since JRE is available for Java 8 only and is considered heavily outdated, I recommend using JDK from `https://jdk.java.net/18/`. You can also use newer version of JDK. Look for GA (general availability) version. On Linux you may also use distribution's package system to install Java. On MacOS I recommend using Brew.sh (`https://brew.sh`) package system and while you are at it, you may also install ProjectLibre using Brew. We provide short installation script for your convenience.

Listing 10.1: Install JDK and ProjectLibre on Mac using Brew (use in terminal)

```
brew install --cask oracle-jdk
brew install --cask projectlibre
```

Remember that after installation you will have to maintain installation of JDK. If you used package manager, it is trivial, for example brew has update and upgrade commands to do that automatically for all formulae and casks installed using Brew.

Listing 10.2: Update formulae and casks installed using Brew (use in terminal)

```
brew update
brew upgrade
```

Please note, that if after update system doesn't find outdated packages, it is pointless to call upgrade, as there is nothing to upgrade.

On Windows both installation and maintenance is manual.

As of time of writing this text (May 2024), latest available version of Project Libre was 1.9.3 and latest GA version of Java was 22.

According to the authors of ProjectLibre new version of this software should be available somewhere later in 2024, so it may be a good idea to watch for its availability on its homepage.

After downloading and installing of the software, we may start to use it. After opening ProjectLibre we have to create new project as we do not have existing project yet.

When creating new project, we have usually specify the project name, project manager and possibly some notes for us, see fig. 10.3. In fact writing these informations is not required, but it is considered as a good practice. Also different software suites support different project properties. For example MS Project allows you to specify what kind of calendar will the project use (common office format – 8 hours a day, 5 days in week, or will it be 24/7).

In this chapter we will try to recreate the example from previous chapter, so lets call the project: Family house – construction. We will specify ourselves as project managers. You also may or may not want to add your own note. Clicking on OK will create the project.

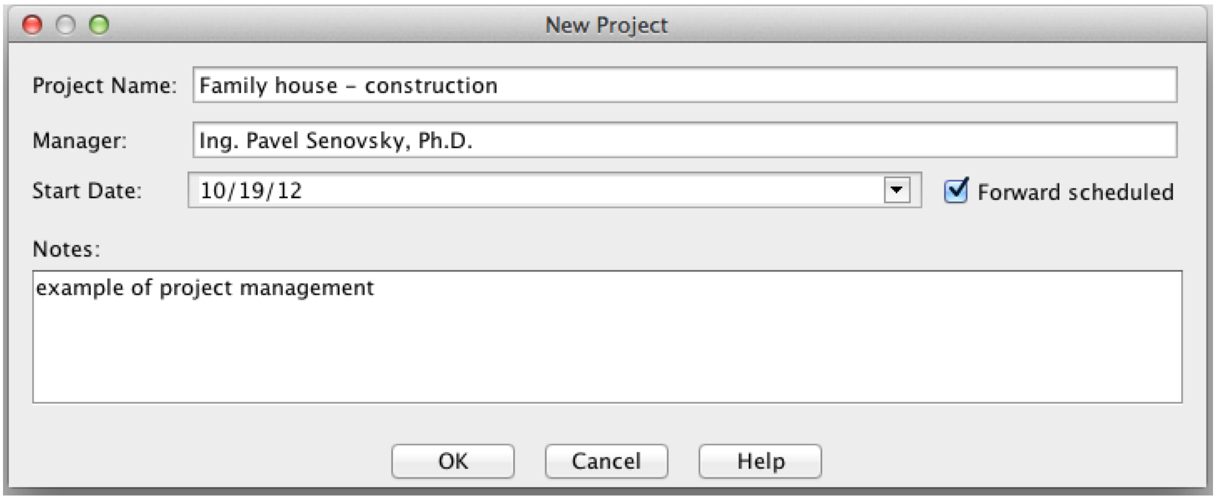


Figure 10.3: Create new project

The new project is automatically opened in Gantt chart mode, which allows us to specify the project tasks and the connection between them. Out of this information it automatically created the progress chart of the project.

We have already specified project tasks in table 10.1, with its name, connection to other tasks and its length. Lets with task definition – in Gantt chart we create the task by writing down its name into column name. When you create the task note that ProjectLibre automatically adds additional information to it. It guesses when the task will start (on the day of the beginning of the project), that its length will be approximately one day.

Activities have many properties we can use to manage the project:

- activity name

- length
- start and finish dates (software recomputes it automatically based on the relation between the activities we define)
- predecessors or follow up activities (used to define relation between activities)
- resources
- and many more - ProjectLibre even supports using custom properties

Also note that this information is automatically used for process chart on the right side of the screen. There are also other columns, which are empty so far – predecessors and resources, allowing us to specify the connection between the tasks and to assign human or material resource to the task.

Lets change length of the tasks, because right now it is one day with question mark. Question mark means uncertainty – so we are basically saying, that the task will require approximately one day. From table 10.1 we know that this is not the case, so lets make the corrections.

The new length can be assigned in days, weeks or months. Gantt chart presumes, that the value is in days, so writing 5 = 5d = 5 days = 1w = 1 week (this holds for office calendar). The time unit used to describe length of the task can be specified in days (d), weeks (w) or moths (m). Be aware that the week may mean something different than you think. When normally speaking about the week, we usually mean Monday – Sunday (or Sunday – Saturday depending, where are you from), the meaning of the week in project management is different.

The difference is in what calendar is used – is it normal office calendar with working hours 7 - 15:30 Monday to Friday or is it different? Meaning that the week can have 5 days or 7 days for different tasks using different resources. In project management we also do not know exactly when we will start with the task as its predecessors may have finished earlier or later then expected, so the week in project management is described by number of working days only – with floating beginning and the end.

The information on tasks in table 10.1 and the way we have defined our tasks in Gantt diagram also differ by tasks identification. In table we identify the task alphabetically using capital letters A – J, in Gantt chart the tasks are identified by the number of the row they are written on (1-10). Defining the connection between the task would be slightly harder for us to do, se we will use capabilities of the software to insert new column for us to write the letter identification in it.

Adding new column is easy. We just click on the column where we want to have new column. Then we right click on it to display context menu allowing us to insert and remove the columns. We will insert one. As you can see (well if you are trying it on the computer :-), there is plenty of possibilities to choose from as the template of the new column. We will choose Text1 – which is general template allowing us to insert any text into it. After inserting the column, we will change its name by right clicking on the column and choosing Rename. The new name will be ID Letter. Now we can write down letter IDs from table 10.1 into our new column.

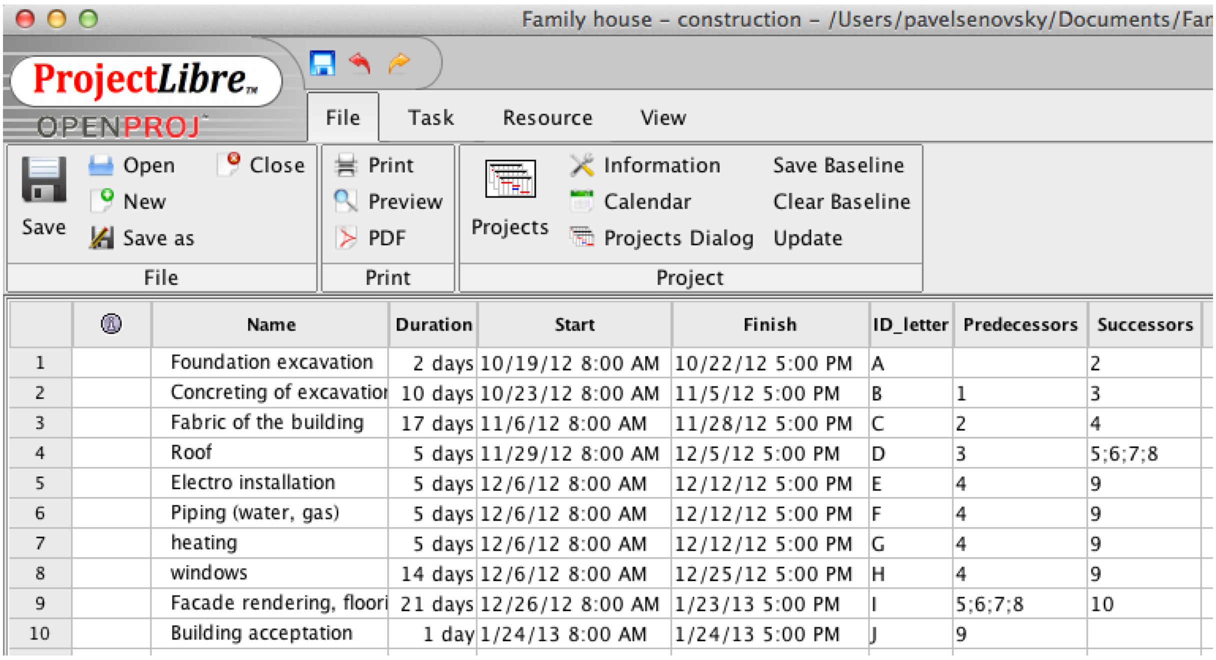
Now the only information missing from table 10.1 are the connections between the tasks. In Gantt chart we already have predecessors, so we can use these, but if you rather preferred usage of the followers – you can, by defining new column with successor template. The software will provide automatic translation to the other one.

In this text, I will use predecessors. Do not forget that we will have to identify the task by number, not by the letter in the new column we created. Also we will need to assign more then one predecessor for task I – we write all IDs in predecessor column and separate them semicolons. Well, the separation by using of comma works too – ProjectLibre just automatically replaces the commas with semicolons. In the end your Gantt chart should look similar to the figure 10.4.

By definition of the predecessors or successors we specified the connection between the tasks and the progress chart now makes some sense, see fig. 10.5.

On the progress chart you can see that the tasks have different sizes, are of different color. The size of the task is related to the task length. So as we defined it, we have it, but once defined it doesn't mean that we stop working with it – on the contrary, our work as project managers has just started. Now we have to closely follow happenings of our project and how the tasks are finished. The software supports us by allowing us to follow percentage of completeness of the task.

To change the percentage we use the mouse. We move the mouse over the beginning of the task. The mouse pointer will change into percent symbol and the arrow pointing right. Now click and pull to right to increase percentage.



		Name	Duration	Start	Finish	ID_letter	Predecessors	Successors
1		Foundation excavation	2 days	10/19/12 8:00 AM	10/22/12 5:00 PM	A		2
2		Concreting of excavatio	10 days	10/23/12 8:00 AM	11/5/12 5:00 PM	B	1	3
3		Fabric of the building	17 days	11/6/12 8:00 AM	11/28/12 5:00 PM	C	2	4
4		Roof	5 days	11/29/12 8:00 AM	12/5/12 5:00 PM	D	3	5;6;7;8
5		Electro installation	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	E	4	9
6		Piping (water, gas)	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	F	4	9
7		heating	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	G	4	9
8		windows	14 days	12/6/12 8:00 AM	12/25/12 5:00 PM	H	4	9
9		Facade rendering, floori	21 days	12/26/12 8:00 AM	1/23/13 5:00 PM	I	5;6;7;8	10
10		Building acceptation	1 day	1/24/13 8:00 AM	1/24/13 5:00 PM	J	9	

Figure 10.4: Gantt chart – task definitions

Similarly we can change the beginning or the end of the task as needed. The remainder of the tasks will adjust itself as needed without the need to micromanage it.

And lastly we have the color of the task. On our chart we have red and blue tasks. Task in blue are these which do have a time reserve. By time reserve we mean, that there is time available for us to lengthen the task without endangering whole project completion.

So the time reserve and manipulation with non-critical tasks allows us to free resources necessary to finish the critical (red) tasks on time.

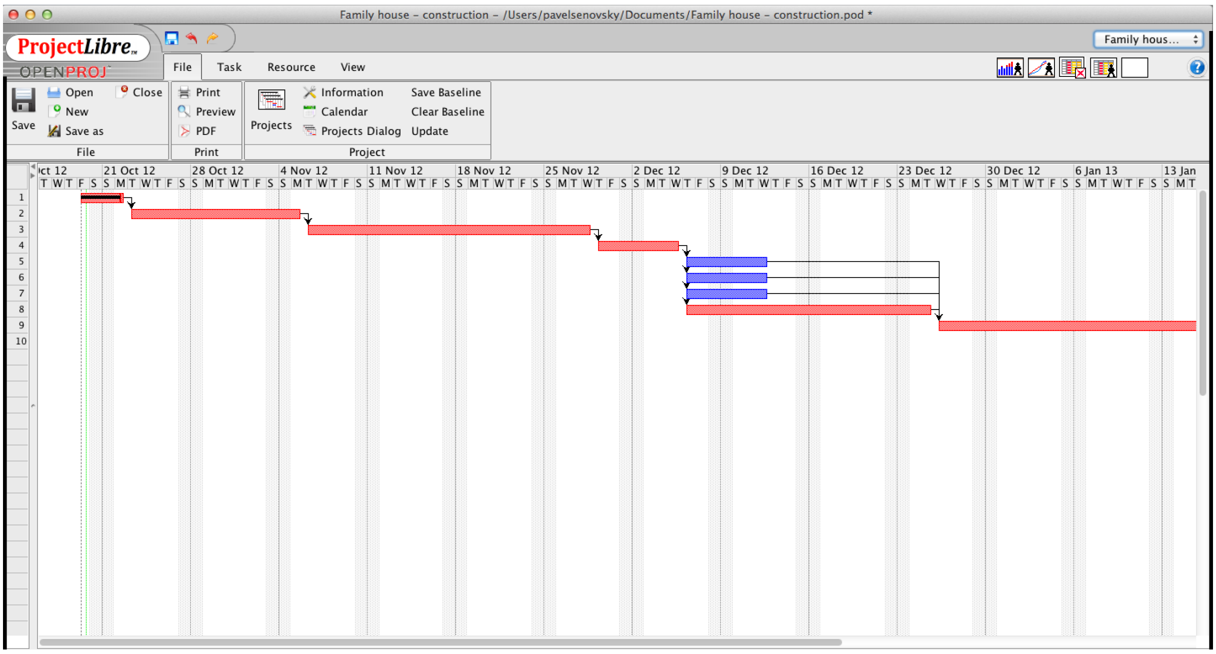


Figure 10.5: Gantt chart – progress chart

What would happen if we increased length of noncritical activity (i.e. F) to match or even exceed activity on critical path (H) which is being worked on in parallel with our activity? From fig. 10.5 we know that they start at the same time.

If we increase duration of the F activity to match duration of H, that it will also become activity on critical patch. The activity will become critical. Originally critical activity H will remain critical, under these circumstances.

But if the duration of activity F exceeds duration of H, situation will change. F will become critical, but H will be ne longer critical. Also since the critical path is longer now, the finish date of the project as whole will be pushed forward by the time difference between activities F and H finish dates.

Alternative way of looking on the progress chart is by using network diagram. In network diagram we see the tasks as the nodes of the network. Connection between the tasks allow us identify which tasks must be completed before others may begin. Network diagram will be generated automatically based on the tasks definitions in Gantt chart.

Start network diagram on Task tab -> tool Network. You may see screenshot of our network on figure 10.6.

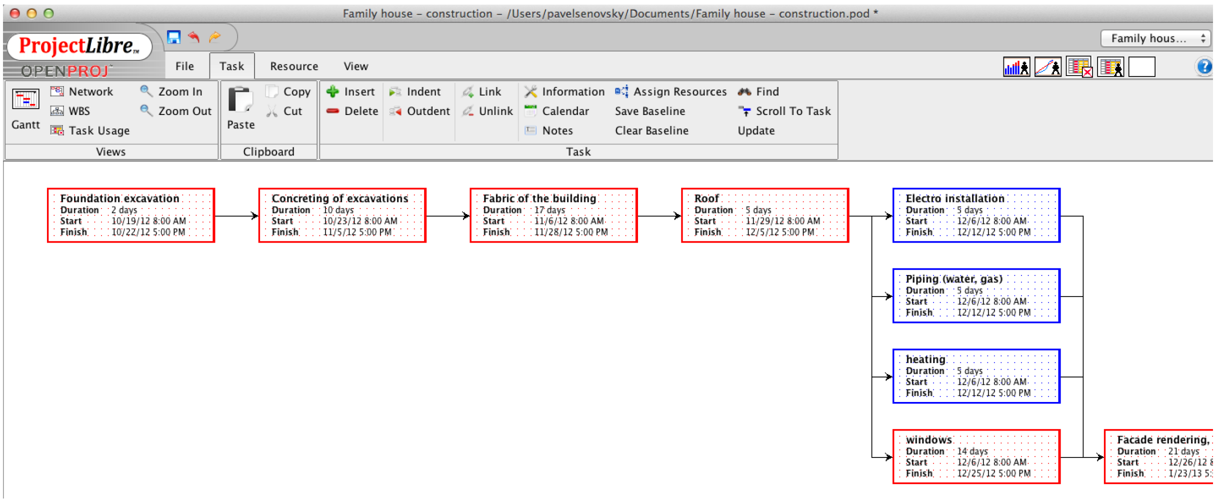


Figure 10.6: Project's network diagram

So why should we use both network diagram and progress chart when both basically show the same things? Well in fact they, but both in slightly different ways. Network diagram allows us to better see the network of the tasks structure, while progress chart allow us to better see and manipulate with reserves and manage the project when it starts.

We are talking here about the resources, can the software tools help us with these too? Yes it can. We can manage the resources available for the project on the resources tab (first tool from left on the tab).

Lets make three resources – worker 1, 2 and sand. For the resources we specify the type. The resources can be humans available for the work, it can be also the material. Obviously resources of the different types have also different properties.

For human resources we choose type *work*. To such resources we can assign contact email and group them for example by the teams they are working in. For human resources we state max. units. Automatically software assigns 100% max. unit for the resource. Usually we interpret it so, that we have available 8 hours of resource's work. Depending on the resource usage we may have more or less available. Max. units should not exceed 150%, as we would be getting closer to the slavery work which is forbidden by law in most countries.

Each human resource has its standard pay on per hour basis and overtime pay again on per hour basis for the cases when the usage of the resource exceeds max. units. Overtime pay is usually significantly higher then standard pay.

Cost per usage is the cost connected to the usage or the human resource, but can't be considered income of the resource. For example I might need to transport workers on the site, which costs me money – that would be cost per usage.

Accrue at column lets us set when we will pay the resource for his work. We may do it as prorated, at the beginning or the end. Prorated would mean, that we will pay the resource every day. In practice we may use the scenario of usage of diggers to prepare sewer network. In such case we pay them on the end of each work day (if paid on the beginning they may not come to work).

At the beginning we pay in cases when there is some kind of resources procurement necessary on side of the resource. On the end means that we will pay after we check and approbate the work.

Every human resource may have its own calendar – standard, 24 hours and night shift.

In case of the material resources many columns do not have any sense, so we do not use them. What is the meaning of the remaining columns? Standard rate should be interpreted as the rate at which we procure unit of the resource. The units may be tons, kilograms, price for the piece, square meters, ... Cost per use is similar to its human resource's counterpart – usually we need to move the material on site before we can use it and it costs money.

Example of the resources definition is on figure 10.7.

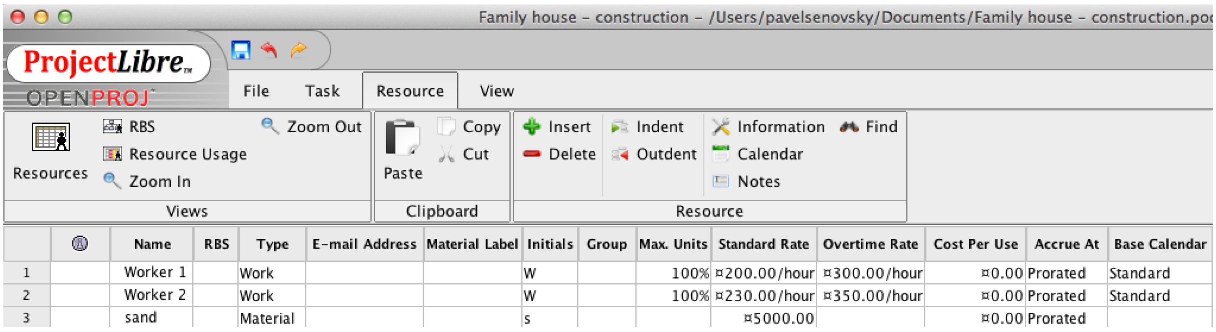


Figure 10.7: Resources definition

Now lets go back to the Gantt chart. We will assign the resources to the tasks. We can do that in two ways - we either click into resource name column or in properties of the activities. We either directly fill in the resource used by activity or double click it to open dialog windows allowing us to set up the resources in more granular manner, see fig. 10.8.

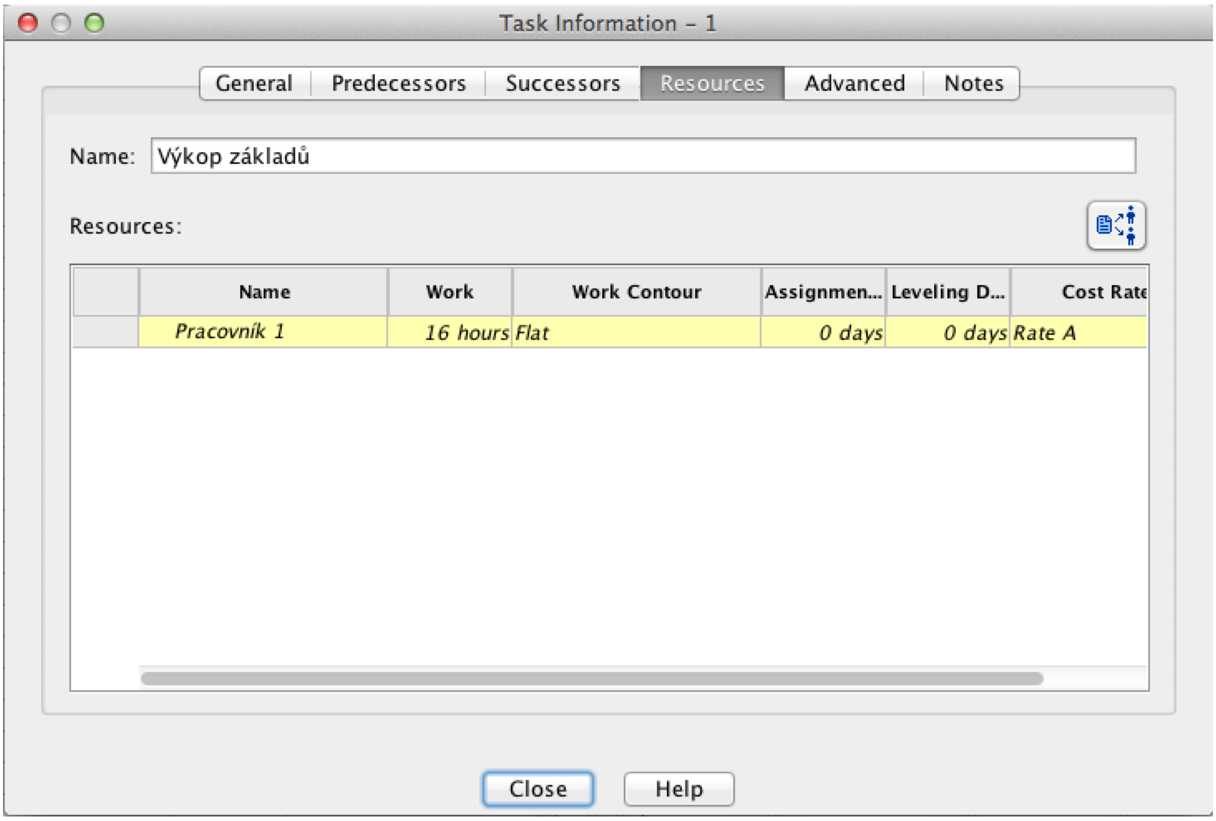


Figure 10.8: Information on activity in ProjectLibre

Lets try to assign the worker 1 to all the tasks. You may speed the definition process by copying resource name in Resources column.

Lets see how exactly are we using the resource. First we will look at the resource's histogram. You may start it by clicking on histogram icon on upper right side of your program's window, or look at the figure 10.9.

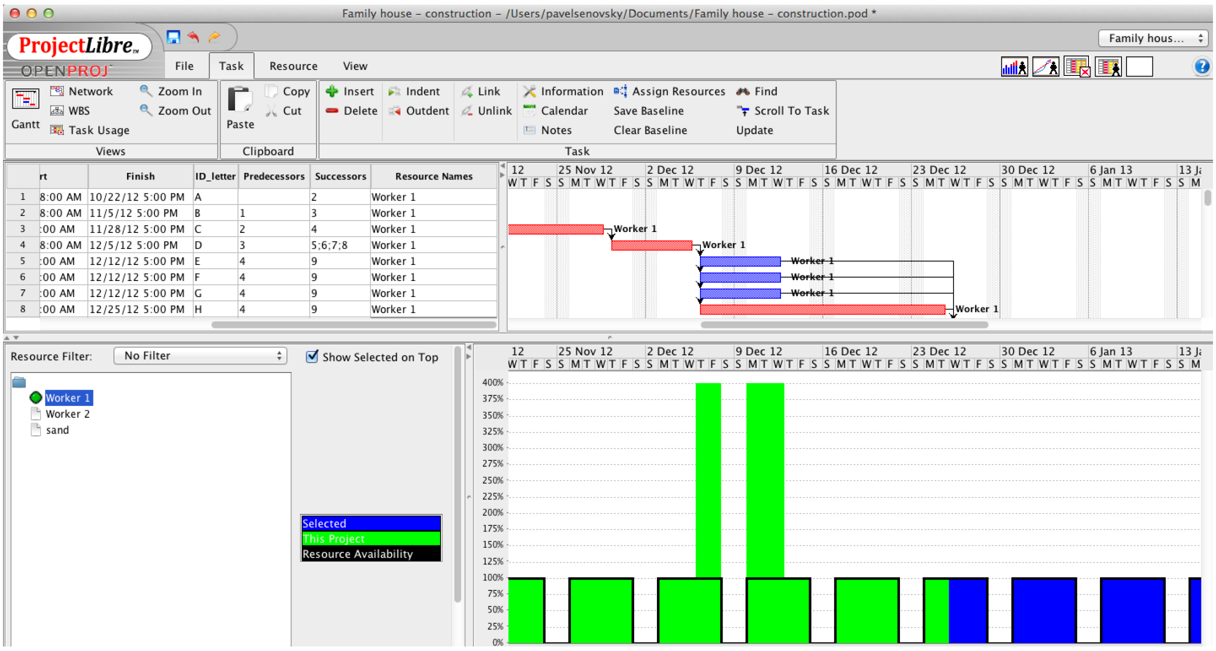


Figure 10.9: Resource's histogram

The histogram is connected to the Gantt chart, to navigate through it you have to mo the Gantt chart, histogram will move automatically. As you can see the problem is in tasks starting 6th December, we have four of them and we assigned worker one to all of them, which leads to our brutal overuse of the resource – we plan to use it on 400%, which is of course not possible.

Histogram thus helps us identify the problematic tasks in our project and allows us to solve the problem before the problem manifests itself in real world.

We may look at the resources also by assigned working hours choosing Resource usage, see fig. 10.10.

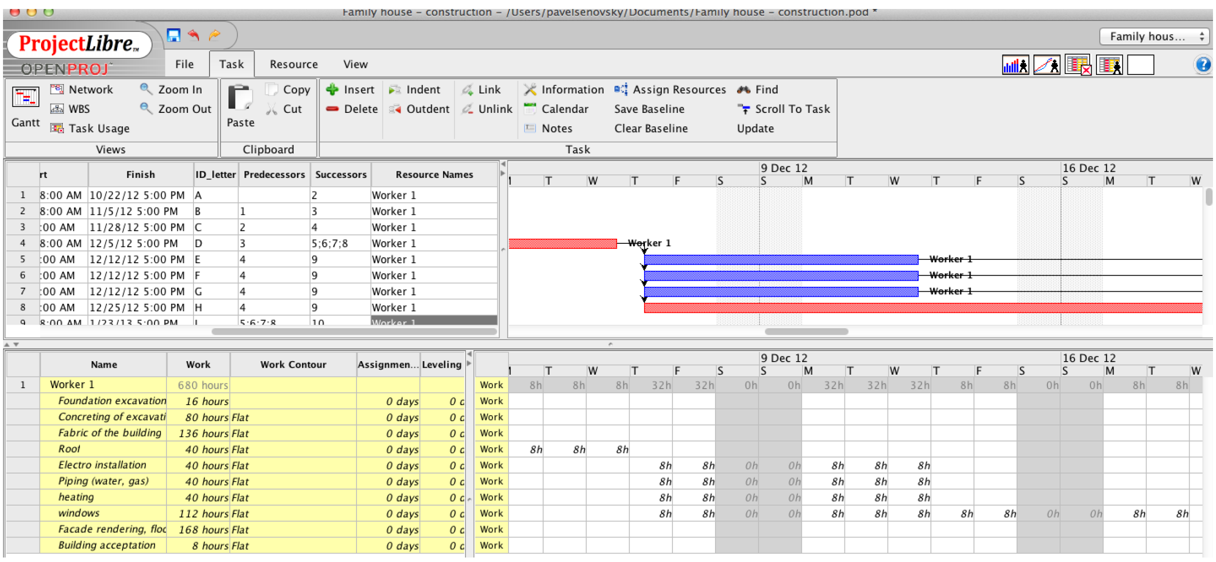


Figure 10.10: Resource usage

And that's all, you will se in this textbook on project management. The software itself has more tools allowing you to better track monetary resources and many more aspects of the project management.

Project management
Create new project of your choice, if you do not want to think – create project for your study program this year, look at all of the terms, semester projects, assignments...

10.3 Optimization in other network models

Project management has special needs, but the knowledge of CPM method may be used in different types of networks as well. For example we may want to analyze road (rail) network or some kind of pipe network. These networks need different types of answer – we may search for shortest road between two nodes or critical path from capacity point of view etc.

Such types of task are solvable if the modeled network closed – there are no outside connections to or from it. Another limitation of the model is that what comes in the first node must also go out from last node.

That means that this type of algorithms is unable to solve problems of networks where we experience significant losses in transported medium. Typical example could be drinking water pipelines in cities, especially the 60+ years old, where water loses can be 40% or more.

Owners of such infrastructure do the best to maintain it and upgrade it whenewer possible, but that does not make water loss insignificant and the network modelable using these tools.

Let's see how we can solve some simple problems algorithmicaly. First type of problem is searching for **maximal flow in network between two given nodes**. Problem with this kind of problem is that the edges between the nodes usually have different capacity. And to solve this problem combinatory would be for large networks virtually impossible as the difficulty of the problem will grow exponentially with number of nodes.

We can apply different approaches to solve it, for example **Ford-Fulkerson algorithm**, which works in following way [52]:

Inputs: Graph G with flow capacity c, source node s and the sink node t.
Output: flow f from s to t, which is maximal

1. $f(u,v) \leftarrow 0$ for all edges of the network $(u,\ v)$
2. while there is path p from s to t in G_f, such that $c_f(u,v) > 0$ for all edges $(u,v) \in p$:
   (a) find $c_f(p) = min[c_f(u,v)|(u,v) \in p]$
   (b) for each edge $(u,\ v) \in p$
      i. $f(u,v) \leftarrow f(u,v) + c_f(p)$, send flow along the path
      ii. $f(u,v) \leftarrow f(u,v) - c_f(p)$, the flow might me "returned later"

Imaging how Ford-Fulkerson's algorithm actually works is rather hard. You can see an example of Ford-Fulkerson's algorithm in Ford-Fulkerson Demo [53].

Software support for physical network computations
Computing the physical network using Ford-Fulkerson's algorithm, or any other by hand is rather hard, even for small networks. That's why there are many software tools which employ the algorithms and virtually separate us from the algorithm itself. You can find them in modern Global Positioning System (GPS) and many more applications.

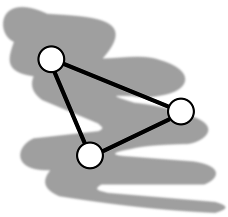
We will use R computational environment, we used already in previous chapters to help us.

Another type of the problem could be *searching for shortest path between nodes*. For this type of problem algorithm of Dutch scientist Dijkstra is often used (*Dijkstra algorithm*). The algorithm is as follows [54]:

1. Create a distance list, a previous vertex list, a visited list, and a current vertex.
2. All the values in the distance list are set to infinity except the starting vertex which is set to zero.
3. All values in visited list are set to false.
4. All values in the previous list are set to a special value signifying that they are undefined, such as null.
5. Current vertex is set as the starting vertex.
6. Mark the current vertex as visited.
7. Update distance and previous lists based on those vertices which can be immediately reached from the current vertex.
8. Update the current vertex to the unvisited vertex that can be reached by the shortest path from the starting vertex.
9. Repeat (from step 6) until all nodes are visited.

Both algorithms work similarly – they reduce the network into the tree graph, for which the number of computations needed to find the answer is manageable. Dijkstra algorithm is also usually applied by specialized software.

Djisktra's algorithm is still used up to day to find shortest path. It is considered still very efficient for networks up to 100 nodes. For the larger networks it is mostly recommended to use much more efficient Johnson's algorithm.



Usage of Djisktra's/Johnsons algorithms in MCA
Some MCA methods use these methods to remove inner loops when analyzing the network of interconnections represented by adjacancy matrixes MCA usually uses to derive the ranking of the alternatives. This process is being calle *network simplification*.

There are about dozen of various algorithms used for different types of tasks. You can review them more in depth for example on Wikipedia in article List of Algorithms [55], which has dedicated section to it.

Questions

1. What are limitations of the network models?
2. What kind of problems are we able to solve by the models?
3. What is the difference between Ford-Fulkerson's and Dijstra's algorithms?
4. What is common for these two algorithms?

10.4 Numerical computations in R

We can use R for both computations and visualization of networks. Core environment does not support this type of computation, so we need to install proper packages, which will allow us to use this functionality.

One of more popular packages is *igraph* [56]. We can install it from CRAN repository. If you tried examples from MCA advanced models chapter, you already have installed the package. Otherwise you can install it using following code:

Listing 10.3: Installing igraph in R

```
install.packages("igraph")
```

We can read the graph using *read.graph* function. We need to specify network structure in external file, which we pass on the function as one of its parameters. Function supports many input formats:

edgelist, pajek, graphml, hml, ncol, lgl, dimacs, graphdb and gml. Package maintainers try to add support for additional formats in new versions of the package as they emerge. If format parameter is left empty, function presumes edgelist format, which is simplest format available for network definition purposes.

Before we use some of these formats, we will need to define the network using *adjacency matrix*. We actually used this approach already in advanced MCA models to visualize outranking relations between the alternatives.

This time we will define network from fig. 10.11, which you could already see on the lectures including demonstration of in hand computations using different methods.

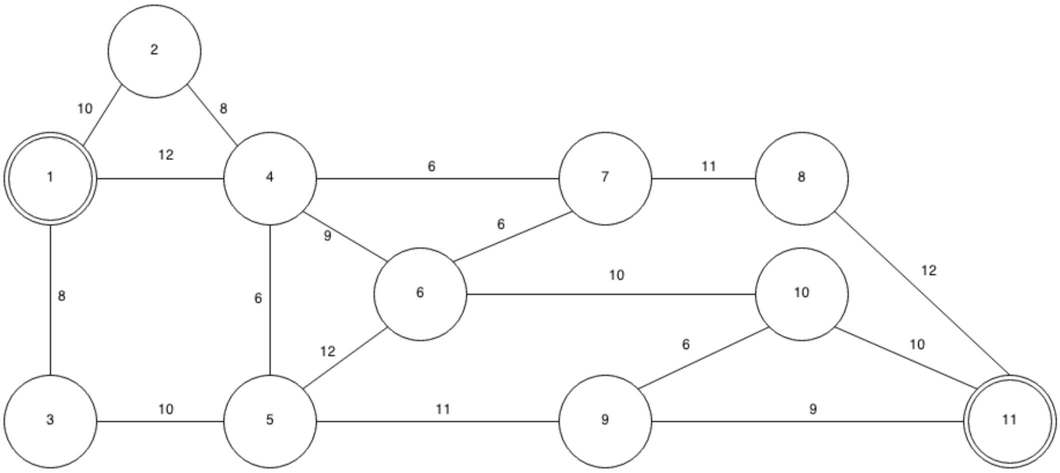


Figure 10.11: Basic network for computation realization using mark-up algorithms (courtesy of Gros [11])

Listing 10.4: Define network in igraph using adjacency matrix

```
library('igraph')
adj_matrix <- matrix(
  c(0, 10, 8, 12, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 8, 0, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 10, 0, 0, 0, 0, 0, 0,
    0, 0, 0, 0, 6, 9, 6, 0, 0, 0, 0,
    0, 0, 0, 0, 0, 12, 0, 0, 11, 0, 0,
    0, 0, 0, 0, 0, 0, 6, 0, 0, 10, 0,
    0, 0, 0, 0, 0, 0, 0, 11, 0, 0, 0,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 12,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 9,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11,
    0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0),
  nrow = 11, ncol = 11, byrow = T
)
net <- graph_from_adjacency_matrix(adj_matrix, mode = "max", weighted = T)
plot(net, edge.label = E(net)$weight)
```

We will create matrix from vector by "pouring" the vector by rows (set parameter byrow = T). Our network has 11 nodes, so the adjacency matrix needs to be of 11:11 size. The graph is not oriented, so we can little bit cut corners. Notice that lower left part of the matrix consist of 0 only. In case we set graph mode to directed, we would need to fill in values these too to specify from which node to which node the flow actually is.

We can interpret adjacency matrix simply as table, where the rows and columns represent network's nodes and table fields with nonzero value will define network's edges.

Since we are working with simple undirected network graph, it might be easier to define network using *edge list* format.

Result will be similar to fig. 10.12 basically regardless of what format we will use for network definition. Please note that networks on fig. 10.11 and fig. 10.12 are structurally same. Only difference is that network in fig. 10.11 has been made in hand and as such the lengths of the edges do not exactly correspond to states length.

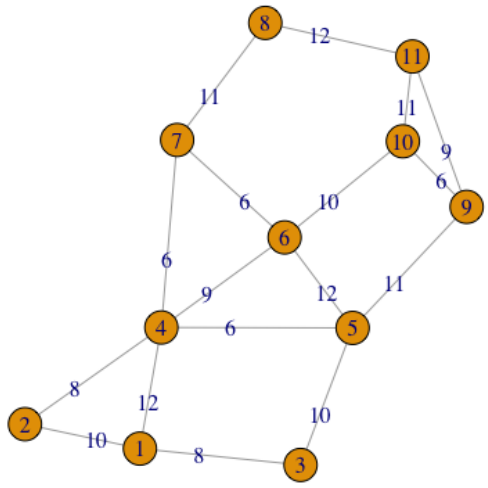


Figure 10.12: Network structure defined using adjacency matrix

Edge list is very simple format for network definition. We specify the network by defining its edges. Structure of the definition is simple: starting node, end node, weight of the edge. The format doesn't require us to put the edges definition on separate rows, but I recommend doing so for better readability.

Listing 10.5: Defining network in edge list format: start end capacity

```
1 2 10
2 4 8
1 4 12
1 3 8
3 5 10
4 5 6
4 7 6
4 6 9
5 6 12
7 8 11
8 11 12
6 10 10
10 11 10
5 9 11
9 10 6
9 11 9
```

We do not have room in the course or in textbook to describe all supported data formats, so we will limit ourselves to studying *GraphML* only. For the rest we provide only directions to other study materials, should you require it: pajek [57], for lgl and ncol data formats consult [58], dimacs [59], graphdb [60], gml [61] and dl [62].

The GraphML format is interesting from several different perspectives. It is based on Extensive Markup Language (XML), and thus is somewhat self-documenting, easy to interpret and has some very interesting properties.

Our network would look like this in GraphML:

Listing 10.6: Using GraphML to define the network

```
<?xml version="1.0" encoding="UTF-8"?>
<graphml xmlns="http://graphml.graphdrawing.org/xmlns"
  xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
  xsi:schemaLocation="http://graphml.graphdrawing.org/xmlns
  http://graphml.graphdrawing.org/xmlns/1.0/graphml.xsd">
<key id="d1" for="edge" attr.name="weight" attr.type="double"/>
<graph id="G" edgedefault="undirected">
  <node id="n1"/>
  <node id="n2"/>
  <node id="n3"/>
  <node id="n4"/>
  <node id="n5"/>
  <node id="n6"/>
  <node id="n7"/>
  <node id="n8"/>
  <node id="n9"/>
  <node id="n10"/>
  <node id="n11"/>
  <edge source="n1" target="n2">
    <data key="d1">10.0</data>
  </edge>
  <edge source="n1" target="n4">
    <data key="d1">12.0</data>
  </edge>
  <edge source="n1" target="n3">
    <data key="d1">8.0</data>
  </edge>
  <edge source="n2" target="n4">
    <data key="d1">8.0</data>
  </edge>
  <edge source="n3" target="n5">
    <data key="d1">10.0</data>
  </edge>
  <edge source="n4" target="n5">
    <data key="d1">6.0</data>
  </edge>
  <edge source="n4" target="n6">
    <data key="d1">9.0</data>
  </edge>
  <edge source="n5" target="n6">
    <data key="d1">12.0</data>
  </edge>
  <edge source="n4" target="n7">
    <data key="d1">6.0</data>
  </edge>
  <edge source="n6" target="n7">
    <data key="d1">6.0</data>
  </edge>
  <edge source="n7" target="n8">
    <data key="d1">8.0</data>
  </edge>
  <edge source="n8" target="n11">
    <data key="d1">12.0</data>
  </edge>
  <edge source="n6" target="n10">
    <data key="d1">10.0</data>
  </edge>
  <edge source="n10" target="n11">
    <data key="d1">10.0</data>
  </edge>
  <edge source="n5" target="n9">
    <data key="d1">11.0</data>
  </edge>
  <edge source="n9" target="n10">
    <data key="d1">8.0</data>
  </edge>
  <edge source="n9" target="n11"/>
</graph>
</graphml>
```

We will use *read.graph* function to read the graph definition and then plot it using *plot* function.

Generated graph image will be only similar to the one we made by hand (fig. 10.11), but all its properties are going to be correct (unlike the graph drawn by hand).

Structurally both networks are same, the generated one takes better into account length of the edges. we can also influence looks of the network by setting various plotting parameters.

We can compute shortest path by using function *shortest.paths*.This function has three parameters - network to be analyzed and first and last node. Apart of length, we may also be interested in the path itself. We can get this one using *get.all.shortest.paths* function, which has same parameters as shortest.paths, but returns the nodes on shortest path. This function does not provide information on overall length of such path.

Ford-Fulkerson algorithm is not supported by igraph, but it supports other algorithms to solve problems around the flow. For example *graph.maxflow* computes maximal capacity of the path between two nodes. To do that, network must be defined including information on capacities, which our network does not have.

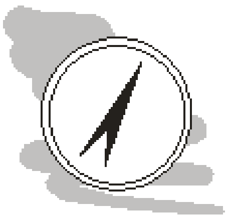
Whole example could look like this:

Listing 10.7: Shortest path computation in R

```
library(igraph)
net <- read.graph("/Users/pavelsenovsky/Documents/skripta/modelovani 3vyd/skripty/sit.graphml", "
    graphml")
plot(net)
shortest.paths(net, 1, 11)
get.all.shortest.paths(net, 1, 11)
```

Note, that igraph doesn't allow us to specify what algorithm should be used to compute shortest path. It is by design, to shield the user against complexities of all the algorithms and their nuances. It is expected, that this type of information should posses the package developer and (s-)he needs to implement some kind of rationale into the package itself.

For sake of completeness, the shortest path computation in igraph package uses Djisktra's algorithm for small networks, but for large networks (over 100 edges) it uses *Johnson's algorithm*. Function chooses appropriate computational algorithm automatically.



Graphviz

In previous edition of this textbook, in this section program *Graphviz* [63] has been discussed. The program is intended for network graph visualizations. Considering functionality offered by R this section of text seemed to be little bit redundant. That is why this section of text has been moved do Annex 2 for network enthusiasts.

Last type of the algorithms we will use in this section of text is *Kruskal's algorithm*.

While in previous cases we begun the computation by defining the network, Kruskal's algorithm solves other type of problems. We start will know node positions only (no edges exist in the graph at the beginning). Out goal will be to construct optimal network, that will interconnect all defined nodes. To express problem in more proper mathematical way, we need to *derive minimal structure of the graph interconnecting all the nodes.*

To be able to fulfill minimal set requirement, we will need some kind of optimization criterion to limit its size.

We call this minimal graph structure *spanning tree*. General outline of the algorithm is simple:

1. compute all possible/allowable connections between the nodes and compute values of its optimization criterion (often cost or length) - these will serve as potential edges of the graph
2. order potential edges by value to the optimization criterion from best to worst
3. repeat following step until all nodes are connected into network
4. from the non-selected edges of graph G, choose edge (in order), which does not create the loop with already selected edges

Computation in R is possible using *optrees* package [19]. Apart from Kruskal's algorithm it also supports other algorithms such as Prim's and in Czech Republic developed Borůvka's algorithm. We provide the computational code below:

Listing 10.8: Example of spanning tree using Kruskal's algorithm (courtesy of [19])

```
library('optrees')
nodes <- 1:4
arcs <- matrix(c(1,2,2,
  1,3,15,
  1,4,3,
  2,3,1,
  2,4,9,
  3,4,1),
  ncol = 3, byrow = T)
G <- getMinimumSpanningTree(nodes, arcs, algorithm = "Kruskal")
```

In the listing there are possible relations between the nodes defined using arcs matrix in which in first columns there is starting node, in second end node and in the third length, cost or other optimization criterion of choice. The function requires the criterium to be formulated in such way that we optimize it by minimization. Resulting graph is available in fig. 10.13.

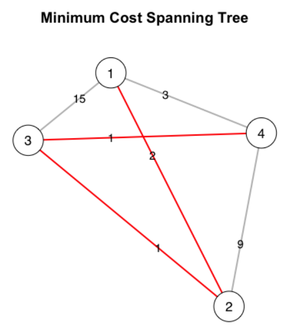


Figure 10.13: Minimal spanning tree derived using Kruskal's algorithm (in red)

Our example presumes, that the possible/allowable edges have been prepared beforehand. What if it is not so, what if we really start with nodes positions only? Presuming that the optimization criterium is (or is connected to) the distance between the nodes, we can generate easily possible combinations of the nodes and compute length of the vector they form.

Let's consider simple example with four nodes at coordinates A = [0,0], B = [1,0], C = [1,1] a D = [0,1]. We will be working with the square. Optimal solution for the problem would be logically constructing graph from 3 out of 4 sides of this square to form minimal spanning tree.

Solution for this problem is provided in listing bellow and the result is available on fig. 10.14.

Listing 10.9: Generating edges by combinations of the nodes for Kruskal's algorithm

```
library('optrees')
nodes <- data.frame(rbind(
  c(0, 0),
  c(1, 0),
  c(1, 1),
  c(0, 1)))
colnames(nodes) <- c("X", "Y")
a <- NULL
for(i in 1:(nrow(nodes)-1)){
  for(j in (i+1):nrow(nodes)){
    t <- sqrt((nodes[i,1]^2 - nodes[j,1]^2)^2 + (nodes[i,2]^2 - nodes[j,2]^2)^2)
    a <- c(a, i, j, t)
  }
}
arcs <- matrix(a, ncol = 3, nrow = choose(nrow(nodes), 2), byrow = T)
nodes2 <- 1:4
G <- getMinimumSpanningTree(nodes2, arcs, algorithm = "Kruskal")
```

Logic behind it is as follows: the costs are often connected to distance. For example if we need to connect the nodes by wire, then there is per meter price for wire. If the node symbolize cities and

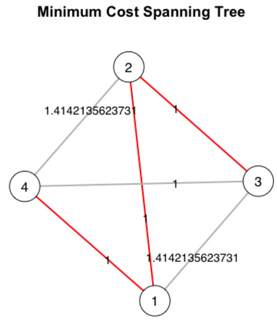


Figure 10.14: Graph generated by Kruskal's algorithm (in red) generated based on combinations of the nodes

edges some kind of pipeline, there is per kilometer price connected to it, etc. In both these examples our requirement to minimize the optimization criterion holds.

Even if the measure doesn't hold, than we can easily rescale the measure to be minimized.

Summary

Working with networks is kinda complex. That's why it is important to always understand what kind of problem do we need to solve. Do we need to manage the project, or are we interested in some properties of physically realized network, such as road, railway, pipelines, etc.?

To support project managements there are many software available. Good examples are MS Project, ProjectLibre and similar. These can help us to follow activities in the projects as they are realized, plan them both time and resource wise.

In physical networks we solve typically different types of problems, so we will need to use different type of tools. There are again available many packages, but using R and its igraph package can be recommended generally as a good starting point.

There are various algorithms available for various network problems. One of most often used is Djisktra's algorithm (alternatively Johnsosn's algorithm) to find shortest path. Ford-Fulkerson's algorithm can be used to find maximal capacity between two points in the network.

Analyzed networks are limited to certain degree. The networks need to be closed and we presume that there is no significant loss of transported medium in the network. Principally both algorithm operate similarly, they reduce whole network into the tree, which is much more easily manageable.

Kruskal's algorithm allows us to design new network using known nodes, constructed so, that it will conform to optimal size based on criterion of choice.

10.5 Network Models and Critical Infrastructure

By CI we mean as per crisis law (240/2014 Sb.) [64]) infrastructure whose: *interruption or limiting functionality would have significant impact of security of the state, endanger basic needs of its citizens, public health or states economy.*

Example
Define your own network, generate it graph using software and compute shortest path between two nodes.

Self evaluation questions

1. Describe way the mark-up algorithms work (for example for finding shortest path)
2. How do we solve shortest path problem in R?
3. Describe function of Kruskal's algorithm.

Networks and critical infrastructure
In this section we will focus on adding security dimension on Critical Infrastructure (CI) to place whole problematic of network models in much broader scope.

Considering that the CI is extremely broad problematic, we have no chance to cover it all. Because of that we will focus only of some interesting features that are relevant to what we already learned about the network models and what will be useful during your further study to better understand CI security or the way problems spread through such networks.

Lot of these infrastructures is networked, like telecommunications, various networks in energetic, traffic networks, etc. Network analysis allows us to explore some important properties of such networks. From this perspective we may also define CI also like complex system with strong intra- and inter-sectoral links. To get overall view about behavior of such networks we can't analyze its elements, like nodes, one by one, because complex system is more then just sum of its parts.

For analytical purposes network models play tremendous role. We can define such network similarly to eq. (10.1) [65].

$$G(t) = N(t), L(t), f(t) : J(t) \tag{10.1}$$

Where t is time, N node in network, L edge, f topology specification and finally J is an algorithm describing node and edges behavior in network. Interpretation of building parts of eq. (10.1) is dependent on the network, which we try to model. For example for railway network the nodes may represent train stations or sliding rails, edges can represent railway itself.

For long time it was presumed, that CI networks are randomly distributed. Barabasi [66] based on study of node distribution of the Internet proved, that this is not the case. Nodes on the Internet and in many other networks use topology we cal *scale-free networks*.

These networks are typical by its high tolerance to failure of the random nodes. Error (outage) caused by such failure usually does not have any measurable impact on the infrastructure as whole. But in such network there are also special nodes, called *hubs*, to failure of which the network is very sensitive. If the hub fails, impact on the network is usually very significant.

We can clearly see differences between random and scale-free network on fig. 10.15.

The prevailing view today is that most, if not all, critical infrastructure networks are scale-free networks.

Mathematically we can specify by degree of distribution of the links between the nodes, which has exponential characteristics (10.2) [65]:

$$P(k) \sim k^{-\gamma} \tag{10.2}$$

where $P(k)$ is probability, that the node is in vicinity of other nodes in network, γ is distribution coefficient, usually $\gamma \geq 1$. This coefficient must be measured empirically on case per case basis.

Non-randomness in organization of such networks can make them vulnerable to targeted attacks. So, if the scale-free (CI) networks are not randomly distributed, there must be some kind of master

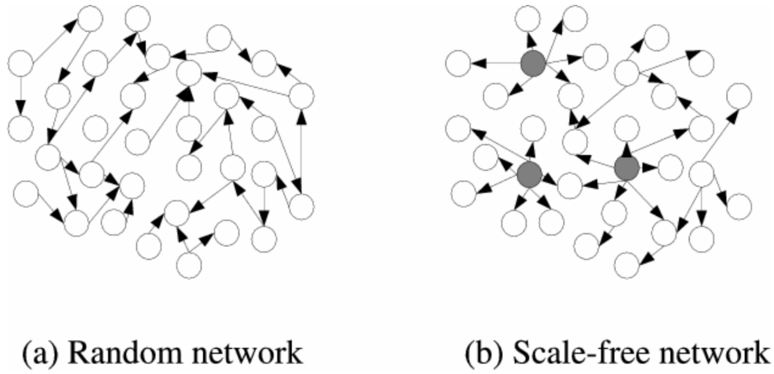


Figure 10.15: Random vs scale-free network (courtesy of [12])

plan behind them, which (or who) introduced the organization into the network, right? Well, not exactly, or maybe yes, but in different way than you may think. The organization, we are seeing in scale-free networks today is a side product of long-term repeated application of simple optimization principles. That in turn led to "organizing" of the network. Since the organization was not a goal and nobody is directly responsible for it, we may also say, that the organization emerged gradually on its own. We call such process *self-organization* or *emergence effect*.

So, what exactly is that self-organization? Nice demonstration has been developed by Resnik [67], who developed simplified termites behavior model. We can visualize the model, see fig. 10.16. Left part of the figure shows randomly generated living space of the termites (red squares) and stones (black squares), the termites are going the "build" their termite mounds. In right part of the figure we see state of this space after many iterations of the algorithm.

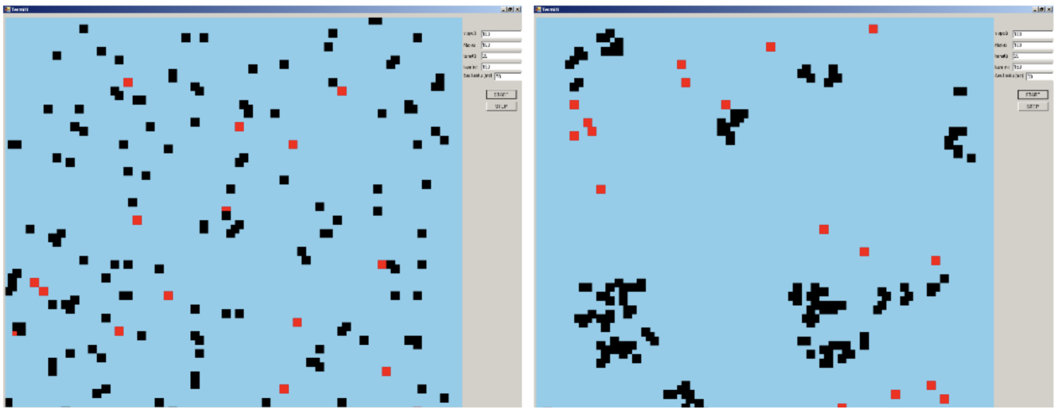


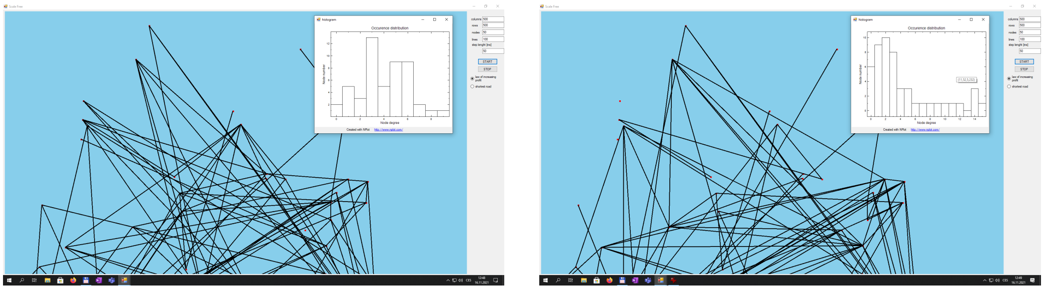
Figure 10.16: Simple model of termites behavior, left starting (random) state, right "end" (self-organized) state later (generated using [13])

The later (iteration wise) are we in computation the higher level of organization is visible in the model. Randomly distributed stones start to concentrate on "piles". Since we are using termites analogy, we can call them termites mounds. These mounds are nonrandom. We cannot say in advance, where they are going to form, only that they will form ... every time. Rules of the model are very simple. Resnick used only four simple rules [67]:

1. termites randomly move through its living space
2. if the termite finds the stone, it will pick it up
3. if the termite is holding the stone and finds another one, it will drop it
4. termite randomly move through its living space (repeat from step 1)

So whole complex organization is result of repeating only these four rules.

Termite model is obviously not network model, the self-organization principle remains same. For scale-free networks is typical principle of repeated investments. We could formulate it in following



(a) Randomly generated network (b) Application of repeated investments

Figure 10.17: Applying law of repeated investments for generating scale-free network (generated using [14])

way [65]:

1. randomly select edge in the network
2. randomly select node from network, which is not in the edge
3. replace original edge by new one formed between two nodes with largest amount of connected edges
4. repeat from step 1)

Also in this case we can demonstrate principle in simplified form, see fig. 10.17. Application of the approach will gradually transform originally random network to organized one. Approach prefers creation and gradual increase in importance of hubs. Note also the distribution of nodes degrees at beginning and at the end of simulation.

By the node's degree we mean number of links (edges), the node is connected to other nodes in network. At beginning the distribution is approximately random. That is clearly visible in bar charts. See approximate bell curve typical for normal distribution of probability. By application self-organizing rules, this distribution starts to change. Now it is more like poison's distribution. We can interpret that as a preference of small number of highly connected nodes in network on one side and high number of the nodes that are connected to only one other node.

Summary

This section of text was intended more as inspiration then anything else. Pursuing knowledge in variety of fields may prove to be beneficial in other areas.

We learned some interesting information on critical infrastructures networks. We know now that they are mostly scale-free networks. These are typical by existence on small number of highly interconnected nodes. We call them hubs. Scale-free networks are highly resistant to failures of random nodes, but may be vulnerable to failures of hub nodes.

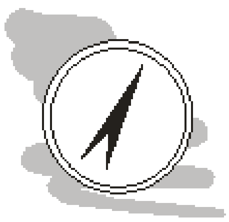
Into this high level of organization CI networks typologies evolved gradually on their own using repeated application of self-organizing principles.

Self-evaluation questions

1. Define critical infrastructure
2. What are difference between random and scale-free network?
3. What is self-organization?
4. How are distributed nodes in scale-free network?
5. Explain principle of repeated investments.

Chapter 11

Balance models



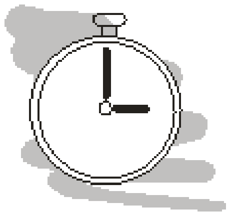
Study guide
When deciding during various emergencies we should come from deep, in past prepared knowledge/information about the organization we are trying to help. One of basic functions of the company (at least the manufacturing one) is to produce the products it is selling.

Balance models, we will deal with in this chapter, are capable to bring into population processes by modeling whole chain of production – allowing us to describe how the raw materials are transformed into the form of half-finished products and these into final products.

Such information is usable for evaluation of emergencies impacts on production capability of the company, so we can develop contingency plans optimizing the post-emergency recovery effort.

After reading through this chapter you will
Know

- What the balance models are
- How do we construct one
- And how can we solve one



Time required to study chapter
For getting through this chapter you will need at least 30 minutes.

11.1 Basics of balance modeling

Balance models are used for exact description of interconnection in production cycle in the company thus allowing us to describe the way the inputs (raw materials, half-finished products) transform on outputs (final products). Balance models are used by economists for long time for planning of production capacity and planning of resources needs of the production.

From technical point of view the models are based on family of models developed in 60-ties of last century by American economist Vasilij Leontief, called *input-output models*. Leontief developed these models to describe and quantify dependencies among various sectors of the economy. Output of one sector serves as input of another sector. Leontiesf validated the models on USA economical data from World War II.

For his work, Leontief was awarded Nobel price for economy in 1973.

Today variety of such models exists, for their common properties we call them *leonties's models*. Balance models are representative of such approach and we can use them on level of one company to better understand various transformations of energies, raw materials, semi-products and products in the production process.

There are also other leontief's models to evaluate possible impacts of critical infrastructure elements disruption. For purposes of CI modeling huge amount if inter-sectoral data need to be collected on regional basis, so we will only briefly discuss these model in section *Security applications of the balance models* available latter in this chapter.

In emergency planning we can use this type of models for modelling of the emergencies impact on our production capability, especially how will take effect disabling or destruction of part of technology in the company on its ability to produce. Based on these results we may plan way of effective recovery from emergencies.

Lets discuss small example of the problem. On fig. 11.1 we have block schema of the simplified production process of the powder fire extinguishers. The structure of the production process is well known before we usually start with balance models – since the organization has to know how to produce the goods to survive on the market, in the first place.

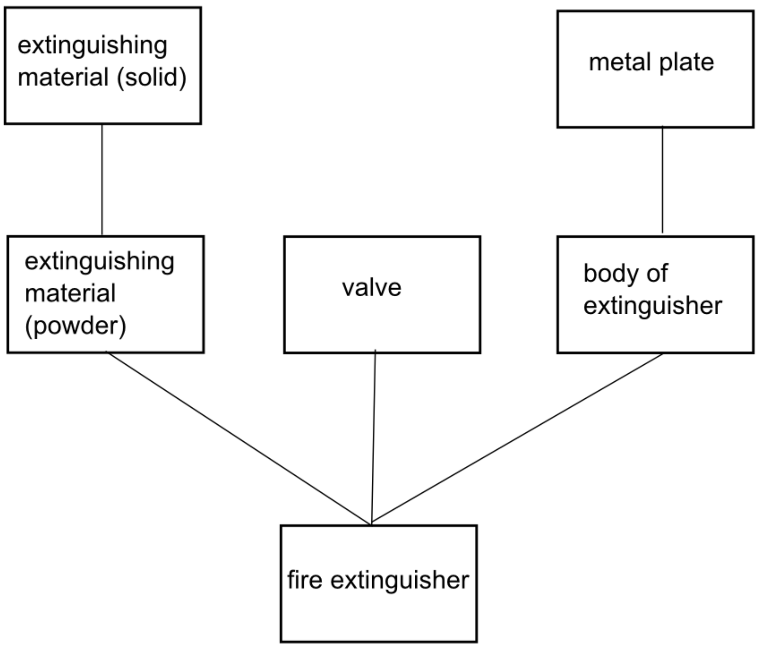


Figure 11.1: Production of the Powder fire extinguisher

Balance models are used to mathematically describe the production process by describing the way the production inputs transform into the goods or half-finished products (semi-product). The difference between the two are in its usage. We use semi-products to produce other semi-products and product, but the (final) products, are finished and are not further consumed in the nodes, they always go outside of the model.

It is possible to express interconnection between raw materials, half-finished products and products by following equation 11.1:

$$x = y + Ax \tag{11.1}$$

where x is overall production of the given product, y is the production expedited outside modeled system, Ax is amount of products used for production of different products and finally A is a matrix of specific consumption of semi-products

To apply the general model eq. (11.1), we need to know the way inputs are being transformed to outputs inside the production process. To do that we need to identify specific:

- raw materials - these are inputs

- semi-products - these are being produced during production process and are being utilized for production purposes (to produce another product). Semi product can be also sent outside of the model
- (finished) products - all of these go outside of the model (they are not used to produce something else in the model)

When we talk about consumption in the production process we mean consumption in the modeled part of the production process. From this point of view we may also see raw materials as a final product of some other production process, which we do not see being produced in our model. Similarly final products may be utilized in other production processes, which we again do not see in our model.

One of basic questions for the modeling is the decision on the scope of the balance model. So we may decide to model single production line, whole organization or even entire supply chain depending on the type of information (purpose) we want to derive from the model.

Equation 11.1 is written in matrix form. If we express the products from figure 11.1, we get following interpretation of the model:

- x_1 extinguishing material (solid)
- x_2 extinguishing material (powder)
- x_3 valve
- x_4 metal plate
- x_5 body of extinguisher
- x_6 fire extinguisher

We can express equation (11.1) using system of equation (11.2). To specify the balance model, we have to state specific consumption of the half-finished products, see table 11.1.

Table 11.1: Specific consumption

Product	**Half-finished product**	**Specific consumption**
Fire extinguisher	Extinguishing material (powder)	10 kg/1 part
	valve	1 part/1 part
	body of extinguisher	1 part/1 part
Extinguishing material (powder)	Extinguishing material (solid)	1 kg/kg
Body of extinguisher	Metal plate	0,25 plate

$$\begin{aligned} x_6 &= y_6 + 10x_2 + x_3 + x_5 \\ x_2 &= y_2 + x_1 \\ x_5 &= y_5 + 0,25x_4 \\ x_1 &= y_1 \\ x_3 &= y_3 \\ x_4 &= y_4 \end{aligned} \tag{11.2}$$

This equation system is easily solvable, when we understand that the production values are defined by commissions and half finished products may be completely substituted by its balance equations. Simple production systems are even solvable by hand, for more complex scenarios we usually prefer using spreadsheet.

Equation (11.1) may be also expressed in this way (11.3):

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,25 & 0 & 0 \\ 0 & 10 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \tag{11.3}$$

The balance model higher is very simple. In practice we often formulate several models describing different aspects of the production process. We split it into several smaller, more manageable models. Event these small models may consist of hundreds variable. Out illustrative example is very simple as it consist only from one final product, two semi-products and three raw materials - together six variables.

Apart from using basic balance model to quantify number of required products and semi-products, we usually use also additional model to compute raw materials needs, energetic requirements and technological limitations.

All these additional models require computed product balance. So the computation always starts with computing (11.1). From this we may derive raw materials needs which are required to produce them, see eq. 11.4.

$$Bx = s \tag{11.4}$$

where x is a vector of the overall production of the given product, s is vector of the overall need of the raw materials to produce all products x and B is a matrix of the technical coefficients describing raw materials usage to produce product x.

Similarly capacity model (11.5) for the production can be used.

$$Cx = f \tag{11.5}$$

where x is vector of the overall production of the given product, f is vector of the requirements on production machines, etc. and C is a matrix of coefficients describing the needed capacity of given machine to produce unit of product x

As you can see the model will not answer the question if the there is enough capacities available to fulfill the production needs of the company. The results have to be interpreted – we get the information on how large the required capacities are. It is up to us to know if we have large enough "real" capacities and if not to solve the problem. Perhaps we can rent capacities elsewhere, the model by itself would not know.

We interpret models described by equation (11.1) and (11.4) in same way.

When constructing and using the model our limitation is only our fantasy. We can work with alternative scenarios, incorporate results into other models. We can model dynamically to explore whole fan of possible scenarios. We can do many things.

For sake of completeness we provide also computation in MS Excel (see the box around) and in R. Structure of the model is available on fig. 11.2.

The R computation uses matrix approach as described on previous pages.

Listing 11.1: Computing 4 products, 6 raw materials balance model from fig. 11.2

```
products <- c('product 1', 'product 2', 'product 3', 'product 4')
rmaterials <- c('material A', 'material B', 'material C', 'material D', 'material E', 'material F')
A <- matrix(c(0, 0, 0, 0,
  0, 0, 0, 0,
  0.3, 0.15, 0, 0,
  0.2, 0.35, 0.5, 0), nrow = 4, ncol = 4, byrow = TRUE)
colnames(A) <- rownames(A) <- products
E <- diag(4)
inverseEA <- solve(E - A)
colnames(inverseEA) <- rownames(inverseEA) <- products
# requirements outside of model
Y <- c(20, 40, 10, 150)
# overall production of the products
X <- inverseEA %*% Y
# raw materials consumption
B <- matrix(c(0.15, 0, 0, 1.1,
  0, 0, 0.3, 0.15,
  0, 0, 0.05, 0.3,
  0, 0.14, 0.11, 0,
  0, 1.01, 0, 0,
  0.45, 0.32, 0.42, 0.2), nrow = 6, ncol = 4, byrow = TRUE)
colnames(B) <- products
rownames(B) <- rmaterials
# raw materials requirements
S <- B %*% X
```

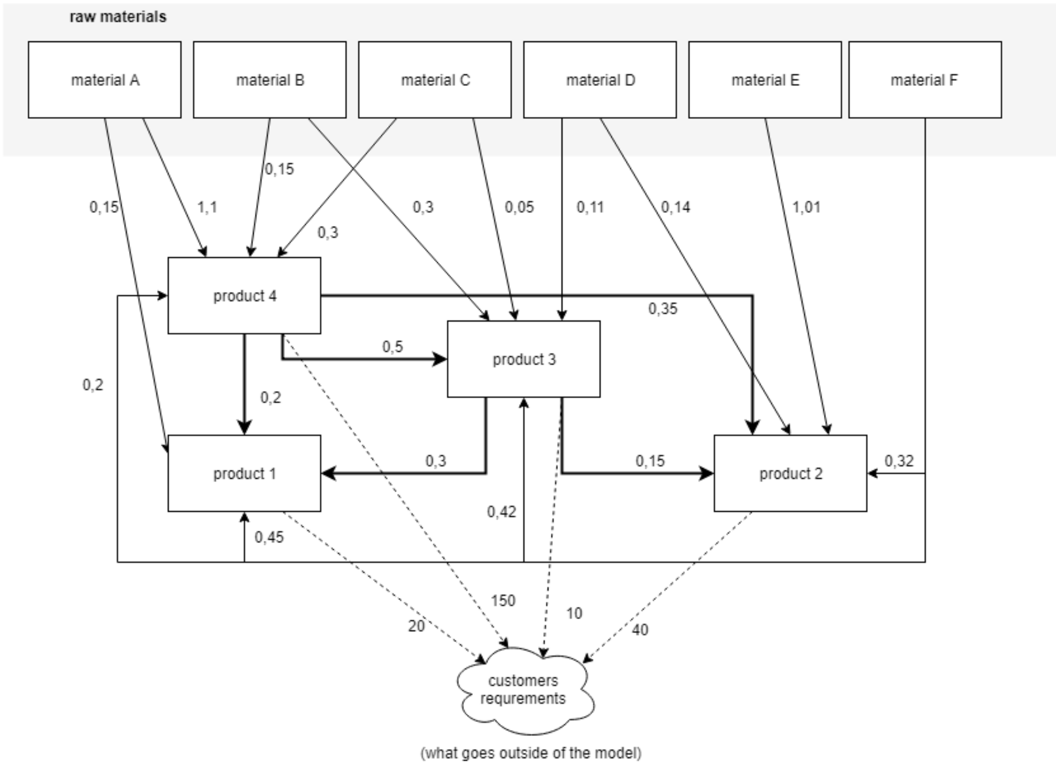


Figure 11.2: Structure of example balance model (courtesy of Gros [11])

Balance models
Example of balance model is uploaded to http://lms.vsb.cz in XLS format (MS Excel). The example is solved in two different ways (equal result wise).

Questions

1. What are balance models good for?
2. Explain balance equation of the product.
3. Can you specify parts of the balance model (what can by computed by it)?
4. How do we interpret the model (what information does it compute and how do we work with it)?

11.2 Application of balance modeling to security problems

From previous section we already know, that the purpose of the balance models is to allow us understand balances structure describing transformation of materials and energies in modeled system. That information can be relevant for planning purposes as it provides different perspectives to:

- Bussines Continuity Planning (BCP)
- in accident management planning - for quantification of the accident on the production process
- recovery planning after the emergencies
- and possibly other types of plans

We can add presumed impacts of the emergencies by introducing boundary conditions to the model. Then we can use model to quantify effect on production process. This type of information will allow us to identify parts of the process where the organization can benefit most by better planning and investing in protective measures.

As a student of security related study program, you already have (or should have) relatively in depth knowledge on risk. If we consider one of simplest models risk models $R = P \cdot C$ defining the risk as probability (P) of consequence (C) occurrence, we need a way to quantify the consequence. Possible damages to technology and buildings can be estimated as a costs to repair or replace the damaged, but what of a time the organization will be unable to produce?

Balance models are one way to shed some light to the problem. There is also another benefit. These models are often routinely used in Enterprise Resource Planning (ERP), so the results of such models are usually easy to communicate with organization's management. In turn this can allow us te more easily find support for necessary investment into protective measures.

For applications in CI some input-output models have been developed. One of more popular is Input-output Interoperability Model (IIM). The model has been developed by Setola et al. [68] in 2009. From mathematical point of view this model is almost same as classical I-O models, see eq. (11.6).

$$x = Ax + c \tag{11.6}$$

where x is vector of production in sectors of interest in economy, A is consumption matrix allowing us to model relations (dependence) between the sections and finally c is a vector of external demand, which represent the production going out of modeled system.

The benefit of IIM is that we can use it in simplified form, which considers only the intensity of the relations between sectors of CI. In such case we may limit $x \in < 0; 1 >$. For modeling of cascade effects, the disruption propagation across the sectors of CI, $x = 0$ will mean complete interruption of the "communication" to dependent sectors, while $x = 1$ will mean that the disruption in sector will have no effect on operation of dependent sectors (disruption does not propagate to these sectors).

For IIM model we can compute *dependency index* (11.7).

$$\gamma_i = \sum_{j=1}^{n} a_{ij}^{*} \tag{11.7}$$

Technically we just sum the rows of matrix A.

Dependency index expresses robustness of infrastructure considering its interoperability with other section. It represents maximal interoperability in infrastructure i in case, that other sectors of CI are not functional.

The lower the decline in γ_i (when $\gamma_i < 1$), the better the infrastructure can maintain its operation and provide services to other infrastructures.

Summing columns of matrix A we can get the value characterizing influence of the sector (11.8), also known as *influence gain*.

$$\rho_j = \sum_{i \neq j} a_{ij} \tag{11.8}$$

High values of ρ_j mean, that the interoperability of j sector introduces significant degradation of the system, when its disturbed.

Today, there are also available more sophisticated modeling tools, allowing us to chain partial models one after the another to get full picture on consequences of modeled event to the region.

Example of such system is HAZUS [15], which is being developed and extensively used by FEMA.

Model chain is illustrated on fig. 11.3.

HAZUS is based on geographical data managed by Geographic Information System (GIS). It allows estimation of possible polygons of expected consequences of the modeled event (i.e. hurricane) and integrates it with stored information on infrastructure present in the area.

Model of hurricane has been used in fig. 11.3. In left part of the figure, colored polygons are used for different wind speeds. Together with information on the infrastructure, the model can choose appropriate vulnerability curve to evaluate what kind of damages will the wind cause to the infrastructure.

Chaining the models together - maps, polygons characterizing the event, consequence to infrastructure computation, IIM to see how the disruption will spread provides complex information to support strategic decision making for local governments and institutions like FEMA to form public policies, standards and organizations to prepare for such events.

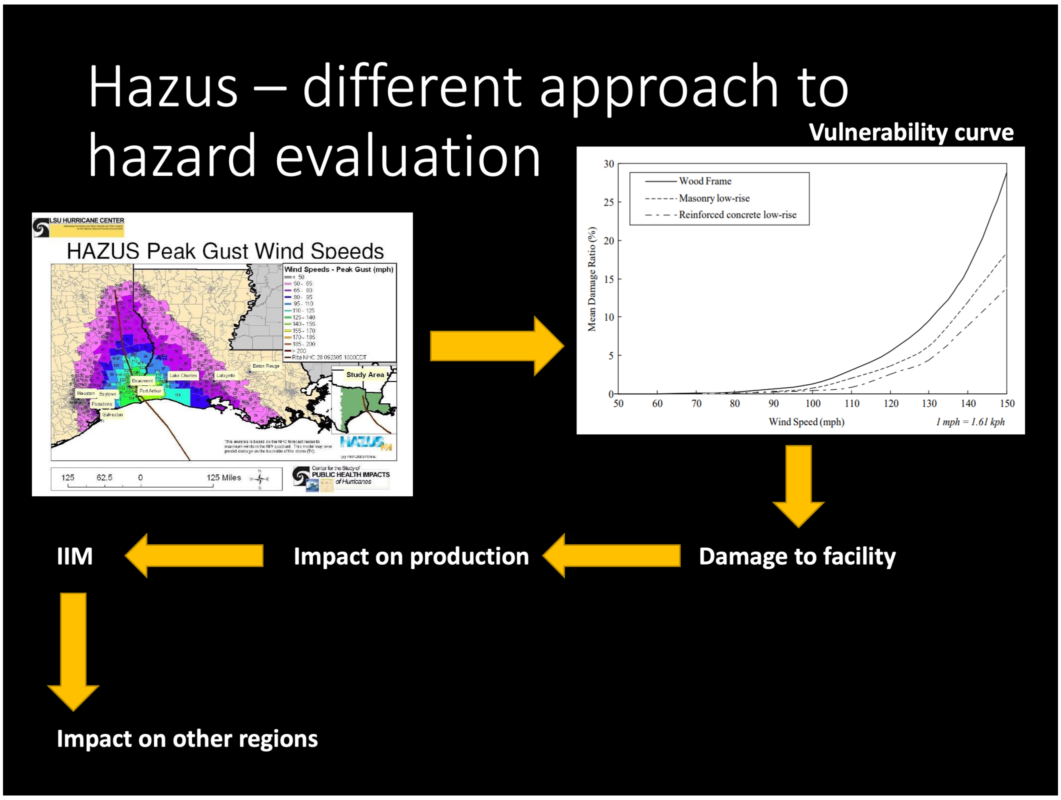


Figure 11.3: HAZUS - model chain (courtesy of [15])

HAZUS itself is available free of charge. To get the copy the registration is needed with specification of the purpose. It is a logical choice, if we consider, that the models allowing us to plan our effort to minimize the impact of the event, can be also used to plan the event to be maximally effective.

To operate it, you will need ArcGIS and geographical information - maps, infrastructure, etc. for the region at appropriate resolution level. Most of the countries do have these dataset available in some form.

Vulnerability curves

Vulnerability curves are one of most important tools we have to estimate consequences of accident and disruptions of various kind. They map strength of the effect with potential to create damages (i. e. wind speed) to actual damages. Main disadvantage is, that we need to construct them separately for different types of events and different types of infrastructure.

To do that extensive information on damages from previous occurrences of event is required.

Even as vulnerability curve construction is not easy, there are many available in existing literature. During your study, you can gain basic outline of these in course *Consequence modeling for emergencies* or in textbook to this course [69][a].

[a]at present time in Czech only

Summary
Balance models are application of input-output models used for modeling of the relations between inputs and outputs inside of production process. Balance models consist of:

- product model,
- raw material model
- capacities model.

First we compute product model as all other models use it as its input.

For purposes of emergency planning we can use this type of models the better understand consequences of disruption to the production process as result of some kind of accident or other type of event.

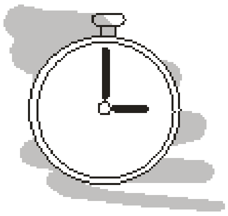
Chapter 12

Linear Programming

Study guide
Another type of problems, we have to solve, are those described by system of constrains expressed by equations. For such type of problems we are searching for solution inside the constrained solution space minimizing or maximizing some objective function.

After studying this chapter you will know
How

- to solve linear programming problem using simplex method



Time required to study chapter
For getting through this chapter you will need at least an hour.

Linear programming falls into group of algorithms for mathematical solution of optimization problems called *mathematical programming*. Linear programming is called linear, because we solve the problem using system of linear equations.

In linear programming we work with one basic objective function which we optimize. We usually do that by maximizing or minimizing system of the linearly expressed constrains of our problem. Using these constrains we limit the space of possible solutions – so we may actually come to some conclusion.

We may express objective function using following equation (12.1).

$$optimz = \sum_{j=1}^{n} c_j x_j \tag{12.1}$$

Where z is objective function which we maximize or minimize (i.e. max z)
c_j evaluation of variables in objective function (for example price, variable costs etc.)
x_j optimized variable

To limit the solution space of the objective function z we may use three types of the constrains. You can see its mathematical expression in equation (12.2).

$$\begin{array}{rcll} \sum_{j=1}^{n} a_{ij}x_j & \leq & b_i & i = 1, 2, ..., k \\ \sum_{j=1}^{n} a_{ij}x_j & = & b_i & i = k+1, k+2, ..., k+p \\ \sum_{j=1}^{n} a_{ij}x_j & \geq & b_i & i = k+p+1, k+p+2, ..., k+p+s \\ x_j & \geq & 0 & j = 1, 2, ..., n \end{array} \tag{12.2}$$

Where

a_{ij} technical coefficient of the model, these are set independently for each model and they are constants of the model

x_j optimized variable

b_i right sides of the constrains, these are set in dependence on model (capacity constrains etc.)

The optimized variables must have zero or positive value. This constrain is based on fact, that we usually can't produce negative amount of whatever. Allowing for negative values would lead to non-standard problem, which would not be solvable by linear programming (at least not by method we will use).

First lower then or equal constrain ($\leq$) denotes the situation, that the solution of the presented problem is constrained by availability of some kind of goods or material and the amount of it is limited. The equal sign ($=$) denotes the situation, when there are known production processes (or similar), where we know exactly how they do come out. For example we must know exactly how much of raw materials is needed to produce unit of the desired product.

Finally greater then or equal ($\geq$) describes the situation, when we try to introduce into model some kind of obligation – for example we have already agreed to supply certain amount of the goods – that amount would be the minimal value and we can go only higher from there.

So what are the problems which are solvable by this method? Primary use of this method is in economy for optimization of the manufacturing program (for sake of profit), but it is general method, which is usable generally – for example for optimization of venting gas from one source using several venting stations, for optimization of transportation of the good using different trucks with different capacity etc.

Linear programming is also a common type of algorithms used by other methods to achieve their goals. For example in following chapters of this textbook we will discuss problematic of localization models and basics of game theory and both of these can be solved using linear programming (but net necessarily by primary algorithm we are going to learn in this chapter).

So how exactly do we formulate the problem and how do we solve it. As first step it is necessary to formulate the model using equations (12.1) and (12.2) – as system of objective function and its constrains, as in the next example:

$$maxz = 2x_1 - 3x_2 + 4x_3 \tag{12.3}$$

$$\begin{array}{rcl} 4x_1 - 3x_2 + x_3 & \leq & 3 \\ x_1 + x_2 + x_3 & \leq & 10 \\ 2x_1 + x_2 - x_3 & \leq & 10 \end{array} \tag{12.4}$$

Such system of equations is solvable using variety of available algorithms as:

1. primary algorithm (simplex method)
2. dual algorithm
3. ...

We will use only the simplex method also known as primary algorithm as most famous representative of linear programming algorithms for solution of this type of problems.

Following text is rather harsh, especially if you heard of linear programming just few moments ago, that's why, you should try nice tutorial by Stephen Waner [70], which is in English and will interactively lead you step by step to learning the method.

So let's solve the problem using simplex method. First step is to transform the constrains to equations. We do that by adding additional variable d_i if the operator was $\leq$. If the operator was $\geq$, we have to formally transform it to $\leq$ by multiplying whole equation by -1. Our example would look as follows.

$$\begin{array}{rcl} 4x_1 - 3x_2 + x_3 + d_1 & = & 3 \\ x_1 + x_2 + x_3 + d_2 & = & 10 \\ 2x_1 + x_2 - x_3 + d_3 & = & 10 \\ -2x_1 + 3x_2 - 4x_3 + z = 0 & & \end{array} \tag{12.5}$$

Additional variables, if computed in optimal solution as not zero represent amount of resource b_i (see (12.1)), which will remain, which will not be used. System of equations higher can be transformed to matrix form (see table 12.1).

Table 12.1: Working table

x_1	x_2	x_3	d_1	d_2	d_3	z	
4	-3	1	1	0	0	0	3
1	1	1	0	1	0	0	10
2	1	-1	0	0	1	0	10
-2	3	-4	0	0	0	1	0

With each table there is associated just one *basic solution*. Basic solution is one of virtually infinite number of possible solution. To find such solution we search for clean columns, in which there is only one nonzero value in the row. Variables in these columns are active, while the remaining variables are not active. Basic solution of our table would be as follows (see table 12.2).

Please note that by equalizing the inequalities we introduce many additional variables we need then compute together with the original variables (x), which we are trying to optimize in first place.

From basic mathematics we know that we are able to compute at maximum number of variables lower or equal to number of equations. Since we add a lot of additional variables there is a good chance that suddenly we have more variables then we have equations, meaning we will not be able to compute them at once (in single iteration).

Idea of basic solution is to give us a computation which will provide result respecting set constrains and also way of identifying which variable can be computed in the iteration. For all other variables we will presume that they are equal to 0 and will not be part of basic solution.

This is also why it is so important to formulate as much constrains as the solved problem allows to. The more constrains we formulate, the faster we produce the results.

Table 12.2: Iteration 0 – working table: basic solutions

x_1	x_2	x_3	d_1	d_2	d_3	z		**Basic solution**
4	-3	1	**1**	0	0	0	3	$d_1 = 3/1 = 3$
1	1	1	0	**1**	0	0	10	$d_2 = 10/1 = 10$
2	1	-1	0	0	**1**	0	10	$d_3 = 10/1 = 10$
-2	3	-4	0	0	0	1	0	z $= 0/1 = 0$

As you can see in table 12.2, only variables d_1, d_2, d_3 and z are active all other variable are inactive (x_1, x_2 and x_3) and we set them to zero. Active variables form up basic solution for this iteration. For example we compute the value of d_1 dividing right side of equation with value for d_1 in its column so $d_1 = 3/1 = 3$.

Basic solution for this iteration would be:

$$\begin{aligned} d_1 &= 3 \\ d_2, d_3 &= 10 \\ x_1, x_2, x_3, z &= 0 \end{aligned} \tag{12.6}$$

Note that in basic solution of first iteration we have only our additional variables (d), we added to the computation to equalize the inequalities. All our regular variables (x) are inactive and thus equal to 0, meaning that also $z = 0$.

Such solution is compatible with stated constrains, but can hardly be considered optimal. Fortunately we can continue our computation with another iteration to get better results.

To do that we need to transform table after chosen pivot. We search for pivot using two simple criteria:

1. *Choosing of the column for pivot* – we choose the column with most negative number in objective function (last row of the table). The lowest number is -4 in x_3 column.
2. Pivot has to be always positive number with highest testing ratio. Testing ratio is the ratio of right side of the equation and value of the column. We compute testing ratios in table 12.3.

Pivot is in the intersect of the chosen column and row. In table 12.3, it is highlighted in bold italique (in the croscection of the highlighted row and column).

In next step, we have to adjust the table so, that the pivot column is zeroed out, excluding pivot row (see table 21). Pivot row remains the same.

Table 12.3: Testing ratio

x_1	x_2	x_3	d_1	d_2	d_3	z		**Testing ratio**
4	-3	**1**	1	0	0	0	3	$3/1 = 3$
1	**1**	***1***	**0**	**1**	**0**	**0**	**10**	**10/1 = 10**
2	1	**-1**	0	0	1	0	10	
-2	3	**-4**	0	0	0	1	0	

Table 12.4: Iteration 2

x_1	x_2	x_3	d_1	d_2	d_3	z		**adjustment**
3	-4	0	1	-1	0	0	-7	- row 2
1	1	1	0	1	0	0	10	
3	2	0	0	1	1	0	20	+ row 2
2	7	0	0	4	0	1	40	+ 4 * row 2

Because there are no negative numbers in effectiveness function row, the basic solution of this iteration is also the optimal solution of the task. If there was a negative value in last row, we would repeat the whole process over and over again.

Our solution is:

$$\begin{aligned} x_1, x_2 &= 0 \\ x_3 &= 10 \\ d_1 &= -7 \\ d_3 &= 20 \\ d_4 &= 40 \end{aligned} \tag{12.7}$$

Of course, it would be highly ineffective if we had to compute the solution by hand, that's why we usually use linear programming calculator, such as [71]. Usually the programs for mathematical computations such as MathLab, SciLab or R support linear programming too. There are also smaller single purpose programs specialized on linear programming such as Finite mathematics utility: simplex method tool [72].

The equation system for the problem is written directly into window of the WWW browser of your choice, see fig. 12.1. See definition and results ... yes they are different based on type of algorithm and the way it is implemented.

Notice tableau part of the fig. 12.1. It shows tables the algorithm computed. Each of these represents one iteration of the computation. You could even derive basic solution based on these, if you really wanted. But there is no need for that as the app provides end result in solution text field.

Provided solution may, or may not be logically correct. For example the application computed that $y = 1.75$, but what if such solution did not have logical explanation. For example we could be producing some products (x and y). We can't produce 1.75 units of a product, it must be either 1 or 2.

In other words the linear programming is problem agnostic. It does not realize the rationale behind the system of equations we provided. Which means that we need to work with provided solution, interpret it and adjust it when needed. We can't mechanically take the result and use it.

We can also demonstrate computation of the linear programming problem in the R. We will use same example, but this time around we will compute it using *lpSolve* package [73]. Remember to install it first using install.packege() function, or via GUI in RStudio.

Listing 12.1: Linear programming exmaple solution using lpSolve package

```
library(lpSolve)
f.obj <- c(2, -3, 4)
f.con <- matrix(
  c(4, -3, 1,
    1, 1, 1,
    2, 1, -1),
  nrow = 3, byrow = TRUE)
f.dir <- c("<=", "<=", "<=")
f.rhs <- c(3, 10, 10)
```

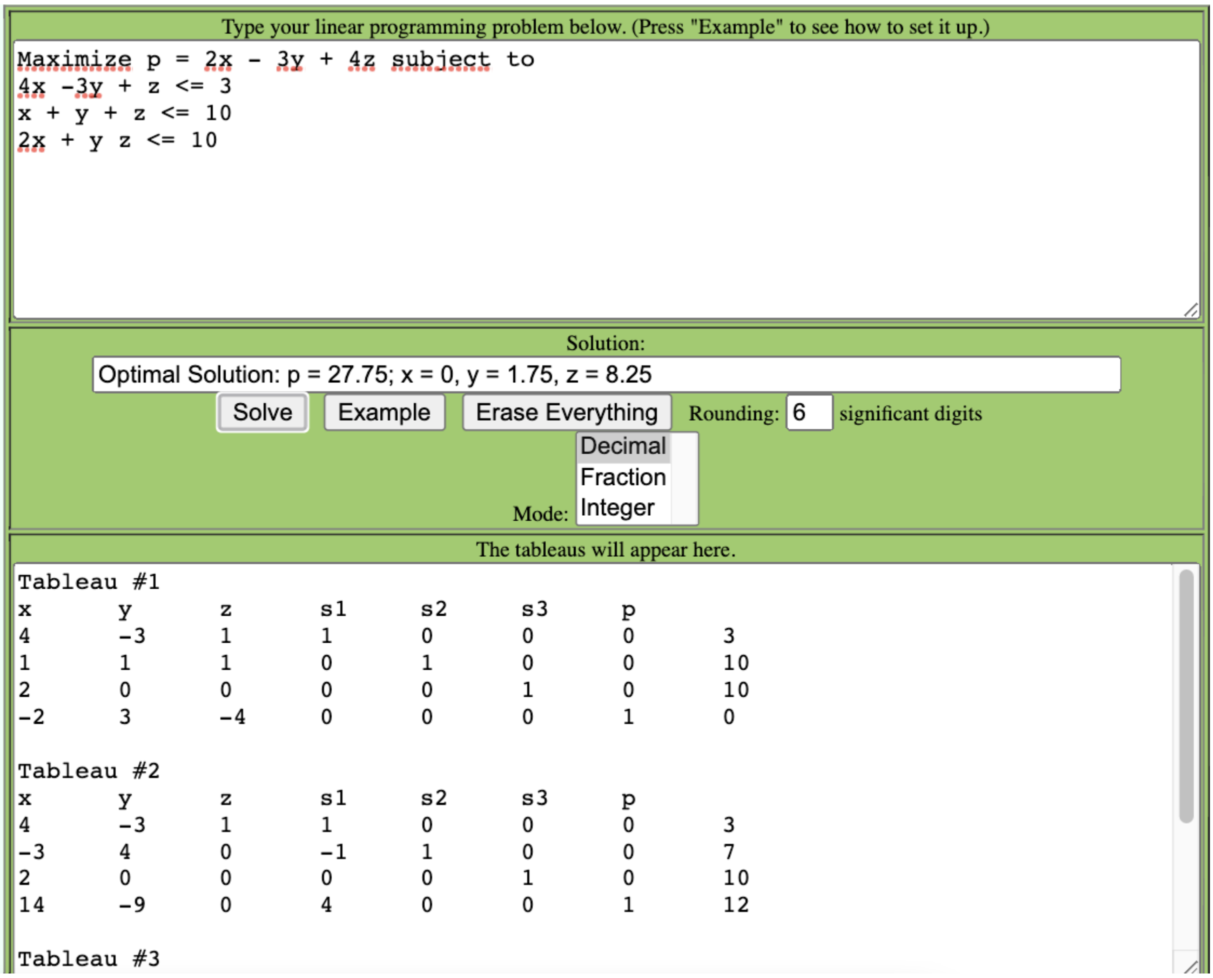


Figure 12.1: Finite mathematics utility: simplex method tool

```r

result <- lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)
result$solution # Variables final values
# sensitivities
result$sens.coef.from
result$sens.coef.to

result$duals # Duals of the constraints and variables are mixed
# Duals lower and upper limits
result$duals.from
result$duals.to
```

Both web app and lpSolve came with exactly same solution: $x_1 = 0, x_2 = 1.75, x_3 = 8.25$. If both approaches yield same results, why bother with "programming" in R. Main reason is that it provides much higher flexibility. It allows us to compute much larger tasks and also explore the results in much grater detail.

For example it allows us to analyze sensitivity of the solution to the changes of the coefficients. For that we can use sens.coef.from and sens.coef.to to find lower and uppoer bound of the solution. It allows us to specify range of the objective's function coefficients, for which the computed solution is still optimal.

In our case $x_1 \in < -1.0e+30; 9.25e+00 >$, $x_2 \in < -1.2e+01; 4.00e+00 >$ and $x_3 \in < 1.0e+00; 1.00e+30 >$. The computation using the package is purely technical. That means that the *lp()* function responsible for solving the program and computing the sensitivities, has no knowledge of true meaning of what has been computed. If the coefficients represented some production process, i.e. number of products 1, 2 and 3 we should produce to maximize the profit, then values < 0 have no meaning. Sensitivities may be computed way outside of the ranges, that are possible in practice (in interpretation of the model).

That is also one of reasons we had to explicitly set lp() to compute it using compute.sens = TRUE parameter, as the function does not compute sensitivities by default.

So what does it mean? How do we interpret it? To answer this question, let's consider much simpler task to compute composition of the vegetables the farmer should plant in the spring. Let's limit selection to only two vegetables, se we can interpret the results in 2D, using graph. This example is adapted from Ong [16], which discusses the problem in much more detail.

It also provides introduction to the R and examples in statistics a and solving of optimization problems. While at the time writing of this textbook, the resource was in draft state only, what was available was very good, so the resource[1] can be recommended for further study.

Back to our example

$$\begin{aligned} maxp &= 0.15x_1 + 0.40x_2 \\ x_1 + x_2 &\leq 200 \\ 0.20x_1 + 0.70x_2 &\leq 100 \end{aligned} \tag{12.8}$$

Optimal solution for the problem would be $x_1 = 80$ and $x_2 = 120$, with sensitivities for x_1 coefficient $\in < 0.1142857; 0.400 >$ and x_2 coefficient $\in < 0.15; 0.525 >$. Graphically we can see the solution on fig. 12.2.

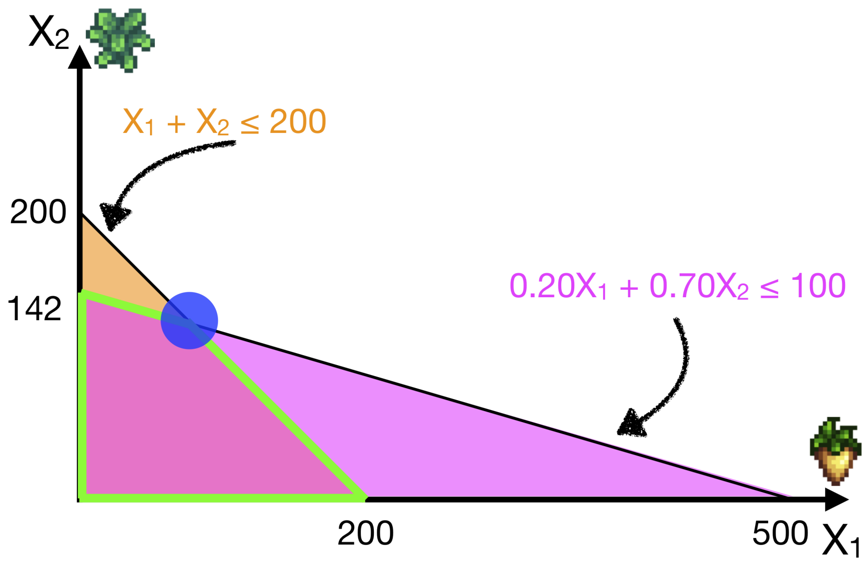


Figure 12.2: Farming problem - constrains and optimal solution visualization (courtesy of Ong [16])

Since we are working in 2D we can interpret the constrains easily as the lines which limit possible solution space. Green highlighted area represent all possible solutions for which the constrains hold. Blue point then represents optimal solution.

If one of the coefficient in objective function changes so drastically, that it would go out of range identified by sensitivity analysis, the solution would shift. If the coefficients in our example represent unit selling price of the vegetable, that selling price would be only an estimate due to demand and availability. If coefficient for x_1 changed from 0.15 to 0.1, which is outside of range, the price for vegetable x_1 would be so low, that it would no longer be profitable to sell it. So all production would shift to x_2.

Another form of sensitivity analysis is analysis using *shadow prices*. Ong [16] defines shadow price of a constraint as the change in the objective function value per unit-increase in the right-hand-side value of that constraint with all other things being equal (ceteris paribus). Let's use farming example once again and change one constrain from $0.2x_1 + 0.7x_2 \leq 100$ to $0.2x_1 + 0.7x_2 \leq 101$.

We can see the difference on graph, fig. 12.3. The line representing the constrain moved slightly right, but that also slightly moved the optimum.

[1] https://desmond-ong.github.io/stats-notes/

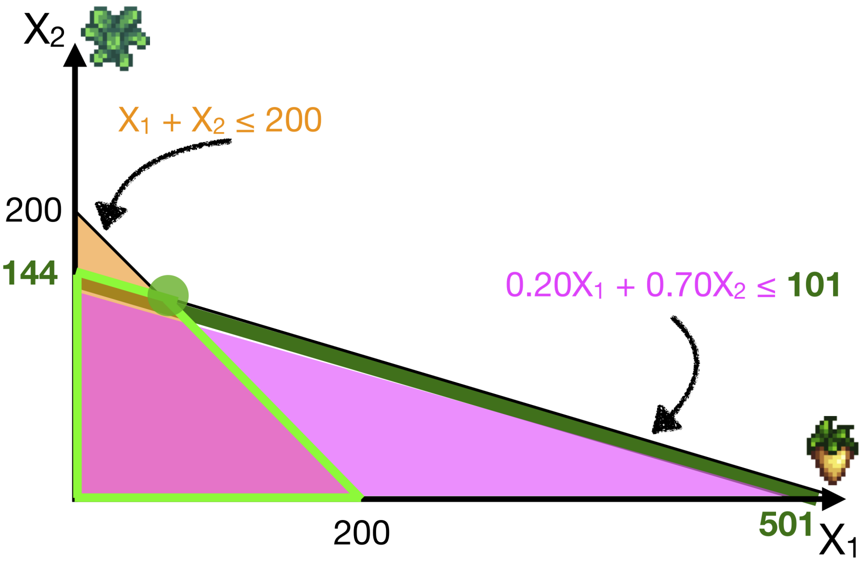


Figure 12.3: Farming problem - shadow price demonstration (courtesy of Ong [16])

New optimum would be $x_1 = 78$ and $x_2 = 122$, leading to increase of profit of 0.5 to 60.5. Computed 0.5 is the shadow price.

Now let's go back to R and see what kind of answers provides lpSolve's lp() function. We call shadow prices by calling property duals of computed results. $duals provide vector with number of elements equal to 2x number of constrains used in linear programming problem.

First three represent shadow price for the three constrains, in our case 1.75, 2.25 and 0.00. The last three numbers (-7.25, 0, 0) are shadow prices of implicit non negativity constrains $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$.

Linear programming task

Two students Ann and Charles work X and Y hours weekly. Together they may work only 40 hours a week. Considering rules for the brigade-workers, Ann may work maximally 8 hours longer then Charles, but Charles may only work 6 hour longer then Ann. Additional constrain is $18 \leq 2y + x$. Try to find optimal solution for situations:

1. If hour pay of Ann is 15 EUR and Charles's 17 EUR - find maximal combined pay
2. If hour pay of Ann is 17 EUR and Charles's 15 EUR - find maximal combined pay
3. If both of them get 16 EUR - find maximal combined pay

Linear programming task - solution
Ann 15 EUR (y) and Charles 17 EUR (x)
Variable represents hours each of them will work.
Effectiveness function: max $z = 15x + 17y$
Both may not exceed 40 hours a week: $x + y \leq 40$
Ann may work 8 hours longer then Charles: $x - y \leq 8$
Charles may work 6 hours longer then Ann: $-x + y \leq 6$
Additional constrain: $x + 2y \geq 18$
Optimal solution: Ann 17 hours, Charles 23 hours, they will earn 646 EUR

Solution second sub task:
Try to construct the constrains yourself.
Result: Ann 24 hours, Charles 16 hours, they will earn 648 EUR

Solution third sub task:
Result: Ann 17 hours, Charles 23 hours, they will earn 640 EUR

Linear programming task 2
The company produces two products (X and Y) and for this, it is using two machines (A and B). For production of each unit of product X, it is necessary to use 50 minutes of machine A and 30 minutes of machine B. For producing unit of product Y we need 24 minutes of machine A and 33 minutes of machine 2
At the beginning of the week, there are 30 units of product X and 90 units of product 90 B in the store. Available time of machine A is 40 hours and B 35 hours.
Demand for product X this week will be 75 units and Y 95 units. Company's politic states that we maximize sum of products X and Y in store at the end of the week.

Linear programming task 2 - solution
Produce 45 units of product X and 6 units of product Y (in store at the end of the week 51 units)

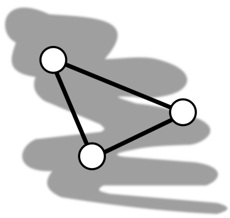
Not sure if you can solve the linear programming problems?
Good news, the Internet is virtually inexhaustible source of the examples, you can practice on. Solve them by both R and Simplex method tool.

(Or choose one of these which suits better your needs or personal preferences.)

Summary
Linear programming is used for solving of optimization problems expressed by objective function (which needs to be optimized) and its constrains expressed as a system of linear equations or inequalities.

To solve it, we often use so called *primary algorithm*, also known under name *simplex method*. The methods works with iterations. Each iteration has connected one solution called *basic solution*. With each iteration we improve the solution, getting closer and closer to the optimum.

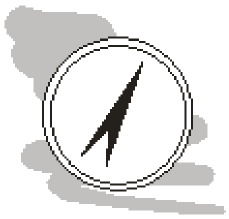
Most of the times we compute linear programming problems using software. It is both faster and also allows us to get additional information, for example for sensitivity analysis.



Connection to other models and approaches
One of reasons we choose to discuss the linear programming in this text book is its versatility. Since the model is entirely agnostic to meaning of the coefficients and variables of the model, it is possible to transform lot of problems to the problems of linear programming. We will see that in last two chapters focusing on Game theory and Localization models.

Chapter 13

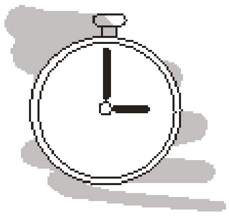
Basics of the Game Theory



Study guide
So far we focused on the decision making based on "hard", measurable data, but most decisions take place in dynamic environment, where other people are trying to achieve their goals. Game theory provides us with understanding of how these interactions between the "players" actually work.

After studying this chapter you will know

- what is game theory a what is it good for
- how to solve simple problem
- various model situations and its specifics



Time required to study chapter
For getting through this chapter you will need at least 30 minutes.

Up to now we described the decision or preparation to decision as process using objective "hard" characteristic, when there was no opponent to the decision with different goals then we. The decision simply hasn't been made against human being. Such situation is not entirely common. So we need a tool to deal with this type of situation too.

Theory describing these decision situations is called *game theory*. Decision situation is described here as a *game*, where all decision makers are *players*. For first demonstration of the theory, we usually use theoretical problem called prisoners dilemma.

The prisoner dilemma is following: *Two prisoners are awaiting their trial. They are being kept in isolation in order to prevent them to coordinate their answers to the inquiry. So they sit alone in their cells and think what should they say during questioning. Should they cooperate or not? If the prisoner A frames prisoner B and prisoner B will say nothing, prisoner A will get free while prisoner B gets hard sentence. If both prisoners will frame each other, both of them will get hard sentence. If both prisoners will not speak at all, they will both get low sentence. Finally if prisoner B frames prisoner A and prisoner A will not speak, prisoner B will get free while prisoner A will get hard sentence. What should both prisoners do?*

Clearly finding the optimum is not simple. The outcome for the prisoner depends on their choice in combination with the choice of other prisoner and they have no control over him and even don't know, how they have chosen.

To better understand possible strategies prisoners can take, can be written down in tabular form. See table 13.1 for detail representing point of view of prisoner A.

We can formulate inverse table for prisoner B, but for this type of the decision it is not necessary.

Table 13.1: Decision strategies – prisoner A

		Prisoner A	
		don't cooperate	**cooperate**
Prisoner B	**don't cooperate**	normal sentence	low sentence
	cooperate	hard sentence	hard sentence

The goal of both prisoners is to maximize their utility in situation, when they do not have information on which strategy the opponent chooses. So we choose one based on what we think the opponent will do. Such decision is not entirely rational.

The goal of the game theory is to cultivate the game so the players do not necessarily choose for the strategy maximizing their utility without considerations of other player, but the one witch is optimizing their utility. We are searching for optimal strategy.

If we expressed the evaluation in numerical form, the numbers would form up matrix of the profits A about witch we could write the following (13.1):

$$\underset{i}{max} \; \underset{j}{min} \; a_{ij} \leq \underset{i}{min} \; \underset{j}{max} \; a_{ij} \tag{13.1}$$

Gros writes about the situation following [11]: ... *highest from minimal profits, which can first player secure can't be larger then highest profit, which the second player tries to minimize.*

In situation when there is a point representing equality of both sides of (13.1) we say that the matrix of the profits has a *saddle point*. This point represents optimal strategy, the player should choose if he wants to optimize his profit.

Saddle point has some interesting properties. First it identifies optimal strategy (net necessarily same) for both players. If the plaers both take their strategy, their utility will be exactly represented by utility of saddle point and will be same for both of them.

Interesting situation would arise if one of the players chose to change his/her strategy, while the other player would still hold strategy prescribed by saddle point. In such case the "wins" of player holding saddle point (optimal) strategy, while winnings of other player would decrease.

That means for rational player trying to maximize their utility (winnings) to identify and hold to optimal strategy without the need to actively coordinate with other player. Both players will reach same conclusion on their own.

Let's demonstrate saddle point computation on our prisoners dilemma problem. First we have to evaluate the winnings for the strategies in numerical form. In our case lets say that low sentence will be 50 points, freedom 100 points, hard sentence 0 points.

Table 13.2: Searching for saddle point

		Prisoner A		min_j
		don't cooperate	**Speak**	
Prisoner B	**don't cooperate**	**50**	100	50
	cooperate	0	0	0
max_i		50	100	

Saddle point (in table bold) we got this way recommends prisoner A and B not to speak. If the prisoner B knows game theory, he will choose this strategy and both will optimize their utility.

Such simple games are called games of *two players in normal state*. Unfortunately not all games have such simple solution (the saddle point) – in such case we talk about *mixed extension of two players game*. Under these difficult circumstances we do not search for single optimal strategy, but for mix of strategies and probabilities by which we should change them. We search for mixed strategies. All games expressible by matrix have solution in **mixed strategies**.

From other point of view the normal state games are special case of mixed games, where the whole mix of the strategies consist of single strategy.

Such game is transformable into form solvable using methods of linear programming.

If all decision situations have had saddle point we would see the benefits of its usage more clearly. In situation, when we have problems to quantify possible reactions on "multiple fronts" as we are rarely in situation when we have single, clearly defined, "enemy". In reality there may dozens of them all pursuing their agendas. In such situations using game theory is much harder and its benefits are much harder to see. So why are we even discussing this problematics as we clearly have no room for in depth analysis of all aspects of the game theory?

The short answer would be – to change a mind set of you as a student. And to do that we will introduce model types of the games with known outcomes. There are dozens of games, but we will focus just on two of them and then one additional games as a bonus at the end of the chapter.

1. game of chicken
2. stag hunt

Game of chicken in games theory is based on real-world, dangerous game, favored by less responsible youth, which involves two strong cars and racing against each other. The one who swerve "chickens out", letting the other player to win. Not swerving means frontal crash against each other in full speed, which is basically death sentence for both of them.

In the model game both player have same car and are have also same capabilities as drivers and of course, they both want to win the race. Lets quantify possible outcomes of the decision situation:

- win race = 10
- loose race = -10
- no win situation = 1
- crash = -100

With limited strategies as of swerve or race we can construct the matrix of the outcomes, see tab. 13.3. Since there are two players I will put in matrix outcomes from point of view of both players, se we can can better understand the problem.

Table 13.3: Game of chicken

		Racer A	
		swerve	**race**
Racer B	**swerve**	1, 1	-10, 10
	race	10, -10	-100, -100

It is obvious, that both players can not win. They can both die, but that is usually something we try to avoid. Optimal strategies is based on forcing other player to chicken out. Unfortunately the players can't guarantee that they will be able to force other one to swerve so there is no optimal strategy.

Logical choice would be to swerve under such circumstances for both players. Also there is a solution in symmetric Nash equilibrium – provided that both players change their strategies randomly.

In this case the negative results are really negative (we can hardly imagine more adverse negative effect then death), but we may come to different scenario for this type of game.

Lets think about the scenario of collective bargaining in some kind of company. Unions bargain against the management for higher salaries. If they both push – the agreement will not be possible and the worker will go on strike, the company will not produce (and will not pay for worker on strike). If it goes on for too long, the company will go bankrupt and the workers will loose their jobs and everybody looses.

When both sides give something away from their demands the workers will gain something while management has to pay something. Other options lead to the victory of one side and defeat of the other side (either management or union).

Another strategy for chicken game winning is to convince your opponent that you will use certain strategy, no matter what. In our chicken game example it would mean signaling convincingly that you will not swerve when racing and that you will either win or die. Second player then will suddenly see his strategies in very limited way as his possible outcomes for the game shrink to either continue to race and die or swerve and chicken out. He no longer has an option to win the game. Under such circumstances rational player will choose to chicken out.

Interestingly enough, first player needs only to convince other player of his intention, he doesn't need to realize this strategy (come through with the plan).

Another example of chicken game would be cold war's doctrine of mutual assured destruction, which possibly kept the world's leading powers from starting nuclear war.

Since the outcomes of such game are not guaranteed, proposed solutions in previous paragraphs are not "stable" solutions. (Which is especially concerning for cold war example.)

Another type of the game is **stag hunt**. This game is derived from hypothetical situation of stag hunt, where the hunters wait in hiding for the game. But it doesn't show up for long time. If the stag hunt is successful, then all of the hunters will eat, but the game is not guaranteed to come. Then a hare comes instead. If the hunter guns it down, he can eat it, but it is not large enough to fill stomaches of all the hunters. So what should the hunter choose – be selfish or cooperate with others?

Lets construct the matrix of the outcomes. We will simplify the situation for two hunters and again we will bring the point of the view of both players in the matrix, see table 13.4.

Table 13.4: Stag hunt

		Hunter A	
		cooperate	**defect**
Hunter B	**cooperate**	2, 2	0, 3
	defect	3, 0	1, 1

If the hunters cooperate, they will both eat, if one of them defects, then the one defecting gets all benefits of the betrayal but his fellow comrade in hunting will get nothing. If both of them decide not to cooperate (like continuing the hunt on their own), both of get some benefit form the decision, but not as large as in case they cooperate.

Basically the stag hunt situation in real world is the situation when two (or more) parties have to cooperate to achieve common goal. For example when protecting n endangered species two neighborhood countries arrange project to save save the species from dying out. If both of them fulfill their obligations, the species will be saved, but if one of them withdraws its support it will get some benefits as the money, it saved by withdrawing, can be used elsewhere to further other goals of the country. The other country will sink its money in now futile attempt to save the species and fails. If both countries withdraw their support, they may spend money in different areas, but the species dies out.

For last (3rd) edition of the textbook we add bonus game, *public goods game*. This type of game has been described in experimental economy for better understanding of the problem of *stowaway*.

We added the game in reaction to recent COVID pandemic to provide a new interesting game to think about the situation in different context.

The game explores relation and motivations between the individuals contributing to overall benefit of society. Stowaway does not contribute to production of this benefit, but utilizes investment of others in it.

Let's consider following situation, the game participants commit to invest 20 USD for improvement of well-being of society. Multiplication effect ensures, that any investment into the society will increase by 20 % and will be returned to game participants in the form of better services, etc. One of participants changes his mind and decides not to invest despite his prior commitment. Since all society members use public goods, the participant who did not invest will still use both improved public goods and his not invested money.

Since the benefits of the investment are distributed equally, those who contributed will get lower benefits then was their original investment.

You can see the situation graphically on fig. 13.1.

We can thing about COVID-19 vaccination in terms of public goods game. People who decide not to get vaccinated are protected to certain degree by other people who decided to vaccinate themselves, at same time they do not contribute to building immunity of the community and do not share the discomfort connected with the process (side effects of vaccination, arm pain ...).

Also there are many more types of the games. As a good start in studying of its basics you can use *Wikipeadia's List of games in Game Theory* [74] page.

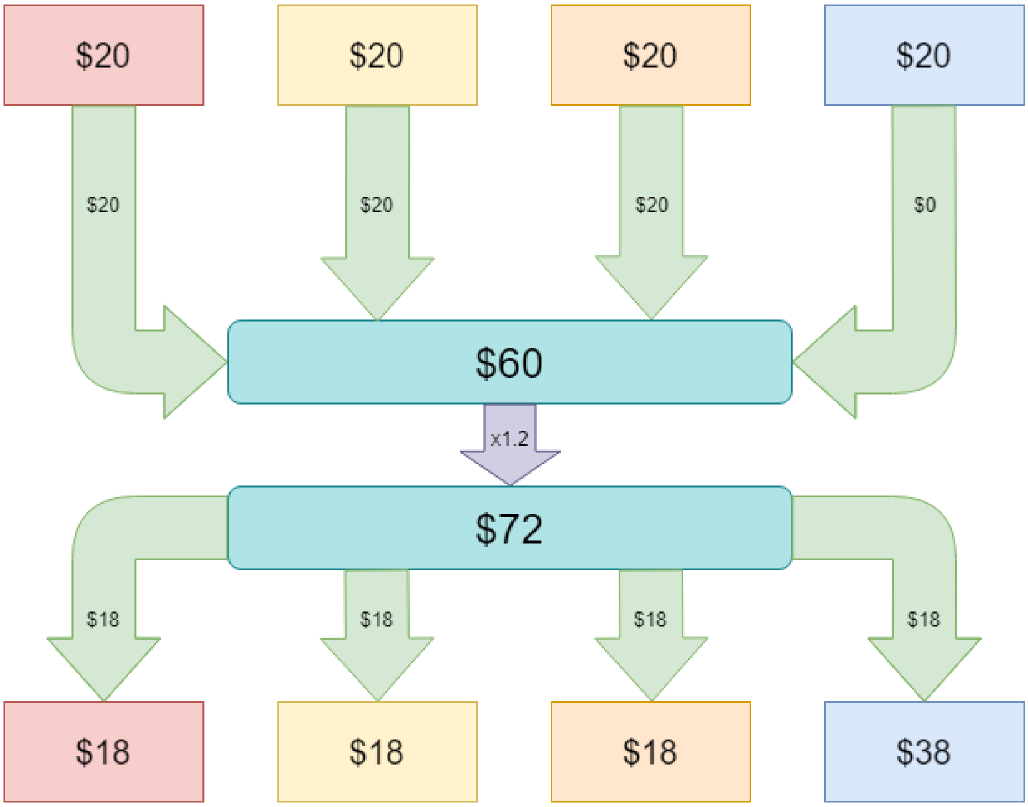


Figure 13.1: Public Goods Game (courtesy of [17])

So what do you think?
What type of the game would be the situation of financial crisis of many countries in the eurozone and European Stability Mechanism? Is it stag hunt or game of chicken? What would be the most important moments moments of such decision?

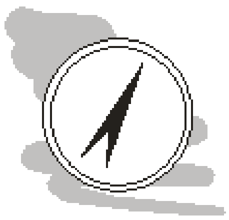
Please note that this question has no "right" answer. It is here for you take perhaps slightly different approach to it - think about the problem.

Questions

1. What is the difference between stag hunt game and the game of chicken?
2. What is saddle point and how do we find it?
3. Do all games have the solution in saddle point and if not what type of the solution do we look for in them?
4. Describe prisoner's dilemma problem.

Chapter 14

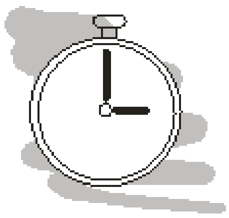
Localization models



Study guide
Another optimization problem is the problem of physical localization of the object in the space.

After studying this chapter you will know

- what are localization models
- how to solve simple localization problems
- how to approach the solution of more difficult problems



Time required to study chapter
You will need at least 30 minutes to study basic theory of localization models.

Localization models exist to help us solve problem of placement of some kind of object in space. We do not place the object randomly, we try to follow some kind optimization criterion usually derived from effectivity of the investment. Typical problem could be to cover some specified are with 5G signal. We can do that placing Base Transceiver Station (BTS) to send the signal. Our goal would be to find optimal placement for BTS's to cover the area with minimal investment. In other words we would try to place minimal required BTS to get maximal coverage.

Solution of the problem depends on the way we define it. Do we place single object, which is going to be communicating with other objects. In network analogy we would be speaking about adding new node in existing network. Problem could be different, we may need to place multiple object. The solution will also differ if we work in 2D or 3D space. We can be limited in selection of places we may use - in this problem we only select best placement from some kind of selection.

Finally do we know how many objects need to be placed? The number itself may be a variable we need to find.

What types of the problem are we talking here about:

1. In the are we have already three manufacturing facilities, company's CEO wants us to find suitable place for storage where the company will temporary store its production from above mentioned production facilities.
2. After the latest floods the area government found out that there are parts of the are, which are not covered by the wireless warning system. The task is to find suitable placement for new units of the warning system. Also the proposed investment should be cost effective (largest coverage with minimum of the warning system units).
3. ...

To find a solution, we have to specify optimization criterion. Such criterion is usually connected to the cost of connection between the already localized objects and the new (proposed) objects. We will compute such cost N_{ij} for each connection between objects i and j.

The costs themselves are usually function transportation distance d_{ij}. Based on presumption that to transport the goods on longer distances costs more then to do the same on short distances. The amount of the goods transported x_{ij} is also important for costs estimation. Third and last value, which affects costs is the type of the vehicle used for transportation $c_i j$. Different types of the transport have different costs of usage.

Mathematically we can express the costs between the existing and new object as in equation (14.1).

$$N_{ij} = x_{ij} d_{ij} c_{ij} = d_{ij} w_{ij} \tag{14.1}$$

Since x and c values are from computational point of view constants, we can substitute them by value w, representing weight of the connection. To solve the problem we will have to optimize d value only.

Connection between new objects, if we have to place more then one object can be expressed as on equation (14.2).

$$N_{ik} = x_{ik} d_{ik} c_{ik} = d_{ik} w_{ik} \tag{14.2}$$

Cost function then may be expressed as a sum of the costs, see (14.3).

$$minz = \sum_{i=1}^{m} \sum_{j=1}^{n} N_{ij} + \sum_{i=1}^{n} \sum_{k=1}^{n} N_{ik} \tag{14.3}$$

If we are localizing one object only, whole sum of N_{ik} equals to zero simplifying greatly the optimization problem.

As we stated higher, the crucial point of the optimization is distance value. There are three basic types of distances. We choose between them based on problem characteristic, or more specifically characteristic of transportation. Lets visualize the various distances, see fig. 14.1.

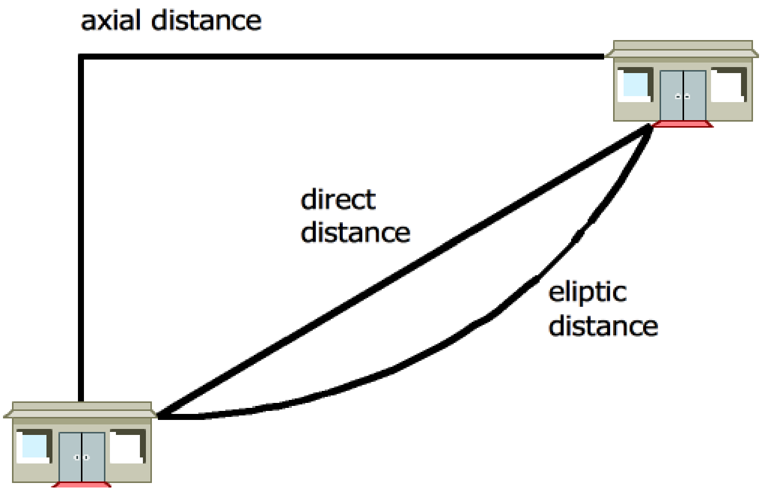


Figure 14.1: Three basic types of the distances

Mathematically we can express them using equations (14.4 – 14.7).

The first distance we usually imagine it direct distance. In localization models this distance is not so common, at least if the problem is to localize the object which should be reachable using road transportation. Only rarely can we use direct route under such conditions. Perhaps if we are working on "macro" scale, for example we could be choosing city to place new logistic centre, then the road network could be perhaps approximated by direct lines.

Usage of direct distance is common for wireless communications, where we are not limited by terrain relief.

The equation to compute the *direct distance* is notoriously well known, see equation (14.4). In localization models we also use its slight modification adding k constant to compensate the fact that in reality the direct route is not entirely direct, there are always small curves on the road, it goes over the hill, etc., which lengthens the path, see equation (14.5).

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{14.4}$$

$$d_{ij} = k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{14.5}$$

Logically the k coefficient must be a number > 1 for eq. (14.5) to make any sense. Eq. (14.4) then becomes a special case of eq. (14.5), where $k = 1$.

Axis distance can be used in situation where we have orderly road system. That is typical for newly build cities or its parts. Look for example on the road systems of the New York, see fig. 14.2. Such distance can be computed using equation (14.6).

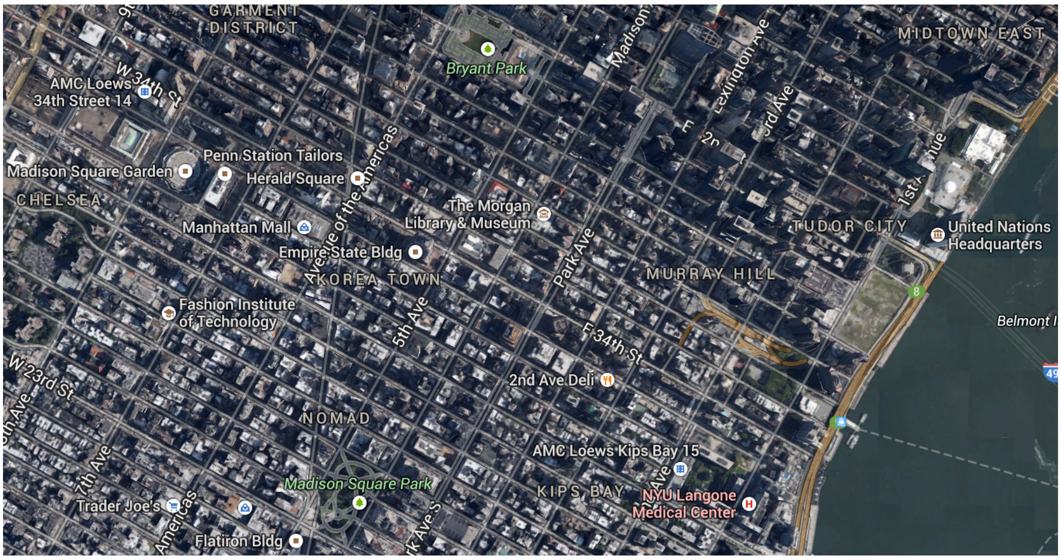


Figure 14.2: Road network of New York (source Google Maps)

$$d_{ij} = |x_i - x_j| + |y_I - y_j| \tag{14.6}$$

Lastly we can use *quadratic distance* as computed in equation (14.7).

$$d_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 \tag{14.7}$$

The most simple example we can demonstrate the problem solution would be localizing one object in 2D using axial distances. We would be optimizing following expression (14.8).

$$minz = \sum_{j=1}^{n} |x - x_j| + |y - y_j| \tag{14.8}$$

Sum of additions can be easily expressed as an addition of the sums allowing us to separate x and y coordinates, see (14.9).

$$minz = \sum_{j=1}^{n} w_j|x - x_j| + \sum_{j=1}^{n} w_j|y - y_j| \tag{14.9}$$

The solution, geometrically, would be in the middle. As we are working with existing road system we are trying to find coordinate for which the sum of the weights w on the right and left (for x coordinate) and higher and lower (for the y coordinate) approximately equals.

We can do that easily in tabular form. Lets find optimal location for new object, so that it may communicate effectively with four other existing objects A-D, see table 14.1 for their coordinates and weights connected to them.

Table 14.1: Localization of existing objects and estimation of the weights (courtesy of [11])

Object	**x**	**y**	**w**
A	1	8	50
B	4	2	100
C	9	2	80
D	8	5	70

Table 14.2: Localization of existing objects and estimation of the weights (courtesy of [11])

Order by X column				
Object	**x**	**y**	**w**	$\sum w$
A	1	8	50	50
B	4	2	100	150
D	8	5	70	**220**
C	9	2	80	300
Order by Y column				
Object	**x**	**y**	**w**	$\sum w$
B	4	2	100	100
C	9	2	80	**180**
D	8	5	70	250
A	1	8	50	300

Now if we want to perform the computation, we have to order the object by X and Y coordinate and choose the coordinate which exceeds half of the sum of the weights, see tab. 14.2.

Under given circumstances the new object shall be localized on coordinated X=8 and Y=2.

When using quadratic distance then the objective function we are optimizing looks as in equation (14.10).

$$minz = \sum_{j=1}^{n} w_j((x - x_j)^2 + (y - y_j)^2) \tag{14.10}$$

We can search for local minima of the function by deriving the equation (14.10) by coordinate variables, see (14.11) and putting them equal to zero. The equation (14.11) has analytical solution (14.12), which is then easily computable using any spreadsheet program.

$$\begin{aligned} \frac{\partial z}{\partial x} &= -2\sum_{j=1}^{n} w_j(x - x_j) &= 0 \\ \frac{\partial z}{\partial y} &= -2\sum_{j=1}^{n} w_j(y - y_j) &= 0 \end{aligned} \tag{14.11}$$

$$\begin{aligned} x &= \frac{\sum_{j=1}^{n} w_j x_j}{\sum_{j=1}^{n} w_j} \\ y &= \frac{\sum_{j=1}^{n} w_j y_j}{\sum_{j=1}^{n} w_j} \end{aligned} \tag{14.12}$$

Finally for direct distance we are optimizing the equation (14.13). Unfortunately partial derivations (14.14) do not have analytical solution, so we have to compute them using numerical methods.

$$minz = \sum_{j=1}^{n} w_j \sqrt{(x - x_j)^2 + (y - y_j)^2} \tag{14.13}$$

$$\begin{aligned} \frac{\partial z}{\partial x} &= \sum_{j=1}^{n} \frac{x - x_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \\ \frac{\partial z}{\partial y} &= \sum_{j=1}^{n} \frac{y - y_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \end{aligned} \tag{14.14}$$

To effectively compute the coordinates we will use the substitution (14.15).

$$f_j(x, y) = \frac{w_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2 + \varepsilon}} \tag{14.15}$$

Epsilon in equation (14.15) should allow us to go around the possible equality of the coordinates, which would lead to situation, when we would divide by zero.

In iterations we go from the point of optimal solution for quadratic distance and compute the next set the coordinates using equation (14.16).

$$x^{(k+1)} = \frac{\sum_{j=1}^{n} x_j f_j(x^{(k)}, y^{(k)})}{\sum_{j=1}^{n} f_j(x^{(k)}, y^{(k)})}, y^{(k+1)} = \frac{\sum_{j=1}^{n} y_j f_j(x^{(k)}, y^{(k)})}{\sum_{j=1}^{n} f_j(x^{(k)}, y^{(k)})} \tag{14.16}$$

Using the old and new coordinated we compute how much the iteration contributed to the improvement of the solution (14.17).

$$\Delta z = z^{(k)} - z^{k+1} \tag{14.17}$$

With each iteration, the improvement in Δz is smaller and smaller. So we are basically optimizing in kilometers, then hundreds of meters, meters, decimeters ... It is usually not effective to reach higher precision, as the benefits of such improvement would be marginal. At that point we simply stop the computation.

Last set of coordinates is as solution.

When computing the localization of multiple new objects, doing se in hand is practically impossible, well maybe it is more impractical then impossible, so we try to use software tools to help us.

Summary

Localization model are optimization model allowing us to find optimal placement for new objects. Optimization criterion is usually derived from travel distance between the objects as distance is often directly related to the costs.

Computation differs based on number of objects we place and relief we take in account - 2D or 3D.

Questions

1. What are localization models?
2. What type of the problems do we solve using them?
3. Are there differences in look on the distances in these models?
4. How do we in general approach the solution of the localization model when localizing single object?
5. Can you specify how iterative method for computation of optimal placement of the object works?

Chapter 15

Regression models

Study guide
Another possibility to get additional information is to analyze numerical data describing some kind of problem, time series and use the information to predict at least important trends in data. Such information can improve significantly decision making.

This chapter is intended as a bridge to other courses and science - in this case statistics.

You can study *Statistics* in detail in course with same name, available for you in 2nd or 3rd study year, depending on what study program you study.

Time required for study
To go trough the chapter you will need approximately 20 minutes.

This chapter is more motivational then anything else. There are plenty of techniques and approaches we can use in statistics and the space available in this textbook is very limited. So consider it as a very short introduction to Statistics a then go and continue your study either in specialized course or pletoria of study materials available. There is for example Hands on ... series of books explaining in condensed form with lot of examples in R problematic of time series [75] or machine learning [76] and there are many more similar publications.

In this chapter we will focus on basics of *regression analysis*. As part of it we will analyze k potential explanatory variables to find out whether they contribute to explaining depended variable. Translated to common English - we have a dataset with numeric data and we will interleave it with curve in such way that it represents data in dataset as closely as possible.

Finding the curve (regression function) will allow us to compute the values which were not part of the original dataset. Regression model in general (linear regression) can be expressed as eq. (15.1).

$$y = \eta + \epsilon \tag{15.1}$$

where y is dependent (explained) variable, η nonrandom part of estimate, ϵ random part of estimate.

Mathematically we are capable to estimate only nonrandom part η. About random part of estimate ϵ we usually presume that it follows normal probability distribution and thus we can presume that the errors introduces by this randomness will not significantly skew the estimate for sufficiently sized datasets.

For η we choose appropriate regression model, for example linear regression $\eta = \beta_0 + \beta_1 x_1$ or $\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, etc. There are also other types of regression models, since this text should serve only as rudimentary introduction and there are whole courses and books dealing with this topic.

General form of the linear model can be expressed in following way (15.2).

$$\eta = \sum \beta_i f_i \tag{15.2}$$

where β is regression parameter, f regressor (function of explanatory variables)

Main goal of regression analysis is to estimate the regression parameters. One of most often used methods used to estimate regression parameters is least squares method. With it we replace regression parameters β with their estimates b. We are searching for such regression function, for which sum of squared error is lowest.

If we are searching for parameters of line (simplest example), we can express the model as (15.3):

$$Y = b_0 + b_1 x_1 \tag{15.3}$$

Sum of error squares can be computed by evaluating differences between data in dataset and the regression model.

We can compute the regression parameters by partially deriving sum of (15.4) by regression parameters, which leads to system of equation we can compute easily.

$$Q = \sum_{j=1}^{n} (y_j - Y_j)^2 = \sum_{j=1}^{n} (y_j - b_0 - b_j x_j)^2 \tag{15.4}$$

After we estimate regression parameters, we need to decide, whether the model sufficiently corresponds to reality. For this purpose we can use so called *determinant index* (15.5) or *correlation index* (15.6).

$$I^2 = \frac{s_T}{s_Y} = \frac{\sum (Y_j - \bar{y})^2}{\sum (y_j - \bar{y})^2} \tag{15.5}$$

$$I^2 = 1 - \frac{s_R}{s_Y} = 1 - \frac{\sum (y_j - Y_j)^2}{\sum (y_j - \bar{y})^2} \tag{15.6}$$

Determinant index $I^2 \in < 0; 1 >$ and can be interpreted as percentage of how well estimated regression model explains variance of measured values of y.

Adequacy of the model can be evaluated using *F-test* and *partial t-pests*. F-test is used for evaluation of null hypothesis stating that apart of parameter b_0 all other parameters are zero, in other words insignificant for explanation of the y. To evaluate the hypothesis we will use testing criterion (15.7).

$$F = \frac{\frac{s_T}{p-1}}{\frac{s_R}{n-p}} \tag{15.7}$$

Null hypothesis H_0 on *significance level* α can be rejected, if $F > F_{1-\alpha}$.

Partial t-tests can be used to evaluate significance of regression parameters one by one. We can use test criterion (15.8) for that.

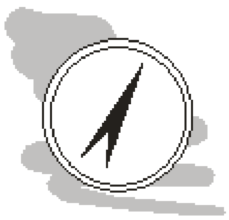
$$t_i = \frac{b_i}{s(b_i)} \tag{15.8}$$

Self test questions

1. Explain hypothesis testing.
2. What is linear regression?

Chapter 16

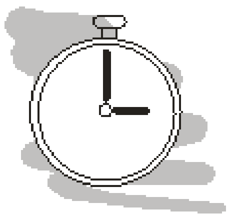
Artificial intelligence



Study guide
Lately artificial intelligence based modes, especially neural networks, gained traction for analytic purposes as well as automation of some processes. In this chapter we will only focus on *expert systems* and *neural networks.*.

Again, similarly to previous chapter, this chapter is only intended as context building for your knowledge. So do not expect to become expert after reading just about two pages of text on this topic.

And again we recommend our students to continue in their studies of this topic by selecting appropriate courses during their study or from wealth of resources available on the topic.



Time required for study
Well how quickly can you read two pages of text :-)? So lets say 2 minutes or so?

Similarly to statistical methods, methods based on artificial intelligence can be very effective support tool for our decision making. While in this chapter we will discuss only expert systems and neural networks, other models are readily available. Especially in area of neural networks we see in last few years extreme increase in usage, up to the point that modern processors have hardware for accelerations of neural networks utilization.

But before that we will start more deep in the history with *expert systems*. These systems were introduced in 60-ties of last century and peaked in 80-ties. End goal of these systems was to create such systems, which enables its user to access knowledge on expert in problem's domain. It tries to solve problem of expert availability at a moment notice.

Expert system presents baseline for decision making. At later time, when the human expert arrives, course can be amended accordingly, if needed.

Each expert system to be usable needs to implement its *knowledge base* in problem domain and logic to process this knowledge. Since computers still are not capable of working with loosely formatted data, the knowledge must be expressed in strictly formalized form, so that the expert system is able to work with it and provide answers.

Encoding of knowledge base is a task for *knowledge engineer*. This task is critical for success of the system and is also extremely hard, even for middle-sized system. knowledge engineer needs to deal especially with following problems:

1. problem domain expert needs to provide the knowledge engineer insight on the way (s-)he works. That is problematic as most experts like to keep their knowledge to themselves and also, even if they would like to help

2. experts might also be unable to provide information in such way, the the engineer is able to formalize it for the system as lot of this knowledge is intuitive and "automated" for the expert (veni, vidi, vici style).

That is also reason why expert system in its original form are rarely used today. Thou, expert systems gave birth to Bussines Rules Management System (BRMS), which are extensively used in large organizations, banks, etc. to automate certain workflows based on defined rule sets which are similar to knowledge base of expert system.

Knowledge in expert system can be divided to declarative and procedural. We also call declarative knowledge as facts. If our expert system was classified the animals, the fact could be: dog has four legs.

Procedural knowledge define the way information will be derived in the expert system. Procedural knowledge is often expressed by rules. The rules use defined facts. For example if the classified creature is dog, then it has four legs.

Expert system during consultation with its user present set of questions to gather missing facts and compares it with the facts stored in its knowledge base to derive answers (using rules). Base on these rules it formulates its answers

Neural networks use different principle. Instead of trying to formally express all necessary knowledge of the expert, neural networks try to emulate the way biological organisms learn and solve problems. Critical for success are the ability to learn and ability to recall it when needed.

Neural network works as a black box, witch after the adaptation process provides answers.

Neural networks are almost as old as expert systems, but opposed to them their golden age is right now. In the past usage of such networks was limited by slow computers. Today hardware based accelerators exist to let us compute extremely large neural networks allowing us to solve lot of very different problems.

When talking about modern neural network we often use term *deep learning*. The depth is achieved by using many layers of neural networks with many different functions.

These are some more known types of deep learning neural networks:

- *convolutional* - for classification problems, image detection, etc.
- *recursive* - for problems like sentiment analysis, language translation, etc.
- *autoencoders* - for data compression, image transformation, etc.

Neural network transforms the inputs to outputs through pushing the signal through many layers of the the network. Links between the neurons in layers are weighted, so the signal changes as it goes though them.

Out of three types of networks mentioned above only autoencoders do not adapt by comparing output of the last layer of the network with true value and using the difference (error) to modify the weights on links of the network.

Adaptation process is iterative. After each iteration error is computed and weights adjusted in hopes to provide better outcome in next iteration. Adaptation process continues either until the global error of the networks is lower then set threshold, or preset number of iterations (epochs) is reached.

Ability of network to adapt depends directly on its type and configuration. Generally we can say, that the larger the network is the better chance it has to adapt successfully. But note that increases in size of then network grow computational requirements to adapt it exponentially. So for complex problem it may be required to pre-process the data and to experiment with network configuration to find combination of parameters which work fine for the problem.

Also note that complex problems are usually not considered solvable on traditional CPU, because the adaptation process would simply take too much time. Instead graphical cards (especially those from NVidia) or accelerators are used for adaptation process. Specialized hardware allows us to adapts in reasonable time order of magnitude larger networks, then those we could adapt on CPU only.

For truly extremely complicated problems, usage of supercomputers may be necessary.

We can see some changes in 2024. Newle released CPU's from Intel, AMD and SnapDragon contain Neural Processor Unit (NPU) cores for acceleration of neural network related computations. At present time these NPUs are not performant enough to allow us to adapt large neural networks on the computer but they allow us to consult one. In other words they are able to help with inference rather then adaptation.

We can expect to move at least part of AI functionality from cloud operating data centers to local computers. Examples of such initiatives are already being announced. For example Microsoft announced its new Copilot+ PC initiative integrating AI deep into MS Windows and many other companies follow the suit.

Self-evaluation questions

1. Try to explain adaptation process of neural network.
2. Explain problems with creating knowledge base of the expert system.

Annex

Annex 1 - Sensitivity Analysis for decision tree 2

Here we present basic R script to print graph for visualization of the costs sensitivity on changes in expected treatment cost during "small" epidemic of influenza.

Listing 16.1: Sensitivity grpah for overall costs

```
x <- 250:310
N <- 0.75 * (0.1 * x + 0.3 * 400 + 0.6 * 600)
plot(x, N, type="b", pch=21,
main="Influence of treatment costs of small epidemic on overall costs",
xlab="trewatment costs [mil. EUR]",
ylab="overall costs [mil. EUR]")
abline(lm(N~x))
```

Annex 2 - Graphviz

One of possibilities to visualize the networks is to use open source tool GraphViz [77]. Structure of the network and its properties, important to appropriately draw the network, must be known in advance. Using general graphical programs to draw the network might be possible, but is inconvenient as the network does not have all true properties of the original network. See for example in hand made fig. 10.11 and automatically generated 10.12). Both are similar but not same.

If the purpose was only to provide illustrative example, the difference would not be significant, but is we used the network as a base for some other analyzes, automatically generated one provides much better value. This is also reason, why such specialized programs even exist. GrapViz is one of many such programs. It is open source and is available for many operating systems, thou not for all of them in latest stable version.

We can work with the program either through the command line (terminal) or using GUI. Terminal interface is useful in situation when we need to generate lot of network visualizations. Batch processing is using some kind of script is usually faster then trying to do the same by clicking through the GUI.

We can define the structure of the network by specifying sequences of nodes linked by edges, for example 1 -> 2 -> 4 -> 7 -> 8 -> 11. We frame the path by identification of the graph, see listing bellow.

If the graph should not be directed we will use "–" instead of "->".

Whole definition for network from fig. 10.11 looks like this:

Listing 16.2: Network structure for analytic purposes

```
digraph G{
  1 -> 2 -> 4 -> 7 -> 8 -> 11;
  1 -> 4 -> 6 -> 7;
  1 -> 3 -> 5 -> 6 -> 10 -> 11;
  4 -> 5;
  5 -> 9 -> 10;
  9 -> 11;
}
```

Easy (relatively speaking), right? GraphViz uses DOT library to generate graph of the network. Our results will be slightly different then network on fig. 10.11, as GraphViz places the nodes on its own using the information on graph structure we provided it. That should not border us, the other, relevant, properties of the network remain unchanged.

We can see the result on fig. 16.1.

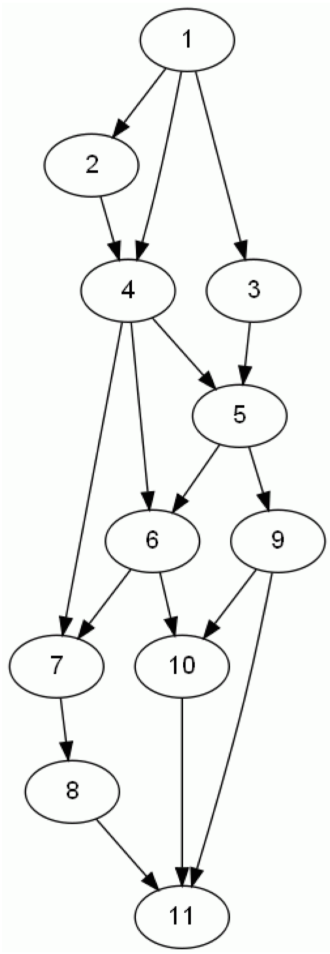


Figure 16.1: Oriented network using DOT language

To solve some problems, such as finding shortest path, we need undirected graph. By replacing oriented lines "->" with unoriented ones "–" (two dashes), we can easily achieve the goal, see listing below. Visualization of this network is available on fig. 16.2. We will use Neato library to generate the figure.

Listing 16.3: Neato generated network structure

```
graph G{
  1 -- 2 -- 4 -- 7 -- 8 -- 11;
  1 -- 4 -- 6 -- 7;
  1 -- 3 -- 5 -- 6 -- 10 -- 11;
  4 -- 5;
  5 -- 9 -- 10;
  9 -- 11;
}
```

Even in this form our network has not enough information to allow us to compute shortest path. We also need to provide information on length of the edges. Amended notation is provided in next listing:

Listing 16.4: Network specification using edges capacity

```
graph G{
  1 -- 2 [len=10];
```

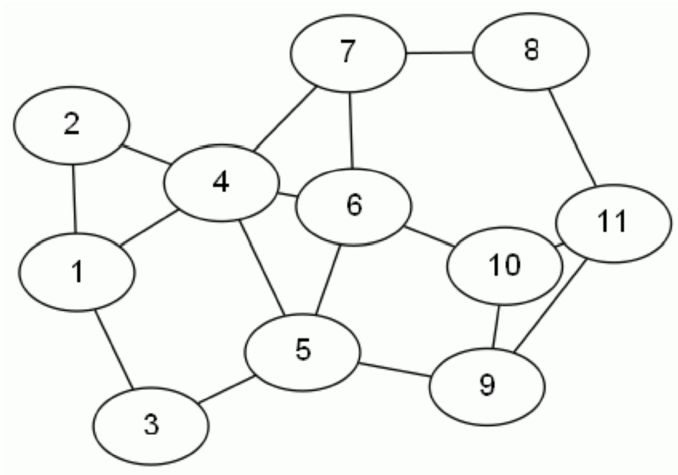


Figure 16.2: Undirected network generated by Neato library

```
    2 -- 4 [len=8];
    4 -- 7 [len=6];
    7 -- 8 [len=11];
    8 -- 11 [len=12];
    1 -- 4 [len=12];
    4 -- 6 [len=9];
    6 -- 7 [len=6];
    1 -- 3 [len=8];
    3 -- 5 [len=10];
    5 -- 6 [len=12];
    6 -- 10 [len=10];
    10 -- 11 [len=10];
    4 -- 5 [len=6];
    5 -- 9 [len=11];
    9 -- 10 [len=6];
    9 -- 11 [len=9];
  }
```

Annex 3 - AHP - YAML file cars.ahp

In AHP chapter we only provided fraction of the file. Now we provide it all. You can also download the file from https://lms.vsb.cz.

Listing 16.5: Full listing of YAML file cars.ahp

```
Version: 2.0

Alternatives: &alternatives
  Accord Sedan:
    Purchase Price: 20360
    MPG: 31
    Residual Value: 0,52
    Safety class: midsize car
    Cargo Capacity: 14
    Passenger Capacity: 5
    Surb Weight: 3289
    crash rating: 4* in side impact front
    60K tire cost: 700
    Brakes Cost: 1x
    Consumer Report: +++
  Accord Hybrid:
    Purchase Price: 31090
    MPG: 35
    Residual Value: 0,46
    Safety class: midsize car
    Cargo Capacity: 14
```

```
    Passenger Capacity: 5
    Surb Weight: 3501
    crash rating: 4* in side impact front
    60K tire cost: 700
    Brakes Cost: 1x
    Consumer Report: +++
  Pilot SUV:
    Purchase Price: 27595
    MPG: 22
    Residual Value: 0,44
    Safety class: midsize car
    Cargo Capacity: 87,6
    Passenger Capacity: 8
    Surb Weight: 4264
    crash rating: 4* in rollover
    60K tire cost: 1400
    Brakes Cost: 2x
    Consumer Report: +++
  CR-V SUV:
    Purchase Price: 20700
    MPG: 27
    Residual Value: 0,55
    Safety class: small SUV
    Cargo Capacity: 72,9
    Passenger Capacity: 5
    Surb Weight: 3389
    crash rating: 4* in rollover
    60K tire cost: 1600
    Brakes Cost: 2x
    Consumer Report: +++
  Element SUV:
    Purchase Price: 18980
    MPG: 25
    Residual Value: 0,48
    Safety class: small SUV
    Cargo Capacity: 74,6
    Passenger Capacity: 4
    Surb Weight: 3433
    crash rating: 3* in rollover
    60K tire cost: 1300
    Brakes Cost: 2x
    Consumer Report: ++
  Odyssey Minivan:
    Purchase Price: 25645
    MPG: 26
    Residual Value: 0,48
    Safety class: minivan
    Cargo Capacity: 147,4
    Passenger Capacity: 8
    Surb Weight: 4385
    crash rating: 5*
    60K tire cost: 2400
    Brakes Cost: 2x
    Consumer Report: +

Goal:
  preferences:
    pairwise:
      - [Cost, Safety, 3]
      - [Cost, Style, 7]
      - [Cost, Capacity, 3]
      - [Safety, Style, 9]
      - [Safety, Capacity, 1]
      - [Style, Capacity, 1/7]
  children:
    Cost:
      preferences:
        pairwise:
          - [Purchase Price, Fuel Costs, 2]
          - [Purchase Price, Maintenance Costs, 5]
          - [Purchase Price, Resale Value, 3]
          - [Fuel Costs, Maintenance Costs, 2]
```

```
          - [Fuel Costs, Resale Value, 2]
          - [Maintenance Costs, Resale Value, 1/2]
    children:
      Purchase Price:
        preferences:
          pairwise:
            - [Accord Sedan, Accord Hybrid, 9]
            - [Accord Sedan, Pilot SUV, 9]
            - [Accord Sedan, CR-V SUV, 1]
            - [Accord Sedan, Element SUV, 1/2]
            - [Accord Sedan, Odyssey Minivan, 5]
            - [Accord Hybrid, Pilot SUV, 1]
            - [Accord Hybrid, CR-V SUV, 1/9]
            - [Accord Hybrid, Element SUV, 1/9]
            - [Accord Hybrid, Odyssey Minivan, 1/7]
            - [Pilot SUV, CR-V SUV, 1/9]
            - [Pilot SUV, Element SUV, 1/9]
            - [Pilot SUV, Odyssey Minivan, 1/9]
            - [CR-V SUV, Odyssey Minivan, 5]
            - [CR-V SUV, Element SUV, 1/2]
            - [Element SUV, Odyssey Minivan, 6]
        children: *alternatives
      Fuel Costs:
        preferences:
          pairwise:
            - [Accord Sedan, Accord Hybrid, 1/3]
            - [Accord Sedan, Pilot SUV, 5]
            - [Accord Sedan, CR-V SUV, 3]
            - [Accord Sedan, Element SUV, 4]
            - [Accord Sedan, Odyssey Minivan, 3]
            - [Accord Hybrid, Pilot SUV, 9]
            - [Accord Hybrid, CR-V SUV, 5]
            - [Accord Hybrid, Element SUV, 7]
            - [Accord Hybrid, Odyssey Minivan, 6]
            - [Pilot SUV, CR-V SUV, 1/4]
            - [Pilot SUV, Element SUV, 1/3]
            - [Pilot SUV, Odyssey Minivan, 1/4]
            - [CR-V SUV, Element SUV, 2]
            - [CR-V SUV, Odyssey Minivan, 1]
            - [Element SUV, Odyssey Minivan, 1]
        children: *alternatives
      Maintenance Costs:
        preferences:
          pairwise:
            - [Accord Sedan, Accord Hybrid, 2]
            - [Accord Sedan, Pilot SUV, 4]
            - [Accord Sedan, CR-V SUV, 4]
            - [Accord Sedan, Element SUV, 4]
            - [Accord Sedan, Odyssey Minivan, 5]
            - [Accord Hybrid, Pilot SUV, 4]
            - [Accord Hybrid, CR-V SUV, 4]
            - [Accord Hybrid, Element SUV, 4]
            - [Accord Hybrid, Odyssey Minivan, 5]
            - [Pilot SUV, CR-V SUV, 1]
            - [Pilot SUV, Element SUV, 2]
            - [Pilot SUV, Odyssey Minivan, 1]
            - [CR-V SUV, Element SUV, 1]
            - [CR-V SUV, Odyssey Minivan, 3]
            - [Element SUV, Odyssey Minivan, 2]
        children: *alternatives
      Resale Value:
        preferences:
          pairwise:
            - [Accord Sedan, Accord Hybrid, 3]
            - [Accord Sedan, Pilot SUV, 4]
            - [Accord Sedan, CR-V SUV, 1/2]
            - [Accord Sedan, Element SUV, 2]
            - [Accord Sedan, Odyssey Minivan, 2]
            - [Accord Hybrid, Pilot SUV, 2]
            - [Accord Hybrid, CR-V SUV, 1/5]
            - [Accord Hybrid, Element SUV, 1]
            - [Accord Hybrid, Odyssey Minivan, 1]
```

```
            - [Pilot SUV, CR-V SUV, 1/6]
            - [Pilot SUV, Element SUV, 1/2]
            - [Pilot SUV, Odyssey Minivan, 1/2]
            - [CR-V SUV, Element SUV, 4]
            - [CR-V SUV, Odyssey Minivan, 4]
            - [Element SUV, Odyssey Minivan, 1]
        children: *alternatives
    Safety:
      preferences:
        pairwise:
          - [Accord Sedan, Accord Hybrid, 1]
          - [Accord Sedan, Pilot SUV, 5]
          - [Accord Sedan, CR-V SUV, 7]
          - [Accord Sedan, Element SUV, 9]
          - [Accord Sedan, Odyssey Minivan, 1/3]
          - [Accord Hybrid, Pilot SUV, 5]
          - [Accord Hybrid, CR-V SUV, 7]
          - [Accord Hybrid, Element SUV, 9]
          - [Accord Hybrid, Odyssey Minivan, 1/3]
          - [Pilot SUV, CR-V SUV, 2]
          - [Pilot SUV, Element SUV, 9]
          - [Pilot SUV, Odyssey Minivan, 1/8]
          - [CR-V SUV, Element SUV, 2]
          - [CR-V SUV, Odyssey Minivan, 1/8]
          - [Element SUV, Odyssey Minivan, 1/9]
      children: *alternatives
    Style:
      preferences:
        pairwise:
          - [Accord Sedan, Accord Hybrid, 1]
          - [Accord Sedan, Pilot SUV, 7]
          - [Accord Sedan, CR-V SUV, 5]
          - [Accord Sedan, Element SUV, 9]
          - [Accord Sedan, Odyssey Minivan, 6]
          - [Accord Hybrid, Pilot SUV, 7]
          - [Accord Hybrid, CR-V SUV, 5]
          - [Accord Hybrid, Element SUV, 9]
          - [Accord Hybrid, Odyssey Minivan, 6]
          - [Pilot SUV, CR-V SUV, 1/6]
          - [Pilot SUV, Element SUV, 3]
          - [Pilot SUV, Odyssey Minivan, 1/3]
          - [CR-V SUV, Element SUV, 7]
          - [CR-V SUV, Odyssey Minivan, 5]
          - [Element SUV, Odyssey Minivan, 1/5]
      children: *alternatives
    Capacity:
      preferences:
        pairwise:
          - [Cargo Capacity, Passenger Capacity, 1/5]
      children:
        Cargo Capacity:
          preferences:
            pairwise:
              - [Accord Sedan, Accord Hybrid, 1]
              - [Accord Sedan, Pilot SUV, 1/2]
              - [Accord Sedan, CR-V SUV, 1/2]
              - [Accord Sedan, Element SUV, 1/2]
              - [Accord Sedan, Odyssey Minivan, 1/3]
              - [Accord Hybrid, Pilot SUV, 1/2]
              - [Accord Hybrid, CR-V SUV, 1/2]
              - [Accord Hybrid, Element SUV, 1/2]
              - [Accord Hybrid, Odyssey Minivan, 1/3]
              - [Pilot SUV, CR-V SUV, 1]
              - [Pilot SUV, Element SUV, 1]
              - [Pilot SUV, Odyssey Minivan, 1/2]
              - [CR-V SUV, Element SUV, 1]
              - [CR-V SUV, Odyssey Minivan, 1/2]
              - [Element SUV, Odyssey Minivan, 1/2]
          children: *alternatives
        Passenger Capacity:
          preferences:
            pairwise:
```

```
    - [Accord Sedan, Accord Hybrid, 1]
    - [Accord Sedan, Pilot SUV, 1/2]
    - [Accord Sedan, CR-V SUV, 1]
    - [Accord Sedan, Element SUV, 3]
    - [Accord Sedan, Odyssey Minivan, 1/2]
    - [Accord Hybrid, Pilot SUV, 1/2]
    - [Accord Hybrid, CR-V SUV, 1]
    - [Accord Hybrid, Element SUV, 3]
    - [Accord Hybrid, Odyssey Minivan, 1/2]
    - [Pilot SUV, CR-V SUV, 2]
    - [Pilot SUV, Element SUV, 6]
    - [Pilot SUV, Odyssey Minivan, 1]
    - [CR-V SUV, Element SUV, 3]
    - [CR-V SUV, Odyssey Minivan, 1/2]
    - [Element SUV, Odyssey Minivan, 1/6]
children: *alternatives
```

Bibliography

[1] R. T. Clemen and T. Reilly, *Making Hard Decisions with Decision Tools*. Cengage Learning, 2004.

[2] B. Ezell, S. Bennett, D. Winterfeldt, J. Sokolowski, and A. Collins, "Probabilistic Risk Analysis and Terrorism Risk," *Risk analysis : an official publication of the Society for Risk Analysis*, vol. 30, no. 4, pp. 575–89, Apr. 2010.

[3] L. Sander, "AHP: Choosing a leader," 2010. [Online]. Available: https://commons.wikimedia.org/wiki/File:AHP_TDHLeadImage.png

[4] "Analytic hierarchy process – car example," 2010. [Online]. Available: https://en.wikipedia.org/wiki/Analytic_hierarchy_process_%E2%80%93_car_example

[5] Jhguch, "Boxplot and a probability density function (pdf) of a Normal N(0,1 σ^2) Population." 2011, page Version ID: 23242975. [Online]. Available: https://commons.wikimedia.org/wiki/File:Boxplot_vs_PDF.svg

[6] J. I. Manoogian, "Cognitive Bias Codex - 180+ biases," 2016. [Online]. Available: https://commons.wikimedia.org/wiki/File:Cognitive_Bias_Codex_-_180%2B_biases,_designed_by_John_Manoogian_III_(jm3).jpg

[7] "Brainstorming." [Online]. Available: http://cs.wikipedia.org/wiki/Brainstorming

[8] "Mind mapping." [Online]. Available: http://alevelproduct.blogspot.cz/2012/09/mind-mapping.html

[9] J. Bethlehem, *Applied Survey Methods - A Statistical Perspective*. New Jersey: Wiley, 2009.

[10] V. Dratva, *Návrh koncepce vzdělávání v oblasti ochrany obyvatelstva a krizového řízení*. Ostrava: VŠB-TU Ostrava, Fakulta bezpečnostního inženýrství, 2015.

[11] I. Gros, *Kvantitativnii metody v manazerskem rozhodovani*. Praha: Grada, 2003.

[12] C. Castillo, *Effective Web Crawling*. Santiago de Chile: University of Chile, 2004.

[13] P. Šenovský, "Termiti - vizualizace," 2007.

[14] ——, "Scale-Free Network - vizualizace," 2007.

[15] FEMA, "HAZUS." [Online]. Available: https://www.fema.gov/hazus

[16] D. C. Ong, *Statistics and Analytics for the Social and Computing Sciences*, 2022. [Online]. Available: https://desmond-ong.github.io/stats-notes/

[17] Jcheming, "Public Goods Game," 2018. [Online]. Available: https://commons.wikimedia.org/wiki/File:Public_Goods_Game.png

[18] T. L. Saaty, "Decision making with the analytic hierarchy process," *International Journal of Services Sciences*, vol. 1, no. 1, pp. 83–98, 2008.

[19] M. Fontenla, *Package optrees - Optimal Trees in Weighted Graphs*. CRAN, 2014. [Online]. Available: https://cran.r-project.org/web/packages/optrees/optrees.pdf

[20] D. Kahneman, *Thinking, Fast and Slow.* New York: Farrar, Straus and Giroux, Mar. 2013.

[21] "Dia [online]." [Online]. Available: https://live.gnome.org/Dia

[22] "Ms visio [online]." [Online]. Available: http://visio.microsoft.com/en-us/pages/default.aspx

[23] "SmartDraw." [Online]. Available: http://www.smartdraw.com/

[24] "Draw.io." [Online]. Available: https://www.draw.io/

[25] "Fbi moodle." [Online]. Available: http://prometheus.vsb.cz/moodle17

[26] J. E. Beasley, "Or - notes." [Online]. Available: http://people.brunel.ac.uk/~mastjjb/jeb/or/decmore.html

[27] "The r project for statistical computing." [Online]. Available: http://www.r-project.org/

[28] P. Šenovský, *MCDASupport Package Documentation version 0.21.* Ostrava: VŠB-TU Ostrava, Fakulta bezpečnostního inženýrství, 2022. [Online]. Available: https://fbiweb.vsb.cz/~sen76/data/uploads/programy/MCDASupport_0.21.pdf

[29] ——, "MCDASupport Package 0.21," Ostrava, 2022. [Online]. Available: https://fbiweb.vsb.cz/~sen76/data/uploads/programy/MCDASupport_0.21.tar.gz

[30] CDF, "SuperDecisions." [Online]. Available: http://sdbeta.superdecisions.com/

[31] C. Glur, "AHP package (CRAN archive)," 2018. [Online]. Available: https://cran.r-project.org/src/contrib/Archive/ahp/

[32] P. Šenovský, "AHP YAML Editor," 2017. [Online]. Available: https://fbiweb.vsb.cz/~sen76/data/uploads/programy/AHPEditor%20v0.1.7z

[33] "Obsidian - Sharpen your thinking." [Online]. Available: https://obsidian.md/

[34] Zettlr, "Zettlr: Your One-Stop Publication Workbench," 2024. [Online]. Available: https://www.zettlr.com

[35] T. Forte, *Building a Second Brain: A Proven Method to Organize Your Digital Life and Unlock Your Creative Potential.* New York: Simon Element / Simon Acumen, Jun. 2022.

[36] H. A. Linstone and M. Turoff, *The Delphi Method: Techniques and Applications*, 2002. [Online]. Available: https://web.njit.edu/~turoff/pubs/delphibook/delphibook.pdf

[37] R. H. Thaler and C. R. Sunstein, *Nudge: Improving Decisions About Health, Wealth, and Happiness*, revised & expanded edition ed. New York: Penguin Books, Feb. 2009.

[38] P. Iamtrakul, K. Hokao, and K. Teknomo, "Public park valuation using travel cost method," *Proceedings of the Eastern Asia Society for Transportation Studies*, vol. 5, no. 1, pp. 1249–1264, 2005. [Online]. Available: http://www.easts.info/on-line/proceedings_05/1249.pdf

[39] B. Roach and W. W. Wade, "Policy evaluation of natural resource injuries, using habitat equivalency analysis," *Ecological Economics*, vol. 58, no. 2, pp. 421–433, Jun. 2006.

[40] T. Buzan, *Mind Map Mastery: The Complete Guide to Learning and Using the Most Powerful Thinking Tool in the Universe.* ReadHowYouWant, 2019.

[41] "Mindmanager." [Online]. Available: https://www.mindjet.com/products/mindmanager

[42] "Freemind – free mind mapping software." [Online]. Available: http://freemind.sourceforge.net/wiki/index.php/Main_Page

[43] "Java download page." [Online]. Available: http://www.java.com/en/download/index.jsp

[44] ČSÚ, "Sčítání lidu, domů a bytů 2011." [Online]. Available: https://www.csso.cz/csu/sldb

[45] *ISO 5807:1985 Information processing – Documentation symbols and conventions for data, program and system flowcharts, program network charts and system resources charts.*

[46] OMG, "Unified modeling language," 2017. [Online]. Available: https://www.omg.org/spec/UML/

[47] MŠMT, "Výkonová data o školách a školských zařízeních – 2003/04–2013/14." [Online]. Available: http://www.msmt.cz/vzdelavani/skolstvi-v-cr/statistika-skolstvi/vykonova-data-o-skolach-a-skolskych-zarizenich-2003-04-2013

[48] ČSÚ, "Složení obyvatelstva podle pohlaví a jednotek věku (04-02) k 31.12.2014." [Online]. Available: http://vdb.czso.cz/vdbvo/tabparam.jsp?cislotab=04-02&&kapitola_id=368&voa=tabulka

[49] "Microsoft project [cd-rom]." [Online]. Available: https://www.microsoft.com/project/en-us/project-management.aspx

[50] "Projectlibre [online]." [Online]. Available: http://sourceforge.net/projects/projectlibre/

[51] "Open workbench [online]." [Online]. Available: http://sourceforge.net/projects/openworkbench

[52] "Ford-fulkerson algorithm [online]." [Online]. Available: http://en.wikipedia.org/wiki/Ford-Fulkerson_algorithm

[53] "Ford-fulkerson demo [online]." [Online]. Available: http://www.cse.psu.edu/~sofya/cse565/lecture-notes/07demo-maxflow.pdf

[54] "Dijkstra algorithm [online]." [Online]. Available: http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm

[55] "List of algorithms [online]." [Online]. Available: http://en.wikipedia.org/wiki/Graph_algorithm#Graph_algorithms

[56] "igraph R package." [Online]. Available: https://igraph.org/r/

[57] V. Batagejl and A. Mrvar, "Analysis and visualization of very large networks," 2022. [Online]. Available: http://mrvar.fdv.uni-lj.si/pajek/

[58] A. T. Adai, S. V. Date, S. Wieland, and E. M. Marcotte, "LGL: creating a map of protein function with an algorithm for visualizing very large biological networks," *Journal of Molecular Biology*, vol. 340, no. 1, pp. 179–190, Jun. 2004.

[59] DIMACS, "DIMACS Implementation Challenges," 2017. [Online]. Available: http://archive.dimacs.rutgers.edu/Challenges/

[60] Ontotext, "GraphDB Downloads and Resources," 2022. [Online]. Available: https://graphdb.ontotext.com/

[61] "Graph Modelling Language," Jun. 2022, page Version ID: 1094073773. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Graph_Modelling_Language&oldid=1094073773

[62] "UCINET DL Format," 2022. [Online]. Available: https://gephi.org/users/supported-graph-formats/ucinet-dl-format/

[63] "Graphviz - Graph Visualization Software." [Online]. Available: http://www.graphviz.org/

[64] ČR, "Zákon 240/2000 Sb. o krizovém řízení a o změně některých zákonů (krizový zákon)." [Online]. Available: http://www.zakonyprolidi.cz/cs/2000-240

[65] T. G. Lewis, *Critical Infrastructure Protection in Homeland Security*, 2nd ed. New Jersey: Wiley, 2015.

[66] A.-l. Barabasi and J. Frangos, *Linked: The New Science Of Networks Science Of Networks.* New York: Basic Books, 2014.

[67] M. Resnick, *Turtles, termites and traffic jams: explorations in massively parallel microworlds*. London: MIT Press, 1994.

[68] R. Setola, S. De Porcellinis, and M. Sforna, "Critical infrastructure dependency assessment using the input–output inoperability model," *International Journal of Critical Infrastructure Protection*, vol. 2, no. 4, pp. 170–178, Dec. 2009. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S1874548209000390

[69] P. Šenovský, *Modelování následků mimořádných událostí*. Ostrava: VŠB-TU Ostrava, Fakulta bezpečnostního inženýrství, 2020. [Online]. Available: https://fbiweb.vsb.cz/~sen76/data/uploads/skripta/model_nasledku.pdf

[70] S. Waner, "Tutorial for the simplex method [online]." [Online]. Available: http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tutorialsf4/unit4_3.html

[71] "Scilab [online]." [Online]. Available: http://scilab.org/

[72] "Finite mathematics utility: simplex method tool [online]." [Online]. Available: http://www.zweigmedia.com/RealWorld/simplex.html

[73] M. Berkelaar, *package 'lpSolve'*, 2022. [Online]. Available: https://cran.r-project.org/web/packages/lpSolve/lpSolve.pdf

[74] "List of games in game theory [online]." [Online]. Available: https://en.wikipedia.org/wiki/List_of_games_in_game_theory

[75] R. Krispin, *Hands-On Time Series Analysis with R*. Packt Publishing, 2019.

[76] B. Boehmke and B. M. Greenwell, *Hands-On Machine Learning with R*. Boca Raton: Chapman and Hall/CRC, 2019.

[77] "Graphviz [online]." [Online]. Available: http://www.graphviz.org/

Glossary

AHP Analytic Hierarchy Process.

AI Artificial Intelligence.

ANOVA Analysis of Variance.

ANP Analytic Network Process.

BCP Bussines Continuity Planning.

BRMS Bussines Rules Management System.

BTS Base Transceiver Station.

CEO Chief Executive Officer.

CI Critical Infrastructure.

CPM Critical Path Method.

CSU Český statistický úřad (Czech Statistical Office).

CVM Contingent Value Method.

DRM Digital Rights Management.

ERP Enterprise Resource Planning.

EWM Entropy Weight Method.

GIS Geographic Information System.

GPS Global Positioning System.

GUI Graphical User Interface.

IDE Integrated Development Environment.

IIM Input-output Interoperability Model.

IQR Inter Quartile Range.

JDK Java Development Kit.

JRE Java Runtime Environment.

LMS Learning Management System.

MCA Multicriterial Analysis.

NPU Neural Processor Unit.

PCA Principal Component Analysis.

RCI Random Consistency Index.

TCM Travel Cost Method.

UML Universal Modeling Language.

WPM Weighted Product Method.

WSM Weighted Sum Method.

WTA Willigness to Accept.

WTP Willigness to Pay.

XML Extensive Markup Language.

YAML YAML Ain't Markup Language.

Index