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Modelling of Decision Processes

textbook
3rd edition



Modelling of Decission Processes

3rd edition

this text has not gone through edition process

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Vysoká škola báňská - Technical University of Ostrava, Faculty of Safety Engineering

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Introduction

Dear student, you are holding in your hands a textbook on the subject of modelling of the decision processes. Basic goal of this textbook is to supply you with various methods for decision support.

This textbook has been derived from 5-th edition of Czech version of it, which has been published first in 2021 in Ostrava and has been prepared for e-learning. As English version of this textbook has been written rather in rush, it is to be perceived slightly more as a syllabus than a full-fledged textbook.

Use the textbook as a companion study material to the lectures. So it is to be used together with your notes and presentations available from LMS (<https://lms.vsb.cz>).

In the text there are some symbols, which are intended to support you while studying. These symbols are:



Study guide

It is used to help you familiarize yourself with basic goals of the chapter, so you know what you should be looking out for :-).



Time required to study chapter

Is estimated time necessary to understand (learn) all of the concepts in the chapter. The time estimation should be looked upon only as rough estimation to help you better organize time. So do not be disturbed if the learning requires more or less time.



Definition, note, important details

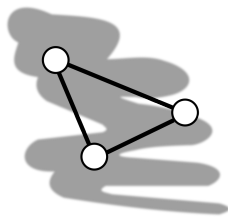
Accompanying this icon there will be text which the author finds so important, that he puts exclamation mark to it. Please pay close attention to it.



Test your knowledge

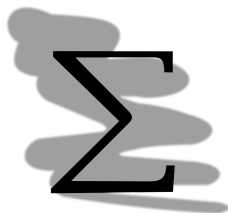
On the end of each chapter there are several questions or tasks you can use to self-evaluate your knowledge. These questions are not exhaustive to the theme of the chapter, so if you know all answers to the presented questions ... it is a good start :-), if not go back few pages and search for answer – chances are you did not understand some important concept.

Also this book has been typeset in LaTeX to introduce some features to improve your learning



Relation to other information

In sections of text with this icon, you will find how the discussed topic relates to other topics either in this textbook or in other courses.



Summary

Summarizes basic ideas of the chapter. This is the basics you absolutely must understand of the chapter.



Take a break

Well done, you worked hard, so it is time to take break.



Example or task

In this textbook, there are many examples but also many tasks available for you to compute by hand or using the computer. Take your time to compute them – it is a good indicator of whether you understand the problematic or not.

experience. The text has list of figures and tables, also it has the index with most important concepts with their location.

I hope that you will find information in this textbook useful.

Pavel Šenovský

What's new in third edition of the textbook

3rd edition is basically rewrite of the textbook. Every part of it was edited for clarity. Multiple chapters have been added. Short, incomplete list of large changes is provided here:

- revisited every chapter and brought it up to date with 5th edition of Czech version of textbook
- added chapter 1.2 to better state context of decision making
- added chapters 2.2 and 2.3 for topics of sensitivity analysis and extension of decision trees for multicriteria analysis
- replaced MCA case study
- added chapter for advanced multicriteria analysis methods including also more indepth information on various normalization methods and ways to estimate criteria weights.
- added chapter for AHP method
- added examples in R for various methods
- added chapter on realization and evaluation of the surveys

- rewritten section on network analysis and visualization in R (chapter 9.3)
- added section 9.4 to explain connections between network analysis and critical infrastructure
- added section 10.2 to explain relation of balance model to consequence analysis, critical infrastructure protection, etc.
- rewritten parts of chapter 11 to compute linear programs using R
- added public goods game to Games theory (relevant for explanation of behavior in society)
- added smaller chapters 14 and 15 to frame problematic of decision making into broader context of statistics and machine learning

Whats new in second edition of the textbook

- added introductory chapter one
- all subchapters now have questions to help you self-evaluate your knowledge on given problematic or tasks for chapters which are more practically oriented
- added practical software related chapter to project management
- added chapter for software support for Ford-Fulkerson and Dijkstra algorithms
- added chapter for software support of networks renderings based on the definition of the network structure
- added discussion of software tools for computation of linear programming problems
- many smaller corrections and changes.

Chapter 1

Introduction to decision making



Study guide

In this chapter we will learn something about decision making. While decision making is part of our every day life – so we could think that we humans are rather good at it. Making a decision is hard, especially in situations which are complicated – we require help and decision support methods are fully capable to supply such help.

After finishing of study of this chapter you will

Know

- What is a decision

Be capable

- to distinguish between different types of decision situations



Time required to study chapter

For getting through this chapter you will need at least one hour ... you know only something small for starter.

1.1 Decision Making

Every day, every hour or perhaps even nearly every moment after we awake and before we go sleep we perform consciously and unconsciously countless decisions. Most of them are small and we make them automatically without even thinking of them, for example when we walk we automatically change the direction of our steps to go around obstacles, we greet people we know and such.

Some decisions are harder, for example we are hungry, we have to decide when and more importantly what to eat. While such decision can be very hard at the times it is not the decision we would usually require assistance of sophisticated decision support tools. Such tools we dedicate to situations with even higher level of complexity – we choose between merchandise of high value or different technologies, choosing between different protective measures ... this is the kind of situations we are going to examine.

All decision situations regardless of what are we choosing between have common points – we choose between choices characterized by different properties of the choices resulting into different outcome of the choice. We are deciding between different *variant* of the problem solution, based on the difference of solution properties. Our goal is to maximize positive (beneficial) properties of the

solution while minimizing negative (net desired) properties. If we are looking at decision making from this perspective we may approach it as *optimization problem*.

Considering predictability of the decision outcomes also forms first property we can use to distinguish between various decision situations. When we know exactly what the results of the decision will be, we call such situation as *deterministic*. Unfortunately most of the times we do not know for sure how the decision will play out. But we are able to find possible solutions (outcomes) and derive its probabilities. Results of our choice are stochastic in this case, characterized by probability.

From this we can logically deduce a related term *uncertainty*. Probability is than a tool we can use to quantitatively express it.

So what is the benefit of using decision support methods and tools? Well, using established method is like using highway instead of making your own way through the thick forest. Decision support methods are documented approaches some else used before us to solve the problem we are solving now. So by using method we may reach goal more easily, with fewer resources and the outcome will come in expected format, that is if we use it as we should – for the problems the method has been verified and are considering possible constrains of the method, if such are known.

Formally established decision support methods do also usually have well documented limitations, making it easier for the user (practitioner) to value results and usability of the method for solving decision problem at hand.

Another benefit is that the methods in general also enable us to make emotional distance between us and our results. Why? Because we all are biased in our decision making. From the tools we choose to analyse the problem to interpretation of the result, affinity to risk taking and perception of what the optimality of the solution actually consist of.

If left unchecked, the *cognitive bias* will surely flaw the decision making potentially leading to suboptimal choices, which otherwise could be prevented. Using formal methods, understanding their process and properties will allow us to approach decision making more defensively, making our choices less error prone and more robust.

Let's some more decision variants and differences between them. We can use these differences as a criterion characterizing various decision making approaches. From this perspective we could divide methods to those using single criterion for characterizing results of decision making, we can call these *monocriterial methods*, and those which need multiple criteria to describe decision making, we call these *multicriterial methods*.

Single by far most often used criterion measure used in monocriterial decision making is the monetary value. It is a logical choice, since we can transcribe lots of things to monetary value: costs, gain, value of property, opportunity costs, well being, for some purposes even value of the life itself. Good example of such method would be *decision trees* we will discuss in next chapter.

Multicriteria analysis is the much more complex. **Multicriterial Analysis (MCA)** will be discussed separately, from different angles. To solve these types of problems, many methods are being used, with mixes success. We will look at some more interesting ones to provide introduction to this topic and also tools to deal with these problems more easily.

Even the way we express the values of the criterions may lead to usage of different methods supporting the decision. Basically we may express the criterion in three different ways:

1. numeric value – as in continuous numerical variable, for example age, charge time, price,
2. ordinal value – may be number or string, for example 0 – no, 1 – yes or
3. freeform answer – meaning that the criterion valuation for the variant will lead to paragraphs of text

We can also use fuzzy numbers to encompass some kind of uncertainty directly to the expression of the criteria values.

It is only logical that different types of the criterion valuation will lead to different methods, which are capable to process them. For example numeric values are processable by statistical methods such as *regression analysis*, while ordinal values are better processed by data mining methods such as *decision trees*.

In this textbook, I try to supply the tools for many typologically different decision situations. So we can say, that the problem typology itself may be used as criterion to distinguish methods supporting our decision.

On the beginning of the chapter we were discussing classical choice – when deciding between different variants as in buying different product. What if the nature of decision (and the difference



Decision Trees

In previous paragraphs we were discussing decision trees, please note that there are two different methods using this name one, which will be discussed in next chapter, is monocriterial decision support tool, while the other can be used in data mining for classification purposes and works best with ordinal values. As this text is not focused on data mining, we will focus on first method only.

Also note that there are many more methods such as statistical methods or methods based on **Artificial Intelligence (AI)**, which will also not be discussed in this text, even knowing that these methods can be very useful in supporting our decision. But perhaps you can study such methods in other courses on Faculty (AI for Security or Computer Science in Security) or abroad.



Portfolio of methods

In your professional life you will have to deal with many different decision situations requiring the usage of different approaches to reach successful solution – so build up portfolio of methods you are familiar with and capable of using. The more different the methods will be the better. Healthy variety in the methods will allow you to solve much broader set of problems.

By the way this recommendation can be applied in any field of study and still hold :-).

of the variants) is not in buying of new product, but rather in change of its operational parameters – setting up in different way. In such cases we talk about *optimization problems*.

Even for optimization very different approaches exist MCA uses distance measures to theoretical optimum for recommendation formulation. But if problem is more geographically formulated, i. e. optimal location problems, there are *localization models* we can use to solve this kind of problem. Is the problem expressible by system of equations/inequations? Then we can use *mathematical programming methods*, such as *primary algorithm*, also known as so called *simplex method*.

Different types of problems require usage of different methods to solve them. So for us to be efficient, we need to build whole portfolio of methods.

1.2 Hierarchy and Structure of Decision Making

We already know in general terms, what decision making is and that for us to do it effectively we need to strive to achieve some kind of *goal or goals* we set for ourselves. Looks simple, right? But the looks can be deceiving. The goals can be achieved by different ways. Some of them may be complementary to each other, while others may contradict or be even mutually exclusive.

The goals will also differ depending on who does make the decision, in what *decision context*. Decisions, with same goal, can be made on different levels. For example: business unit, enterprise, region, state, etc. all make decisions with goal to improve security of the citizens. But naturally tools available at each level is very different. To better understand decision situation, we can express it graphically using *goal hierarchy*.

Let's define problem to demonstrate creation of such hierarchy: *we shall be deciding on measures for improving security in road traffic*.

Goal hierarchy as well as the tools for achieving such goals is available on fig. 1.1.

Goals hierarchy on the figure shows differences in perceptions of the goals. The higher in the hierarchy we are, the more generic the goals are. That corresponds to management system in general, the higher in leadership hierarchy of the enterprise we are the more it is focused on "big picture" rather than mundae details.

Differences in goals can also be related to practitioners groups, in other words the people who will be responsible for decision making at the decision level in question. So in our example we can identify following decision makers:

1. common citizen, wishing to maximize his or hers personal safety

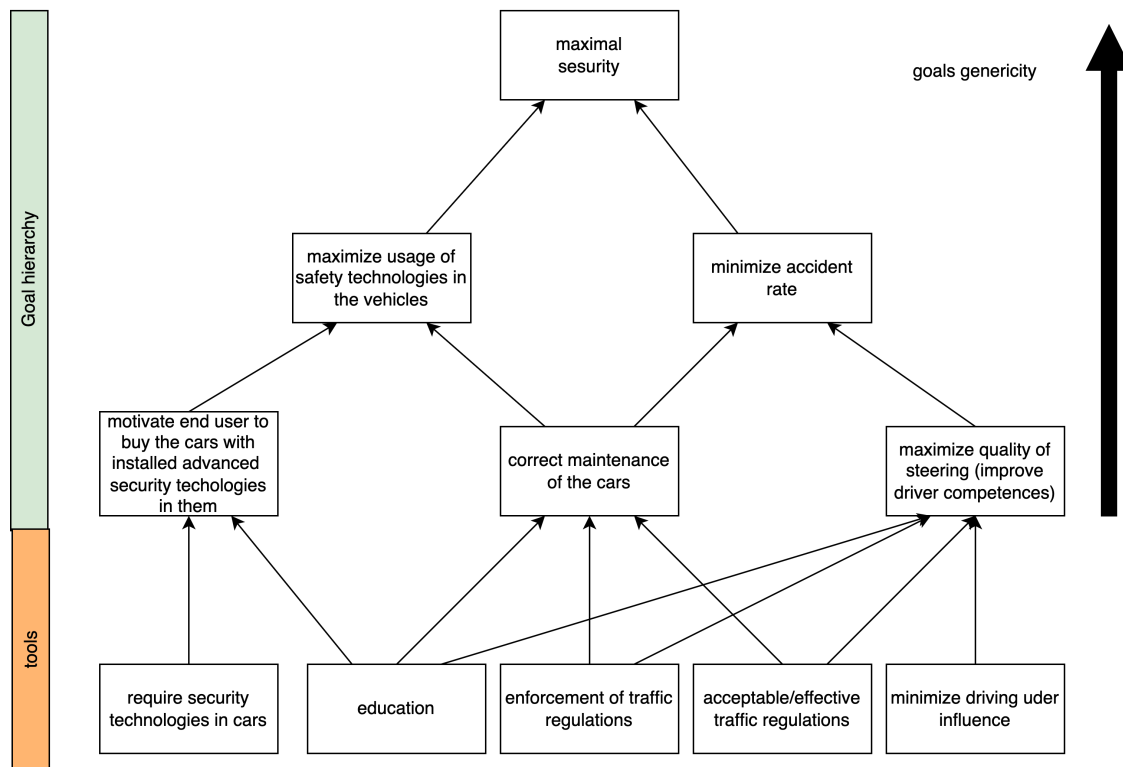


Figure 1.1: Goals hierarchy and the tools available to achieve them (adapted from [39])

2. car manufacturer, who chooses between possible safety characteristics for his new car to be successful on the market
3. Ministry of Transport, which is responsible for legislative and regulation of road traffic
4. driving school
5. technical inspection stations
6. etc.

Each of these groups will have slightly different goals all under general umbrella of maximization of the traffic safety/security. That means that the preferences and the tools these group will have at their disposal and that they will be inclined to use will differ significantly even if the wording of the goal will remain same. This is important, as this very moment illustrates necessity to understand the *decision making context*.

Even if limit our selves to context of decision making in the enterprises, it is necessary to consider various positions on different levels of the management and their influence on decision making process. The decision itself is always in the competence of somebody. This competence is usually derived from job description or can be stated by some kind of inner regulation in the enterprise. In case of state and local governments such competence may be defined by legislation.

Competence to decide does not mean, that such person needs to perform whole decision process. In fact, it is very unusual, if someone even tries to realize the decision in such way. The manager will decide and will hold managerial responsibility. But the materials on which the decision is based upon are usually prepared by one or more experts in the decision problem's domain. The more sophisticated the problem is, the larger team preparing such materials usually is.

Manager can delegate partial decision step on experts or other managers directly under him. This delegation does not diminish in any way overall managerial responsibility.

To improve chances to succeed, the manager evaluates proposed solutions to the decision problem, especially whether the proposition meets quality standards and is practically realizable in organization's environment. if (s-)he identifies any significant problem in the underlying analysis, such analysis is returned to its creator for completion and or other edits.

If no problem with these is detected, the manager chooses variant based on his/hers preferences using the analysis to support the decision and proceeds with its realization.

If the manager had to do the whole process by himself or herself, the level of knowledge necessary for one person would be too high. So the manager needs to delegate. On the other hand, the manager also needs to have good enough knowledge in the problem domain and decision making methods to be capable of interpreting results. Otherwise he or she will be completely reliant on the provided information, without the ability to judge quality of it, while taking full responsibility for outcomes of the realization of such decisions. That is clearly undesirable.

Length of the decision making and size of the team preparing supporting materials is proportional to complexity of the problem. Simple problems might require hours to find optimal solution (decision) using just normal office software, while complex problems might take large teams, months of work and use of specialized software.

Especially for complex decisions it is often necessary to better understand the factors influencing outcome of the decision. There are many methods usable for this purpose. You may already studied some them such as SWOT analysis or Ishikawa diagram. These are so notoriously well known, that we will not discuss them in this textbook. Other popular methods are brainstorming and mind maps, which we will discuss in later chapters. In this chapter we will focus on another method well suited to analyse decision situation from point of view of various factors influencing it. This method is called *influence diagram*.

As the name suggests, influence diagrams are graphical method allowing us to describe characteristics and connection between various factors we perceive as significant for the decision. This perception (believe) can be, but does not have to be accurate. So the influence may or may not be present. Formal specification of these influence allows us to set starting framework for the decision making. We can then focus on detailed analysis of the factors and exclude them (or not) at later time, based on additional information and better understanding of the problem.

Influence diagram with its simplicity, allow us to create good starting point we can base further, more detailed work, using specialized other methods on. That also means that by using influence diagrams we usually do not come to conclusion - identify variants of solution.

Influence diagrams use just few graphically simple symbols (constructors) using shapes common in variety of software packages for graphics such as Dia [2], Microsoft Visio [20], SmartDraw [25], Draw.io [4] and many more.

Constructors of influence diagrams are presented on fig. 1.2.

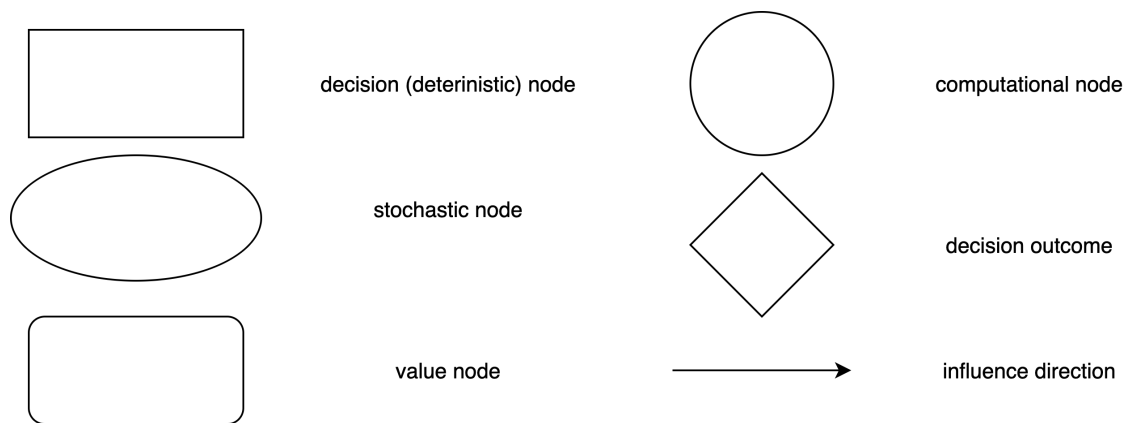


Figure 1.2: Basic constructors of influence diagrams

Let's look at the constructors in more detail. First is the *decision node*. This symbol is used for situations, when the nature of the (partial) decision is purely deterministic. In the other words we are sure how the outcome of this partial decision will play out.

Stochastic node allows us to describe the situations, when the outcomes of the partial decision are not sure. That is useful in situations when the outcomes are not completely under our control, are probabilistic based, etc. As we already know, the majority of the decisions are stochastic in nature.

Value node serves for capture the measures used in computations. You can see simple example in fig. 1.3.

Mathematically we can express it quite simply as $\text{profit} = \text{demand} * (\text{price} - \text{profit})$, for unit of production. The equation itself may be directly written into the node or kept separately, depending on complexity of the computation. In the diagram more interesting are symbols used for demand

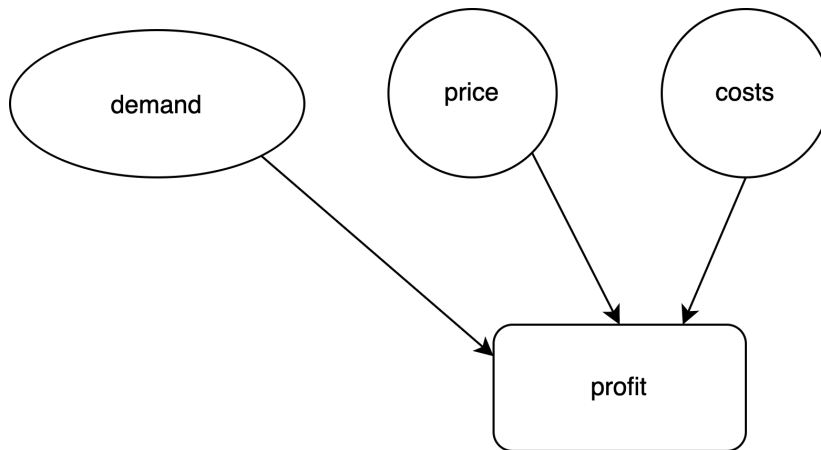


Figure 1.3: Simplistic diagram of profit computation using influence diagram

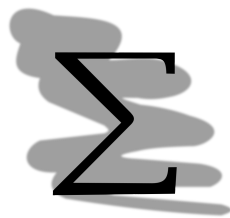
(*stochastic node*) and for both price and costs (*computational node*). Both price and costs can be considered as constant for unit of production, while demand is variable depending on other factors (not explored in the diagram), thus expressed by stochastic node.

Fig. 1.3 represents only fragment of possible decision situation, to be complete it would need to have at least one deterministic node and also one, or realistically much more then one results of decision represented by *decision outcome node*.



Task - experiment with influence diagrams

Choose one of software packages mentioned in this chapter (or choose one on your own) and experiment with it to create some influence diagrams. First choose a problem to model and then decompose it and visualize using influence diagram. Consider for example what influences your decision on course selection for next semester.



Summary

Process of decision making presumes existence of variant solutions, which differ by its utility for decision maker. The decision itself is always influenced by uncertainties regarding outcomes of decision making. Usually we are able to at least somewhat quantify this uncertainty, by establishing probabilities of the outcomes. Decision making context play especially important role in decision making, it defines preference system of the decision maker, which will limit options for possible solutions (limit solution space).

We can visualize decision situation and various influence in play for it using influence diagrams.



Questions

1. What characterizes decision situations?
2. What is difference between stochastic and deterministic decisions?
3. Specify types of criteria we use to describe utility of the solution variant (there are 3 of them).
4. Can nature of the problem, we are trying to decide, have impact on the methods effectively usable to solve it – can you give example?
5. Look back at your study plan and try to find methods usable as support for decision making, you already know.
6. What is influence diagram good for?
7. Try to specify reason to consider decision making context for effective decision making.

Chapter 2

Decision trees



Study guide

In this chapter we will learn of how to use one of simplest tools for decision support – *decision trees*.

After finishing of study of this chapter you will Know

- What decision trees are
- When and how to use them
- what is sensitivity analysis and ho to use it



Time required to study chapter

For getting through this chapter you will need at least one hour and you will need about two hours for calculation of examples.

2.1 Monocriterial decision making

One of most simple tools available for decision support are *decision trees*. Decision trees are oriented graphs, which are by its look similar to the tree. As normal tree, decision tree has one root and branches itself to leafs.

The root of the tree represents a basic choice between variants. The branching then represents partial decisions we may do during the decision making. What do we mean by partial decisions? In our decision we usually do not perform single decision – but a series of small decisions. For example we want to by a car, the basic choice may be between the car producers – Ford, Škoda, Volkswagen but there would be also other decisions leading to our dream car – what type will it be – SUV, hatchback, limousine and what about the engine? To finish the choice we must perform all these small decisions (branching on the decision tree). Then finally we reach the outcome of our decision – *leaf node*.

Writing decision situation into form of the decision tree helps us by visualizing the situation in the easily to understand way so we may focus on partial decisions while not loosing the bigger picture of the decision situation.

Decision trees are monocriterial method, so each variant of the choice is characterized by only one criterion. The criterion itself must be common over the whole tree regardless of the nature of partial decisions. It is logical requirement, otherwise we would not be able to compute the value of the decision as whole.

Such universal criterion is usually only one – money. We are capable to translate many different variables to financial value. For some of them there are even available the tables to make the process easier (damages on fauna and flora, valuation of injuries, cost of building cubic meter of certain type

of building and many more). The optimal solution to our problem then would be the variant with minimal or maximal value of the criterion.

Even with money as common denominator we may face both maximalization and minimalization problems. For example, we usually like to minimize the costs while maximalizing the benefits or profits.

When making a decision or partial decision, we may or may not know the outcome of that decision. Decision in a situation of certainty means that we fully know what will happen, if we make the choice. Unfortunately, that is usually not the case, so we have to deal with some kind of uncertainty in most of the cases.

Decision in a situation of uncertainty means that we cannot state with 100% certainty what exactly will happen, if we make that choice. But usually we are at least capable of establishing probabilities of the outcome.

Both situations are solvable by decision trees, but of different types. In a situation of certainty, we use *deterministic decision trees* and in a situation of uncertainty, we use *stochastic decision trees*. A general example of both types of trees is on the fig. 2.1.

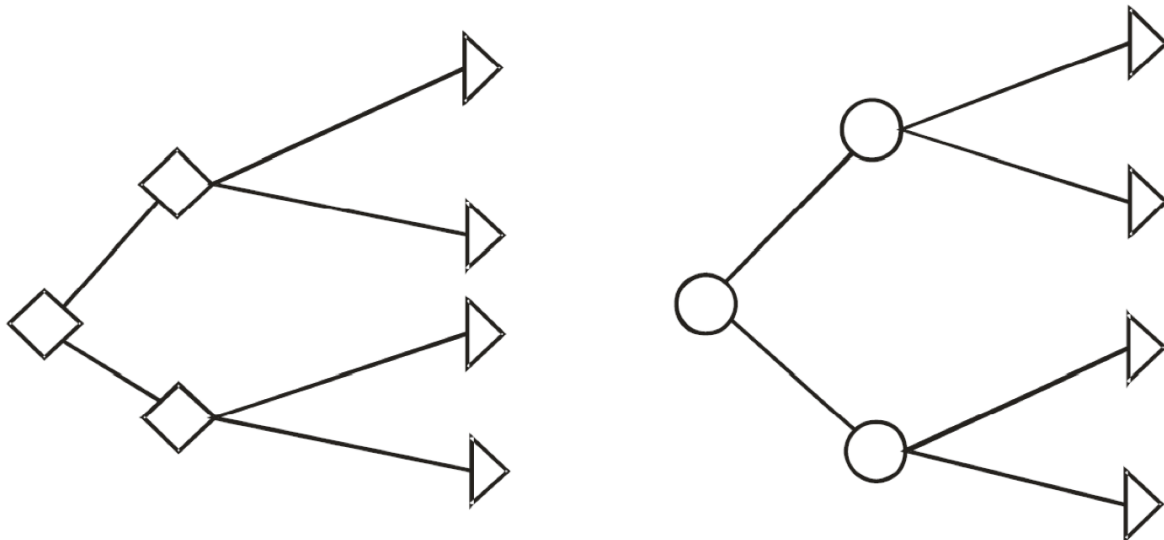


Figure 2.1: Deterministic (left) and stochastic (right) decision trees

Decision trees consist of the nodes and edges as we stated higher. The nodes differ for partial decisions based on whether the outcome of the decision is known in advance. So as we have stochastic and deterministic decisions, we also have stochastic and deterministic nodes. The shape of the nodes differs for both types of decisions.

For deterministic nodes, we are using diamond or square shapes, for stochastic nodes, we are using circle shapes, and for leaf nodes, we are using triangle shapes. The decision tree is stochastic if it has at least one stochastic node. For the decision tree to be deterministic, it has to consist only of deterministic nodes.

The root node is usually deterministic as it represents the decision between basic variants, which are then more developed further down the tree. Also, it makes little sense to create the decision tree using stochastic nodes only – in such a situation, our decision would have absolutely no value, as we would have been fully in the hands of chance.

Position of the node in the X-axis of the graph should also correspond to the time of the decision, the node represents. The nodes representing partial decisions made farther in the future should be farther right than those made in the near future. Position of the nodes along the X-axis is for the sake of better readability of the graph, the computation of the tree is affected by the structure of the tree only, not by its graphical representation.

Also to improve readability of the decision tree, we are often numbering the nodes. Numbering enables us to more easily document the tree or perform more elaborate computations outside of the tree, while making it easy to identify the part of the tree we are working with.

Lets try to finally use it!



Example 1: Deterministic decision tree

Company XYZ must decide whether to or not to reconstruct its business premises. At present time the company is capable to sell its products for 2,5 mil. EUR, during reconstruction the company would have to temporally move into alternative buildings, from which only 1 mil. EUR worth of its products could be sold.

The reconstruction is expected to take one year and will cost 10 mil. EUR. In new business premises the company hopes to sell its products worth 3 mil. EUR per year.

Recommend whether to or not to reconstruct, considering 10 years time horizon.

Let's solve the example together. Considering the situation described higher, we may construct the decision tree similarly to fig. 2.2.

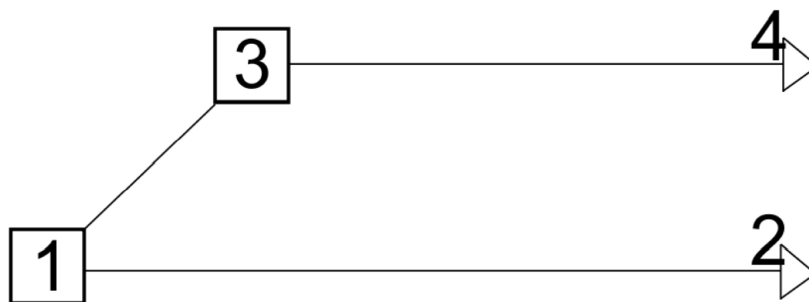


Figure 2.2: Deterministic decision tree construction

The decision tree on fig. 2.2 is rather simple – that is not so surprising as we already know, that the deterministic decision trees do not describe real decision situations well. So the example on fig. 2.2 is typical school problem just to demonstrate the way of computation.

Lets look at the tree. Its node 1 represents our decision to or not to reconstruct business premises. Node 3 only marks one year reconstruction period and does not represent the decision by itself.

We can calculate the net profit in the leaf nodes for 10 years period: **Node 2: we did not reconstruct**

Profit: $2,5 \text{ mil EUR} * 10 = 25 \text{ mil. EUR}$

Node 4: we reconstructed

Profit:

reconstruction period: 1 mil. EUR

Post reconstruction: $3 * 9 \text{ mil. EUR} = 27 \text{ mil. EUR}$

All in all: **28 mil. EUR**

Costs:

Reconstruction: 10 mil. EUR

Gain: $27 - 10 = 17 \text{ mil. EUR}$

So if we reconstruct, the profit will be 17 mil. EUR, while if we will not reconstruct, the profit will be 25 mil. EUR. Since its usually better to have 25 mil. Then 17 mil. - we mus recommend for company not to reconstruct its premises while specified financial constrains remain.

Lets try another example to demonstrate simple stochastic decision tree problem and its solution.

Well, the example 2 is also “shoolary” example – so there are some simplifications to make the computation easy. The largest one is that the vaccination will be 100% effective, could be realized “overnight” for whole population and will will function immediately. Real vaccination needs long (usually over october and november) vaccination campaign, only approximately 25% of the europe’s population is vaccinated (approximately 5% in Czech republic) and the body needs few weeks to create enough antibodies to protect itself effectively. Also the effectiveness of the vaccination is not 100%, but slightly above 60% - so do not take the example to seriously.



Example 2: Stochastic decision tree

Government committee considers economical usefulness of preventive influenza vaccination program. For such program the committee considers “early warning” system, which will cost 120 mil EUR and will permit to detect early start of influenza epidemic. After the early warning, the vaccination program will start. If the vaccination program will not be realized, we estimate the cost of medication 280 mil. EUR with probability of 0,1, 400 mil. EUR with probability of 0,3 and 600 mil. EUR with probability of 0,6. The vaccination program will cost 400 mil EUR and probability of influenza coming is 0,75. We consider the vaccine to function immediately and be 100% effective.

As example 2 is stochastic tree, we are working with probabilities and there are some rules we have to follow. The probabilities themselves may be expressed by one of three ways as probabilities in interval $<0; 1>$ (this is our case), as the percentual chance of occurrence in given time interval or as frequency of occurrences.

For example if the event repeats itself once per ten years, then the probability for time interval of ten years would be 0,1 (1/10) or 10%.

The decision trees work only with probabilities in interval $<0;1>$, so if we had probabilities in different format, we would have to convert them.

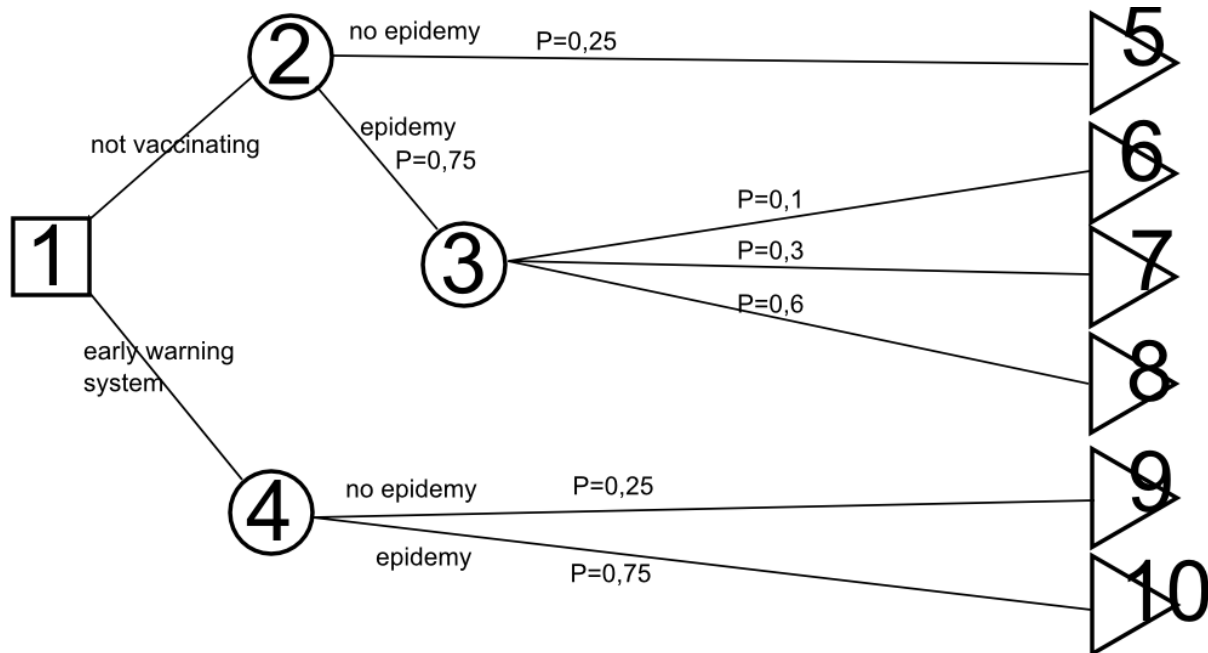


Figure 2.3: Stochastic tree construction

The construction process should be pretty obvious from the fig. 2.3, so let's start computing the gains of variants. The computation process, at least in its first phase will be exactly same as in deterministic tree – so we will ignore the probabilities for now.

Costs:

- Not vaccinating and no epidemic: 0EUR (1-2-5)
- Not vaccination and epidemic with low costs: -280EUR (1-2-3-6)
- Not vaccination and epidemic with middle costs: -400EUR (1-2-3-7)
- Not vaccination and epidemic with high costs: -600EUR (1-2-3-8)
- Early warning system and no epidemic: -120EUR (1-4-9)
- Early warning system and epidemic: -520EUR (1-4-9)

If the decision tree would have been deterministic, we would end at this point evaluating the costs. Unfortunately we have to consider the probabilities too, as we do not have the whole process under our control.

We have to compute the costs of stochastic nodes all the way back to node 1. We will compute the cost of such nodes in such way that we will multiply the costs of branches leaving the node by its probabilities and summarize them.

Cost node 3: $(-280 * 0,1) + (-400 * 0,3) + (-600 * 0,6) = -508$ mil. EUR

Cost node 2: $0 * 0,25 + (-508 * 0,75) = -381$ mil. EUR

Cost node 4: $(-120 * 0,25) + (-520 * 0,75) = -420$ mil. EUR

Considering the nodes 2 a 4 we decide that we should not apply the vaccination program, as the costs connected to it are way larger (-420 mil EUR), then the costs of medication (-381 mil EUR).

You may try to compute some tasks yourself.



Task 1

MDS is company oriented onto geologic evaluation of the land. MDG has the possibility to buy a land for 3 mil EUR. If MDS buys the land, it will perform geologic evaluation of it. Such evaluation costs about 1 mil. EUR and will result:

- Manganese 1% chance
- Gold 0,05% chance
- Silver 0,2% chance

If the one metal is found, no other metal may be found. If the manganese is found, the land may be sold for 30 mil EUR, if gold for 250 mil EUR and silver for 150 mil EUR.

MDG may also pay 750 000EUR for right to realize 3 day preliminary evaluation. Past experience show, that 3 day ecaluation will cost 250 000EUR and increases possibility, that there is metal deposit to 50%. Chances to find the metals is:

- Manganese 3%
- Gold 2%
- Silver 1%

If 3 day test does not show deposit existence, chances that there actually is metal deposit is very low.

- Manganese 0,75%
- Gold 0,04%
- Silver 0,175%

What will you recommend?



Solution: task 1

3 day test and if it is successful buy



Task 2

Company decides, if it is going to take part in selection procedure. They estimate, that only the preparation for selection will cost 10 000EUR. If it is going to participate, they estimate that there is 50% chance that they will advance to second list. In second list it will be necessary to supply more information (estimated cost 5000EUR).

Company estimates, that the cost of the work and material for realisation of the contract will be 127 000EUR. Company thinks about three variants of offer: 155 000EUR, 170 000EUR, 190 000EUR. Probability of success will be: 0.9, 0.75, 0.35.

Should company take part in selection procedure and if yes at what price.

The Examples and task has been taken from [32].

**Solution: task 2**

take part in selection procedure, if selected into short list go for middle price (170 000EUR)

**Other tasks you may want to practice on**

On Moodle supporting module “Modelling of Decision Processes” there are also available other tasks for computations. See if you can solve them all.

The module is available at <http://prometheus.vsb.cz> [5].

2.2 Sensitivity analysis

One of methods to identify whether the variable has some influence is to perform so called *sensitivity analysis*. This type of analyzes is generic, so it can be use together with basically any other method including decision trees. Basic principle is simple, change one variable at time and examine haw are the results affected.

The analytic process follows these steps:

1. identify variables
 2. choose variable from yet not analyzed ones
 3. find possible range of the variable (upper and lower limit)
 4. substitute actual value of the variable with lower and upper limit and compute the model
 5. go back to step 2 and choose different variable
- optional visualize the results using tornado graph (or any other type of graph)
6. interpret the results

So how does the actual sensitivity analysis look like? Let’s go back to simple example of stochastic tree we computed in task 2 in previous section of the text. In tree we have quantified all edges of the graph in order to be able to compute the tree. To do that we needed to establish both utility and probabilities connected to the graph.

Since decision tree is single criterion method, we used costs as this criterion. Formally from statistical point of view we estimated mean value of expected costs. In reality the true value will be higher or lower, although we do not expect it to be a lot lower or higher then this value. If we have large enough dataset, we can even specify upper and lower limit at certain level of expected accuracy probability.

In statistics we often use 95 % percentile. In other words we choose the limits to cover 95 % of variable occurrences in the dataset.

Full statistical analysis requires us to identify used probability distribution for the variable. To do that we need good dataset, from which we can derive required information. With that we have two problems. Data availability may be problematic, so in the real-world situations we often do not have available *large* enough and *good* enough dataset. That is exactly the case for our example. When no hard data is available, we have to make our own, usually based on expert estimation. See tab. 2.1 for our example.

Expert estimations are notoriously imprecise, so in case we had data, its always better to derive required information from it. That was not an option in this case.

Using lower and upper limits from tab. 2.1 we compute the decision thee. First for values of nodes 2 and 4 (their utility will determine the optimality of the solution, see fig 2.3. We performed this type of computation in previous section, so vy can just copy these results: node 2: 381 mil. EUR and node 4: 420 mil EUR.

These values lead from interpretation perspective to decision not to use early warning system for influenza (we are minimizing the costs). We will also use these values to test sensitivity of the solution. For variables A - C we will try to determine if the costs are not greater then 420 mil. EUR. For variables D and E we will try to determine, whether the costs do not fall under 381 mil. EUR.

Table 2.1: Estimate of upper and lower limit for the variables used in task 2 for purposes of sensitivity analysis

label	Variable	Mean [mil. EUR]	lower lim. [mil.]EUR]	upper limit [mil EUR]
A	epidemic (small costs)	280	250	310
B	epidemic (medium size)	400	360	440
C	epidemic (large costs)	600	520	680
D	early warning system	120	100	140
E	vaccination (after warning)	400	360	440

We will star with computation of the upper and lower limit for variant A in eq. (2.1).

$$costs = 0.75 \cdot (0.1x + 0.3 \cdot 400 + 0.6 \cdot 600) \quad (2.1)$$

In equation (2.1) the x is the analyzed variable. In case of variant A we substitute the x with values 310 and 250 mil EUR. And after computation we can clearly see, that the results are not sensitive to changes in treatment costs when the epidemics is small, because the resulting we in both cases lower then 420 mil. EUR.

Using our tree on fig. 2.3, we can derive equations similar to (2.1) for all other analyzed variables. You can find the results in tab. 2.2. For later use there is also added column with difference between upper and lower limit of the costs.

Table 2.2: Computation of the costs when analyzing sensitivity of the variable for decision tree from task 2

label	Variable	costs lower lim. [mil. EUR]	cost upper limit [mil.]EUR]	costs Δ [mil EUR]
A	epidemic (small costs)	378.75	383.25	4,5
B	epidemic (medium size)	372	390	18
C	epidemic (large costs)	345	417	72
D	early warning system	400	440	40
E	vaccination (after warning)	390	435	45

From tab. 2.2 we can see that the decision situation represented by tree from task 2 is not overly sensitive to changes in variables. If the task was real, as opposed to scholarly, we would have better confidence in the validity of the results the tree provided.

If we found out, that the solution is actually significantly sensitive to certain variables changing, we would be motivated to better understand these variables and try to keep them under better control, if possible. Otherwise we also could fail to reach desired outcomes of the solution.

It is possible to present results from tab. 2.2 in graphical form. See influence of changes in treatment costs on fig. 2.4. We used R [23] to generate the graph. Script is provided in Appendix of this textbook. Similarly we could visualize all analyzed variables, but I suppose that for demonstration purposes it is not required.

Graph on fig. 2.4 is not the only possibility we have. If we need to visualize sensitivities of multiple or even all variables the *tornado graph* is often used. The name comes from typical "tornado cone" shape. Formally tornado graph is sub type of bar chart, where on y axis there are variables and on x axis behavior of analyzed variable. The variables in the graph are ordered by size of the difference between model results for upper and lower limit from largest differences to smallest. The ordering leads to typical shape.

We will be using pre-prepared values of Δ from tab. 2.2.

For our example the tornado graph is available of fig. 2.5.

We could also compute sensitivity for various probabilities to get complete picture for the topic. Since we would be basically repeating already demonstrated steps, I will leave the computation to you.

If you want to learn more, we can also recommend the book from Clemen & Reilly ??, which is considered authoritative for this topic.

Influence of treatment cost of small epidemic on the costs (dec. tree 2)

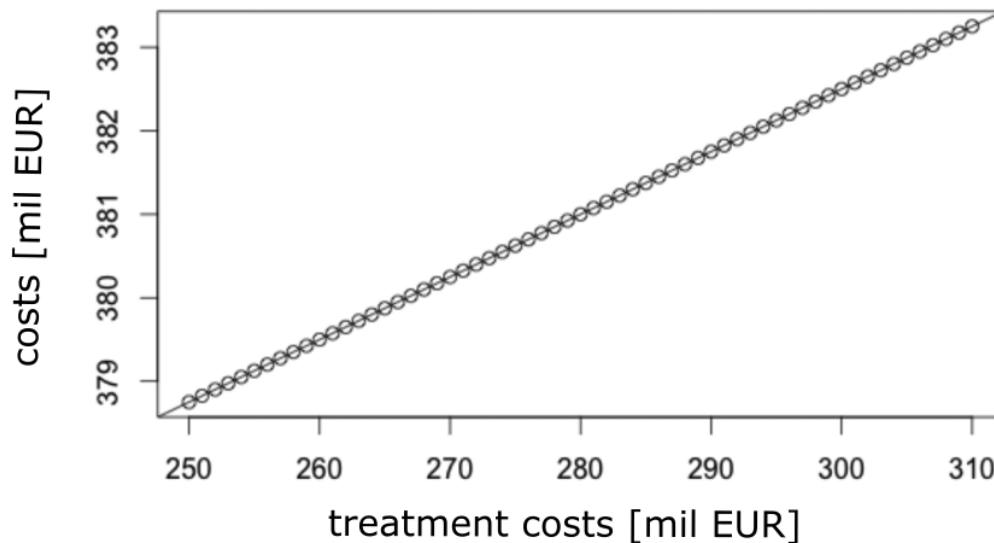


Figure 2.4: Stochastic tree construction

2.3 Using Decision Trees for Multiple Criteria Problem Solution

Technically we can use decision trees also for solving multiple criteria decision problems, but we do not recommend it. Using decision trees for finding solution has a benefit in relative easiness of use, and straight forward interpretation of the results. If we introduce multiple criteria, these benefits vanish.

That's why we will touch on this topic only briefly. Start of the process is same as in single criterion decision trees:

1. derive structure of decision tree
2. quantify probabilities of partial decisions
3. quantify utility of *all criteria* (one at time)
4. compute utility in leaf nodes for *all criteria* (one at time)
5. *establish weight system for the criteria to describe, which of them is more significant for the decision*
6. *normalize utility of the criteria originally expressed in its natural units (so we can compare them)*
7. *apply the weights in normalized utility of the criteria and aggregate them*
8. with this result, we can work as in single criterium situation (compute as standard decision tree)

Differences in comparison to single criterion computation are emphasized using italics. When normalizing, it is important to understand, that it will transform the scale in certain way, depending on chosen normalization method. It will have impact on the computation itself (what will be recommendation in the end).

Normalization methods are discussed in more detail in following chapters.

We also need to establish weight system as basis of aggregation of previously incomparable criteria values. Again there are many ways of how to establish weight and the choice of the process will again influence the results. Without weights we would be presuming, that all criteria are equal in their importance, which is not realistic. Some methods for establishing weights will be introduced in next chapter.

From perspective of easiness of use, considering last few paragraphs, we recommend not to use decision trees for multiple criteria decision problems. **MCA** methods are specialized on this type of problems and provide much more elegant solution, as we will show in next chapter.

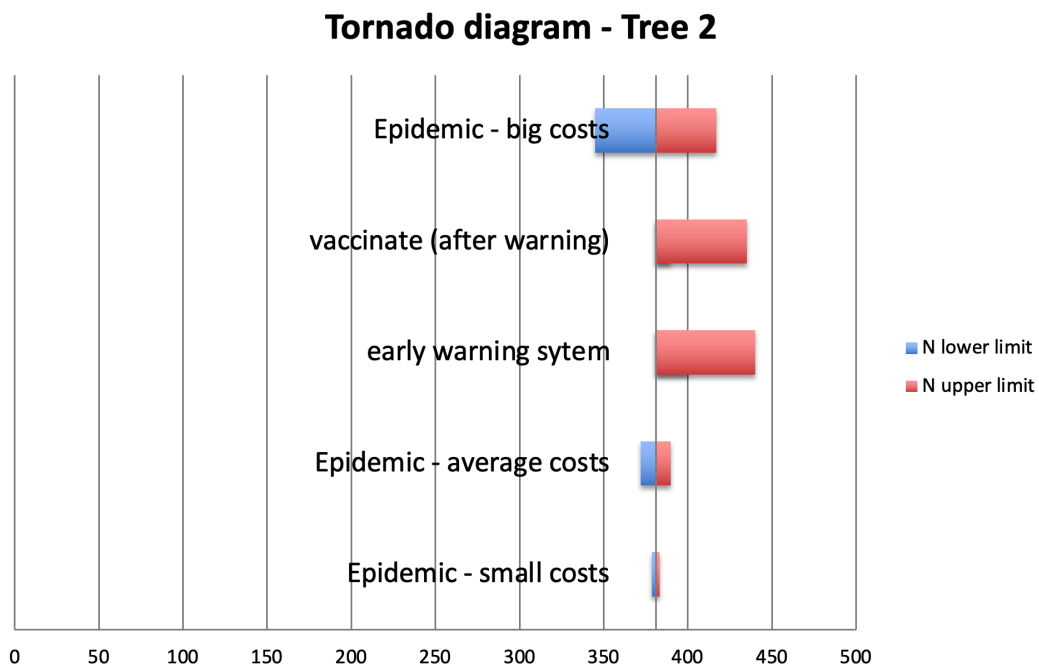


Figure 2.5: Tornado diagram for differences between the variables in decision tree 2



Computation and visualization of the variable's sensitivity

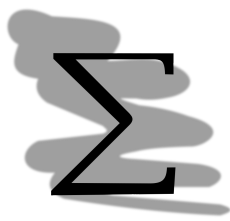
Choose one of "unsolved" variables (B - E) and perform computation of the sensibility using limits from tab. 2.1. Check, that the results correspond to those presented in tab. 2.2.



Software support for sensitivity analysis

Sensitivity analysis does not require usage of sophisticated software, you can perform basics in any spreadsheet, such as MS Excel, LibreOffice Calc or any other. That being said, using more advanced software packages such as R, MathLab, SciLab or other has its own challenges, but also benefits. While there is a higher learning curve associated with them, they also allow for much better control on the results and are generally more versatile. We recommend choosing one (or two) and investing time in learning at least the basics of them.

In the course we will be using R, so it is a logical choice :-). It has a large support community, thousands of libraries for solving various types of problems and is open source, so you do not have to pay for it.



Summary

It is possible to transform some decision problems into the shape of a tree. We call such a tree a *decision tree*. Solving practical problems we use more often a "stochastic" variant of a decision tree, allowing us to value the probabilities of various outcomes of a partial decision.

Although we can use decision trees to solve multiple criteria decision problems, we use them usually in single criterion decision situations, with the criterion often being some monetary value.

After conducting the analysis it is often useful to also evaluate how the proposed solution is sensitive to changes in input variables. If the solution is not overly sensitive to such changes, it increases our confidence in the robustness of the proposed solution. For such purposes we use *sensitivity analysis*. We can visualize its results using a *tornado graph*.



Questions

1. What are the rules for the edges going from stochastic nodes?
2. What are rules for choosing whether the node should be deterministic or stochastic?
3. What is tornado graph and what is its purpose?
4. What is the purpose of sensitivity analysis?
5. In textbook there is described graph of dependence of the result on changes of one variable (model input parameter). Do you think that it is possible to similarly visualize such dependence for multiple variables at same time? How would such graph look like?

Chapter 3

Multi – Criterion analysis



Study guide

In this chapter you will learn something about various methods of analysis of the decision situation, when we are optimizing decision based on multiple criteria. We will focus on decision analysis, which we will examine both theoretically and practically.

After reading this chapter you will understand how to

- Create decision analysis

know

- multi – criterion analysis
- Method of pair comparison



Time required to study chapter

For getting through this chapter you will need at least two hours.



Advanced multicriteria decision methods

In this chapter, only basic outline of multicriteria analysis is presented to introduce you to the decision making framework. For that we use weighted sum method and simplistic way of expressing values of criteria, weight, etc. With use of computers, there are many more advanced methods available. These are discussed in chapter *Advanced multicriteria decision methods*.

Decision tree we were discussing in previous chapter is without any doubt very useful tool, but it is capable to use single criterion only. What if we actually need to use other decision criteria too, perhaps even many more criteria? And what if these criteria are measured using different units. The various criteria themselves may also differ in usability for decision making – some of them may be more important than the others.

Under such circumstances usage of decision trees is out of the question. We have to use specialized type of analysis called **MCA**.

So what benefits do we get in general, when using **MCA**?

- we may compare criteria measured in different units,

- we may assign different importance to the criterions,
- we may evaluate both benefits and risks connected to variants both independently and together.
- MCA allows us to easier achieve objectivity in decision making – to diminish effects of our bias.

So how exactly MCA achieves such different goals?

3.1 Decision analysis

Main goal of decision analysis is to process available information into tabular form and if possible to objectively analyze the problem. There are usually two main goals we can have when using this type of methods:

- identify best/worst option - in general choose one of many different variants
- rank variants

If we consider these goals, they are actually similar, but not same. If the goal is to rank the variants, we need the method to be capable to accurately "understand" the criteria and decision maker's preferences regarding them. So it is more complex problem then the problem of finding just one optimum. Since all methods compute some kind metric usable for, you may find last remark surprising, as we can assign rank to variant based on this metric.

The question is, *is the metric accurate enough to be capable distinguish between the variants*. Good analogy is the the one of distance. With MCA methods we measure their distance to optimal variant. In this regard variant closes to the optimum is the best one. But what about the others?

If our goal is to rank all variants, that distance to optimum is not enough, we need to also consider distances between the variants and that complicates things, especially if some of the criteria are expressed by ordinal values, other criteria may be expressed as continuous variables and questions like how large difference between 2 variants must in such case be, to consider one better then the other.

MCA in its written form is a supporting document for manager's decision. Manager himself is usually not part of the analytic team. Analytic team, especially for complex problem is usually assembled ad hoc, so the team members have large portfolio of specializations to cover whole problem domain. Manager presents the problem to the team and then awaits the results. Manager does not need to be specialist in problem domain and the analytical team has to take this fact into account.

The result – the analysis, has to general enough for manager to understand it and agree with its conclusions (and possibly also constrains of the decision itself).

The analysis itself is made in several steps:

1. Definition of the problem
2. Information analysis
3. Evaluating benefits of variants
4. Evaluating risks of variants
5. Final evaluation – recommendation of variants.

Our first task is to *define the problem*. Correct definition of the problem allows us to specify the criteria, which we can use to evaluate the variants. Definition of the problem may be problematic itself as the original definition is performed by manager outside or analytic team. The analytical team has to make sure that its members understand the problem same way as the manager and each other. Also they have to make sure that there is an agreement on the goals of the analysis.

If such agreement does not exist or is questionable – the problem must be redefined to resolve all controversies and questions regarding the problem. To do that analytical team has to communicate problems with task definition with the manager. The redefinition itself is performed my the manager alone again. Failing to resolve such problem may result in the rejection of the results of the analysis as whole or the need to remake large parts of the analysis at later time – which requires time, human resources and presents additional costs of the decision, we would like to evade whenever possible.

By *analysing of information* we mean gathering all available information significant to problem solution and its evaluation. There are different types of information we can use – *analytical information* describing various interesting properties of the variants and decision situation and *thematic information*.

Analytical informations are the informations adding to our understanding of the problem. They may consist of the values, catalogues, need specification, description of present state, business offers.

All these informations support us on our way to reach the goal the manager defined for the analytic team.

Thematic informations are typologically same as analytical with single, but very important difference – they do not help us in reaching goal. So what are these informations good for? They map alternative routes to similar goals.

For example the manager sets the goal to choose the new car to buy for ... something. Analytical information would be on prices, car producers, type of the engine, size of the car, fuel consumption, safety ... Thematic informations be these on alternative way of financing – perhaps it would be financially interesting not to buy the car “cash”, but lease it or even rent it. What if getting new car was not necessary at all. We might think about overhauling old car.

So the thematic information basically describes the road not taken during decision process. Such information may be useful in case the original goal is unachievable. Even if the analysis is good and its result accepted by the manager, the optimal variant may not be realizable (purchasable, rentable, ...) due to many different circumstances.

The company may change its orientation due to different situation on the market, leaving original goal behind. The company may not have enough money to “make it happen” or the negotiations with bank, producer, contractor, ... may fail on either side. At that moment the company stands before the choice not to pursue the goal at all or choose different approach.

Thematic informations may become analytic information at that moment, forming base of the new decision situation, with slightly different goals.

3.2 Evaluating Utility of the Variant

To evaluate variant benefits we have to define evaluation criteria, we will use and quantify them in some form. Different criteria may be quantified using different units. Such is the nature of the world around us and the way we use to describe it. We know, that the battery in mobile phone will hold X minutes, that the screen resolution may be measured in pixels, the price in Czech Crowns (CZK) or Euros (EUR).

We call such description natural. Unfortunately it is not possible to directly compare the criterions in different units. So that would be our first task after quantifying the criteria.

First we have to create table (matrix) with criteria in its natural units (see table 3.1). To increase the objectivity of the evaluation process, we use neutral labels C1 – Cn instead of criterion names and V1 – Vn instead of variant names. Actually it is not possible to completely elude the connection to “real” names at this point, as we need to gather all necessary data needed for evaluation. But following this principle in other tables can help help us significantly.

Table 3.1: Quantification of the criteria of the variants

Criteria	Unit	V1	V2	...	Vn
C1	[min]	10	12	...	9
C2	-	Good	Average	...	Excellent
...
Cn	[kg]	0,5	0,4	...	0,55

Criteria in tab. 3.1 are expressed in their original measures, we may call these *natural units* of the criterion. When we search for information on the criterion for the variant, we usually find it in these units. Measures in the table express performance of the variant in the criterium. That is why we call this table and matrix derivable from it, the *performance matrix*.

Unfortunately the measures in natural units are usually directly incomparable. To be able to compare them directly we need to transform them into same comparable scale.

Basic method presented in this chapter does not provide any guidance on how to do such transformation. That means, that we are responsible to choosing such method by ourselves, and make sure, that such transformation is both mathematically sound and interpretation-wise makes sense. You can find some more popular normalization methods in chapter *Advanced multicriteria decision methods*.

In this chapter we will choose more intuitive approach and perform manual transformation of the values to measure of our "satisfaction" with the value, expressed as percentage. Benefit of this

approach is that we usually can do such transformation quickly, yet the transformation can have complex thought process behind that.

Unfortunately there are also disadvantages. We often choose such transformation arbitrary, without in-depth reasoning behind it. The result of such transformation is basically expert estimation of the value. As experts (including us) are human beings, we tend to be imprecise, or even wrong at times. We all are also biased. Considering all this, the transformation step can introduce easily huge errors into decision making, which the method has no means of how to deal with.

Another problem is, that the transformation process might be nontransparent for decision maker if the analytic did not describe his or hers thought process in the analysis text. It may be complicated for decision maker to decide, whether the analysis can be trusted.

Some advanced methods (see later chapters of the textbook) have incorporated means to deal with this problem, but not this one.

Transformation to percentages also solves another we did not mention yet. The criteria scales may have different. So some of the criteria we might need to maximize, while other need to be minimized.

Let's say that for our example C1 may represent benchmark for some task performed by computer. Nine minutes as best time will represent 100 %, while 12 minutes (variant 2) will be only 10 %.

Establishing of the percentage of satisfaction will enable us to compare the criteria, as the their unit will be same.

With introducing of percent values, we may transform table 3.1 into form of simple utility (see table 3.2).

Table 3.2: Table of simple utility

Criteria	V1	V2	...	Vn
C1	80	10	...	100
C2	75	50	...	100
...
Cn	60	100	...	50

Now we have criteria with exactly same unit (percentage), so theoretically comparing these values is possible. That is under presumption that the importance of all these criteria is exactly the same. In other words, if we believe that all criteria contribute to finding solution to the problem in exact same way we may just start to comparing them. Needless to say, this is not exactly real world scenario, as we usually regard some criteria more important than the others. We have to find a way to evaluate criteria importance.

Again there are many methods we can use to derive weights. For this chapter we will use *pair-wise binary comparison* method called *Fuller's triangle*, different methods to derive weights are available in later chapters of this textbook.

Pair-wise binary comparison means that we will derive weights based on direct comparison of each pair of the criteria, deciding which of these is important. We presume, that the criteria preferred more often are more important than these which have been preferred less often or have not been preferred at all. Number of preferences will be in this method used to derive importance ranking for the criteria, which in turn may be used as basis to establish weight system.

It is entirely possible, that the criteria preferences (weights) will be part of problem documentation – the team got from manager, but if not, we will have to find another way to explore significance of the criteria. Before we do that let me point out one very important thing regarding criteria evaluating. The criteria need to be independent. This requirement is actually very hard to achieve. So we usually take as sufficient *weak dependency*.

Existence of dependencies in criteria system for decision problem may disproportionately inflate influence of these criteria on overall results. So we can say, that inter-dependent criteria "support" each other.

Weak dependency means that, there is interdependency between some criteria, but the resulting influence change is very small and we presume, that is not significant from point of view of final recommendation.

If we identify dependencies in the criteria system, and are unable to simplify it, for example by removing some criteria (we identified them as redundant) or aggregate the information into single

criterion (they cover some concept inside problem domain, but only partially overlap each other), we must work with *criteria hierarchy*.

MCA method, we are using in this chapter is **Weighted Sum Method (WSM)**. This method does not support hierarchies out of the box. It can be modified to allow them by basically breaking normalized weighted performance matrix into separate submatrices, which are then computed separately and in the end aggregated up in the hierarchy.

General recommendation is - *if you identified hierarchy in your decision problem, and cannot simplify it to avoid it, use decision support method, which supports hierarchies out of the box*. Good go to methods are **Analytic Hierarchy Process (AHP)** or **Analytic Network Process (ANP)**. We will have separate chapter for the method later in the textbook.

So how exactly does such dependence look like? Lets consider following scenario: we are considering buying of the new monitor to our desktop computer or notebook. Among other criteria we would like to evaluate also ecological aspect of such device. When searching for possible criteria we found following candidate criteria:

- energy star certification,
- TCO certification
- easiness of disassembly
- used materials
- environmentally friendly

Now when we know, that we are trying to demonstrate dependence between the criteria, it is rather easy to identify the problem – the monitor is environmentally friendly if it has low power consumption (energy star), low radiation emissions (TCO), is easy to disassemble and its produces is using materials which encumber the environment less then other materials he could legally use.

We may express the situation graphically by forming the hierarchy of the criteria used in analysis, see figure 3.1.

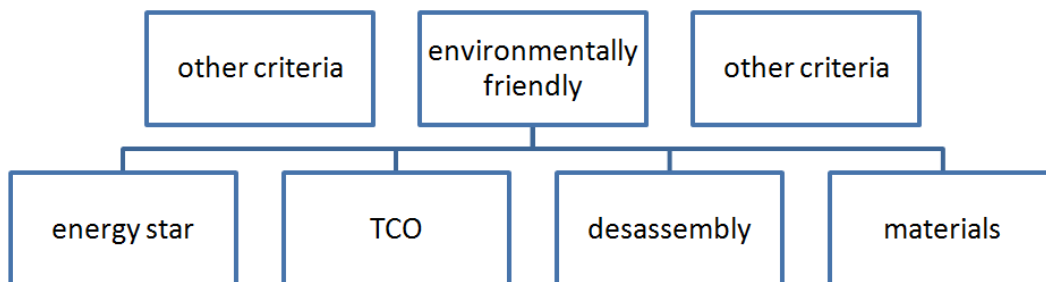


Figure 3.1: Forming up of the criteria hierarchies

To evade the problem, we have to resolve interdependencies, so we use either environmentally friendly or or any other substitute for these four subordinate criteria.

To demonstrate problem of dependencies consider following. We can imagine that the criteria will form index, based on which we will formulate our recommendation. (Which is by the way exactly what we are doing). To each criterium we assign certain percentage which will represent its contribution to overall solution of the problem. In context of MCA we call this percentage (contribution rate) *weight*. In we establish weights based on pair-wise comparison, we will basically create cluster of criteria with very similar weights. But in MCA we will end up adding all of these weighted values, which will lead to artificial inflation of these criteria's influence.



Demonstrate inflation of influence of interdependent criteria

You can try it easily yourself by using criteria from hierarchy on fig. 3.1 using procedure described in following paragraphs. Add additional criteria like screen size, resolution, you can add other criteria as well. Compute weights using Fuller's triangle. Then compute alternative weights for same situation, with only change of aggregating energy star, TCO, disassembly and materials into single criterium - environmentally friendly.

Compare computed weights.

Lets go back to the problem of establishing criteria importance. There are many different ways of how to approach the problem. Perhaps similar problem has been solved by somebody in the company or the report is available in some journal or over the Internet on the topic. So the weights already may exist, we only need to find them. If that is not the case, we need to derive them using available information.

For this purpose we often use services of domain expert(s), see for exaple *Delphi method* discussed in latter chapters of this textbook. One of easiest ways of how to estimate weights is to use *Fuller triangle*.

Let's compare criteria C1 – C5. To do that we need to construct the comparison triangle.

Table 3.3: Fuller triangle for pair-wise comparison

C1				
	C2			
C2		C1		
	C2		C4	
C3		C2		C1
	C4		C4	
C4		C3		
	C5			
C5				

We construct the triangle column after column. Firs column has all criteria in it and we write it down without any evaluation. In the second we compare criteria next to each other, so we compare C1 and C2 – we prefer C2, C2 and C3 – we prefer C2, C3 and C4 – we prefer C4, C4 and C5 – we prefer C5. In third column we compare first criterion with third, second with fourth, third with fifth ... In the fourth column we compare first with fourth, second with fifth and lastly in fifth column we compare first with last.

This approach is generally usable for any number of criteria. Evaluation will always result in shape of triangle.

There are some presumptions for the method. For example we presume, that when comparing the pair, there always is one criterion which is better. In reality that is not always the case. If we identify such situation during evaluation, we will need to simply use different method for establishing weight system.

Having constructed the comparison triangle, we have to interpret the results. Let's make a table for it. We will count number of occurrences in the triangle. We may count occurrences in the first column, but nothing happens if we leave it out, as it has exactly one occurrence of each criterion and that's why it will not influence the final order of criteria by importance, see table 3.4.

Table 3.4: Computation of the weights

Criteria	No. of occurrences	Order	Weights
C1	2	2	2
C2	3	1	3
C3	1	3	1
C4	3	1	3
C5	1	3	1

In table 3.4, there are weights computed as inverse order to criteria order by importance. Most preferred criterion will get highest weight. In some cases, this way weights estimation may be too rigid, so we can modify it to get larger (or smaller) gap between the weights. What we can't, or better said – we shouldn't, modify is the order itself – so that more important criterion can't have lower weights then those behind it.

In case some of the criteria share rank, in our case for example C2 and C4, we can use more granular evaluation. We can set weight of C2 to 3.1 and C4 to 2.9. The explanation could be that when comparing these two, we preferred C2 over C4.

As rule of thumb we need to make sure that out weight modification will not change the ranking of the criteria. Again in our case we must ensure, that the weight of C2 will remain greater then C1,

as C2 was preferred more times and thus is more important for the decision making.

Fuller's triangle method puts together par-wise comparison and weight computation. This perspective is available in tab. 3.5. When you compare results presented in tab. 3.5 and tab. 3.4 you will see, that the results are exactly same. So we can perceive it more as an issue of layout then anything else, and choose between these based on personal preference.

Table 3.5: Fuller triangle for pair-wise comparison with weight estimation

Criteria comparison				critierium	occurrences	rank	weight
1	1	1	1	K1	2	2	2
2	3	4	5				
	2	2	2	K2	4	1	3
	3	4	5				
		3	3	K3	1	3	1
		4	5				
			4	K4	2	1	2
			5				
				K5	1	3	1

Preferences in tab. 3.5 are written in bold.

Applying the weights on matrix of simple utility we get the matrix of weighted utility (see table 3.6).

Table 3.6: Table of weighted utility

Criteria	Weights	V1	V2	M
C1	2	80 x 2 = 160	20	200
C2	3	75 x 3 = 225	150	300
C3	1
C4	3
C5	1
\sum		385	170	500
U	%	77	34	100

Alternative solution for tab. 3.6 is to use normalized weight coefficients. Simplest way of how to normalize them is to take coefficients from tab. 3.5 and divide them by sum of all weights. Resulting weight coefficient would automatically be $w \in \langle 0; 1 \rangle$, with $\sum w = 1$. If the normalized criteria are expressed as % - $U \in \langle 0; 100 \rangle$, then $\sum wU \in \langle 0; 100 \rangle$ for all variants, which allows us to interpret it as % and we do not need to compute theoretical optimum M as used in next step.



Controversies around weight coefficients

In tabs. 3.4 and 3.5 the weights all $w \in \langle 1; 3 \rangle$, this is not the only way of how to compute them. The choice of scales is arbitrary provided that we preserve proportionality of the weights. In our case for example that the criterium with original weight of 3 will retain weight 3 times that of weight of criterion with original weight 1.

For practical reasons it is often recommended to use weights $w \in \langle 0; 1 \rangle$, which corresponds to possibility to interpret it as a percentage. Which is usually perceived as easier to interpret.

When choosing weights we also need to consider the way we are going to use them.

We compute the weighted utility as product of weight and simple benefit for criterion of given variant. We also construct extra maximum variant M , which should represent an ideal variant. Ideal variant fulfils the criterion on 100%, weighted gains then are simply $w \cdot 100$ for this ideal variant.

Ideal variant will help us find, how close are actual variants to this ideal. Computing this, we will get first results of the analysis.

Alternatively we may use maximal value of the criterion for M variant instead of presuming, that the ideal variant will fulfil our expectations to 100%. The reasoning behind this is to try to compare the existing variant to the M variant having best available properties instead of ideal properties.

Choosing best available instead of ideal will lead to higher scores for evaluated variants in comparison to situation, when we perform the comparison to the ideal variant.

For its properties we sometimes call results presented in this table as *preference matrix*. Reasoning behind it is that it holds information on the preferences of the decision maker in form of weights, which were applied on normalized performance of variants in these criteria.

3.3 Risk evaluation

For evaluation of risk we will basically perform same steps as for utility evaluation. We again start with performance of the variants in identified risks. We usually specify risk as a function of probability of occurrence and expected impact such occurrence will have. While it may seem straightforward, there are some caveats, which need to be discussed.

First we need to consider time horizon for the risk - it needs to be same. Consider risk R1 with yearly probability of occurrence of 10 % and risk R2 with 10 % probability of occurrence in 10 years. We can expect R1 to occur ten times more often than R2.

Performance matrix for the risk is presented in tab. 3.7.

Table 3.7: Matrix of simple risks (performance matrix for risk)

Risk	V1	V2	Vn
R1	10%	8%	...
R2	15%	25%	...
Rn
...

Similarly to criteria utility evaluation, the risks may be weighted. If we perceive risk performance matrix in tab. 3.7 as a measure of propability specified for set time period, we may perceive the weight as a measure of impact, forming together simple model of risk. See tab. 3.8 for example. The weights for risks in this case were established in similar way as the weights for the benefits.

Table 3.8: Matrix of weighted risks

Risk	Weights	V1		V2		Vmax	
		P	GT	P	GT	P	GT
R1	1	0,1	0,1	0,08	0,08	1	1
R2	2	0,15	0,3	0,25	0,5	1	2
\sum			0,4		0,58		3
GT	%		13,3		19,3		100

As opposed to utility evaluation, where we could have criteria to be maximized and other criteria which must be minimized, when evaluating risks, we always minimize them.

Similarly to utility preference matrix, we introduce new variant Vmax, with probability of occurrence 100%. We use this variant as etalon to compare our proposed variants.

Again Vmax may be also constructed based on worst available scenario, using highest value for given risk.

3.4 Results and their interpretation

By constructing utility and risk performance matrices we gathered all data required to formulate recommendation.

When choosing of what to recommend we usually follow one of strategies:

1. Maximal
2. Minimal
3. Optimal

Maximal strategy prefers maximization of the utility regardless of possible risks connected to the variant. Our recommendation in this strategy is derived from results computed in table of weighted benefits. We are searching for the variant with highest score.

Minimal strategy represents the situation, when we are completely averse to the risk – so we are minimizing risks regardless of possible utility we could get from realizing the variants. Our recommendation is based on results of the table of weighted risks.

Lastly we can compare risks and benefits optimizing our decision, see table 3.10.

Table 3.9: Gains vs. Risks

Order	1st	2nd
Benefits	V1	V2
Risks	V1	V2

Table 3.10: Final Effect

	V1	V2
Gain (U)	77	34
Risk (R)	13,3	19,3
Final Effect (U-R)	63,7	14,7
Final Effect (U/R)	5,79	1,76

As we can see both minimal and maximal strategy did recommend same variant (V1), optimal strategy may only result into recommendation of this variant too, otherwise we would have to compute final effect of the variant (see tab. 3.10).

As final effect is computed in two different way. In our case both approaches lead to recommending of V1, but in cases when maximal and minimal strategies differ in recommendation it may be different. In such cases we recommend one variant as optimal and the other one as its alternative.

Also it is good practice to write down executive summary for management, which consist of recapitulation of whole analysis and its recommendations.



Mistakes?

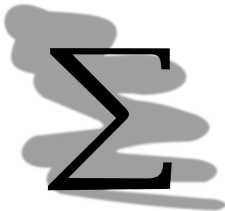
Well if you look back at the chapter, the process of the **MCA** is pretty strain forward. The analysis structure is firm and we use only basic mathematical operations, so we can not make a mistake, can we?

Unfortunately my experiences say that you can and it is not hard at all. It is rather easy to make mistakes both logical and numerical. Typical logical mistake would be to presume that we are maximizing all criteria, another one would be to maximize the risks.

These kind of mistakes come from lack of focus and automatic writing without thinking. Best defense against it is to thing logically proofread parts of the analysis to make sure to constrains hold.

Numeric problems come from bad translation of the criterion's natural units into matrix of simple benefits or simply making a mistake during the tables computation. Best help is to use spreadsheet such as MS Excel, LibreOffice or any other to support you with computations. Using such programs will automatically recompute all tables as required, leaving to you only the task to copy them to the report.

Both types of mistakes come from heavy underestimating of the complexity of the analysis – so do not underestimate complexity of **MCA**.



Summary

In **MCA** we process in following way:

1. collect analytic and thematic information
2. identify possible criteria and risks, which are going to be used in the analysis to measure performance of the variants
3. construct performance matrix for utility of the variants in criteria's natural units
4. normalize it to get comparable measures
5. derive wights for criteria and apply them to get preference matrix for utility
6. specify performance of risks using risk model of choice.
7. compute preference matrix for the risks
8. compute final effect
9. interpret the results and formulate recommendations



Quiz:

1. Define problem with the hierarchies of criteria and how can we resolve it.
2. How can we estimate weights of the benefits?
3. How is working pairwise comparison?

Chapter 4

MCA Case Study



Case study

Consider simple task, we will use to demonstrate construction of the simple decision analysis. Our problem is to choose optimal *e-book reader*.

The reader should will be used by heavy reader requiring maximal comfort when reading. The reader should be using e-ink display, which provides maximal contrast when reading even on direst sun. Physical buttons on the device are preferred for page turning to minimize grease deposit on the display.



Note to organization of the solution

As this task is relatively simple and for demonstration purposes criteria system has been simplified. Also note that all technologies age, chances are, that the reader we are using in the example have been replaced by some newer and better model, so they might seem archaic at this point. For purposes of demonstration that is irrelevant, as basic principles demonstrated here still hold.

Text of the analysis is accompanied by notes, which should provide context for the steps and commentary for you to better understand it. These notes are in this chapter emphasized using italics. These notes are not part of the standard analysis.

4.1 Information

At present time there are about dozen various devices allowing the reader to consume the books. Even for e-ink devices there are lot of options which deffer significantly by the size of the reader, but also other properties. Most reader use following size of the display:

1. small - diagonal 5 inches
2. standard - diagonal 6 inches
3. large - diagonal 9.7 inches

For purposes of the analysis size of e-book readers will be limited to standard size only as it provides best ratio between the usability and reading comfort. 8 inches diagonal, would be ideal as it provides same size of electronic paper as A5 paper format. Large devices with diagonal of 9.7 are significantly heavier and also use older display technologies and do not allow to use front light to improve further contrast of the text on the display.

Differentiation is also on display technologies used. Especially older devices use Vizplex technology which is inferior to newer Peal and Carta displays. Carta displays provide best properties but at the time their availability is limited to only Amazon Kindle devices.

As Vizplex displays are utilized by outdated devices, the analysis will focus on devices with more modern Perl or Carta display.

From usability point of view, ergonomics of user interface is very important. More complex operations are better handled by touch displays, but for reading itself, using physical buttons for page turning is better, to limit display smudges. The reading is also better if the display is illuminated at least somewhat using LED, which subjectively increases text contrast and allows for reading in dim lit environment.

For reading is also important supported formats of the e-books. Format support will limit usability of e-book stores, as these usually only support one format of choice. If the e-book reader supports more formats, it provides the reader with freedom of the choice of what store to use. On the other hand many high quality conversion utilities exist, so the format support does not need to be considered such a large problem. Introduction of another utility to the e-book workflow presents some inconvenience and additional work, which could be otherwise avoided.

Format support criterion will be used as supporting (low weight).

If the device has audio-jack, it can also be used for consuming audio books or music. The quality of sound is questionable for this type of devices. Slot for SD cards, can have its uses as it allows to physically transfer whole libraries between the devices, but for normal use, the available memory is good enough for most uses.

Audio-jack and SD card support will also serve as supporting criteria. On the contrary to that, ability to connect to the Internet via Wi-Fi has significant benefits, as it allows the device to update over the air, synchronize itself with computer without the need to physically connect the device using USB cable. It also allows tighter integration with e-book e-shop to improve user experience of finding and purchasing of the books. Consumption of on-line text content is also enabled by Wi-Fi, but refresh speeds of the display result in not so good experience. So this functionality is considered as secondary at best.

Based on the information we gathered, we choose following devices:

- V1 - Kobo Glo
- V2 - Sony Reader PRS-T3
- V3 - Kindle Paperwhite 2
- V4 - Pocketbook Touch Lux
- V5 - Bookeen Cybook Odyssey HD FrontLight

Note commentaries in previous paragraphs. Specifications of parameters used in decision making is largely dependent on purpose of the analysis. Task formulation itself may be not well formulated, allowing multiple explanations. In such case analyst has only two options. (S-)He can interpret it best as (s-)he can, or inquire on mean meaning, leading to adjusting of problem formulation. The decision making process does not need to be one way process.

Also note, that the analyst commentaries to criteria and variants serves also a purpose. It informs the decision maker on how analyst sees the problem. This is important signal the decision maker can use to quickly identify possible problems with analysis. Proper step for decision maker would be to return the analysis to be amended.

In case the purchase of e-book reader will not be possible to complete, there is possibility to purchase another type of device - tablet. Tablets are more universal than e-book readers. They allow full fledged media consumption and use of large amount of applications, which are not available on e-book readers. This allow the tablet to be used for much broader portfolio of task and workflows. But there are also some negatives connected to the tablets. They are often much pricier, heavier, and require recharging battery much more often than e-book readers.

Last paragraph was written in different way than paragraphs before it. That because it presents different type of information. While first paragraphs described rationale of task and criteria and variants, in other words analytic information, last paragraph explored alternative mindset. Purchase of tablet goes against the task as it clearly say that the device should be e-ink based. This type of information is thematic.

Now for the reason to change our mind. For example originally we planned to purchase both e-book reader and tablet, but we found that for economical reasons we no longer can purchase both. Suddenly the decision problem changes. Thematic information provides starting point for different decision analysis.

4.2 Utility

Table 4.1: Performance matrix (criteria in natural units)

Variants	units	V1	V2	V3	V4	V5
C1 Display	-	Pearl	Pearl	Carta	Pearl	Pearl
C2 weight	g	185	200	206	198	180
C3 touch display	A/N	A	A	A	A	A
C4 hardware buttons	A/N	N	A	N	A	A
C5 front light	A/N	A	N	A	A	A
C6 battery	no. of page turns	2 100	1 700	1 700	1 700	1 900
C7 Wi-Fi	A/N	A	A	A	A	A
C8 price	Kč	3 400	3 800	3 300	3 400	4 200

Legend

Display - Newer Carta provides higher contrast in comparison with Pearl displays. We will evaluate Carta displays as 100 % and Pearl displays as 80 %.

Weight - is closely connected to convenience during long reading periods. The weight difference between lowest and highest values is only 26 g, which is very low value, almost insignificant. That's why we evaluate weight 206 g as 85 %.

Tough display - implemented in all devices we evaluates. Since the devices do not differentiate in this criterium we deem to be irrelevant. We will exclude this criterion from rest of analysis as it does not contribute to decision.

Hardware buttons - for page turns. In the buttons are not present on the device we evaluate it 50 % as it means significant discomfort in using the reader.

Front light - the display is illuminated by LEDs, which significantly increases the reading contrast, allowing reading in dim lit areas. The absence of front light is a major defect in usability, so such device is rated only 20 %.

Number of page turns (with Wi-Fi off): each e-book reader producer has its own methodology for computation of battery capacity. For example for Kobo Glo its manufacturer expect 70 hours, presuming 1 page turn per minute, which lead to estimated value 2100 pages. Other manufacturers do have their own methodologies on how to measure the battery capacity. E-ink displays require energy only to redraw the page. Page turns is ideal criterium to use to compare devices across different manufacturers and their own measurements.

Wi-fi - again is implemented on all devices and again we exclude this from further analysis as it does not contribute to the decision.

Price - is important factor for the decision. Highest value 4 200 is considered to be only 50 %.

Table 4.2: Normalized performance matrix

Variants	V1	V2	V3	V4	V5
C1 Display	80	80	100	80	80
C2 Weight	97	88	85	90	100
C4 hardware buttons	50	100	50	100	100
C5 front light	100	20	100	100	100
C6 battery	100	70	70	70	85
C8 price	94	70	100	94	50

Please note, that the utility doesn't need to be in interval $\langle 0;100 \rangle$. Acceptance values are estimated solely from subjective perception of it, finding the differences between the devices and evaluating performance hit these differences make.

The utility curve may have but does not need to have linear character, any other function may be usable too. For example the price 4 200,- Kč is evaluated as 50 %, but somewhat higher price of 5 000, may be perceived as 10 % or even lower (if evaluated as prohibitively expensive). The analyst chooses transformation function arbitrary depending on his/hers knowledge and experience. It is recommended for the analyst to describe his thought process for these transformation as the transformation will sig-

nificantly influence the results and it maybe something which decision maker might be "hard to digest".

Weight coefficients are estimated based on preference measurement using binary pair-wise method of Fuller's triangle. Pair-wise comparison is available in tab. 4.3.

Table 4.3: Binary pair-wise comparison - Fuller's triangle

C1 display				
	C1			
C2 weight		C1		
	C4		C5	
C4 hardware buttons		C5	C1	
	C5		C2	C1
C5 front light		C4	C2	
	C5		C4	
C6 battery		C5		
	C6			
C8 price				

Table 4.4: Weight coefficient estimation

Criterion	no. of pref.	rank	weight
C1 display	4	1	5
C2 weight	2	3	2
C4 hardware buttons	3	2	4
C5 front light	4	1	5
C6 battery	1	4	3
C8 price	0	5	1

Table 4.5: Utility preference matrix

Criteria	weight	V1	V2	V3	V4	V5	M
C1 display	5	400	400	500	400	400	500
C2 wright	2	194	176	170	180	200	200
C4 hardware buttons	4	200	400	200	400	400	400
C5 front light	5	500	100	500	500	500	500
C6 battery	3	300	210	210	210	255	300
C8 price	1	94	70	100	94	50	100
\sum		1 688	1 356	1 680	1784	1 805	2 000
utility (U)	%	84,4	67,8	84	89,2	90,25	100

4.3 Risk

For purchasing of e-book readers following risks have been identified:

- R1 - premature end of support for the reader
- R2 - rejecting reclamation for the display
- R3 - closing of the e-shop integrated in the reader

4.4 Results

Considering values computed in this chapter variants V3 and V5 look as good candidates for the purchase. V3 is Amazon Kindle Paperwhite 2 and V5 is Bookeen Cybook Odyssey HD Frontlight.

Table 4.6: Performance matrix for risks

Risk	V1	V2	V3	V4	V5
R1 end of support	40	30	5	10	10
R2 reclamation rejection	30	30	5	40	10
R3 e-shop's end	40	40	0	30	40

Table 4.7: Pair-wise comparison for risks

R1 end of support			
	R2		
R2 reclamation rejection		R3	
		R2	
R3 e-shop's end			

Table 4.8: Estimation weight coefficients for risks

Risk	no. of pref.	rank	weight
R1 end of support	0	3	1
R2 reclamation rejection	2	1	3
R3 e-shop's end	1	2	2

Table 4.9: Preference matrix for risks

Risk	weight	V1		V2		V3		V4		V5		Vmax	
		p	R	p	R	p	R	p	R	p	R	p	R
R1 end of support	1	0,4	0,4	0,3	0,3	0,05	0,05	0,1	0,1	0,1	0,1	1	1
R2 reclamation rejection	3	0,3	0,9	0,3	0,9	0,05	0,05	0,4	1,2	0,1	0,3	1	3
R3 e-shop's end	2	0,4	0,8	0,4	0,8	0	0	0,3	0,6	0,4	1,2	1	2
\sum			2,1		2		0,1		1,9		1,6		6
R			35		33,3		1,7		31,7		26,7		100

Table 4.10: Results - final effect

	V1	V2	V3	V4	V5
Utility (U)	84,4	67,8	84	89,2	90,25
Risk (R)	35	33,3	1,7	31,7	26,7
Final Effect (U-R)	49,4	34,5	82,3	57,5	63,3
Final Effect (U/R)	2,4	2	49,4	2,8	3,4

Table 4.11: Ranking

	1	2	3	4	5
per utility (maximal strategy)	V5	V4	V1	V3	V2
per risk (minimal strategy)	V3	V5	V4	V2	V1
optimal strategy (U-R)	V3	V5	V4	V1	V2
optimal strategy (U/R)	V3	V5	V4	V1	V2

Kindle can be considered as safe bet due to the size of its manufacturer Amazon and very good service record by both Amazon and e-book publishers. Cybook scores with innovative technical properties. Investment in it can be seen as slightly more risky as Bookeen is small French company in comparison with Amazon, leading to higher probabilities of problems associated with device service or reclamation.

**Semestral project**

Perform MCA on problem of your choice as your semester project. There must be minimally 5 variants characterized by 5 criteria. Consider also minimally 3 risks.

Upload result of your work on <http://lms.vsb.cz> course Modeling of Decision Processes for evaluation.

Chapter 5

Advanced Multiple Criteria Analysis Methods



Study guide

In this chapter we will broaden our horizon with more advanced methods for **MCA**.

After reading this chapter you will know how to

1. apply various normalization methods
2. use alternative methods for weight coefficient estimation
3. compute formally defined decision problems using WSM (as in previous two chapters) and ELECTRE family methods with support of R.



Time for study

Time required for going through this chapter can be hugely different based on your expectations of it. It can take 20 minutes if you want to just go through it and get "strategic" view of the topic. On the other hand, if you are interested in topic, this rabbit hole goes really deep.

You can for example spend a lot of time on basics of R. Benefit would be to get competences to more effective data manipulation, support of statistical computations and advanced graphing. Variety of libraries exist for all sort of problems, from statistics to AI. In fact its currently impossible to outgrow R, as it is usable on everything from personal computers to supercomputers solving cutting edge research problems.

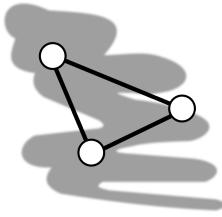
From perspective of decision support, you can also go deep. The library, we are going to use has implemented support for about a dozen of methods not described in this textbook. Armed with basics theoretical knowledge and user manual you can try them and learn a lot by doing so.

5.1 Normalization

By *normalization* we understand transfer of numeric scale in original, in our case natural, units to unified (normalized) scale often in interval $< 0; 1 >$. Many approaches to normalization exist. On following pages we provide some more popular approaches for study.

min-max

Simplest method to use is so called *min-max*. When using this method the normalized value is



Ranking, risk management, ...

Knowing the basics of decision making a good question can also be, where else can we use these principles? Turns out that what we learned is very broadly applicable. In risk management we use risk analysis methods to analyze identified risk for region, organization, unit, technology leading to ranking of the risks. Principles allowing us to compare variants of the decision allow us to compare risks as well.

Methods like WSM serve as base for various screening methods for example in risk mapping or critical infrastructure resilience evaluation. That allows us to better understand true strengths and weaknesses of those methods and better interpret its results.

computed from ratio of minimum on the scale to difference between maximum and minimum. The equation is available in (5.1) for growing scales (for criteria we maximize) and in (5.2) for descending scales (for criteria we minimize). As apparent from the equation, the normalized value will always be $z \in \langle 0; 1 \rangle$.

If we multiply z by 100, we can easily translate the scale to percentages.

$$z = \frac{1 - \min(x)}{\max(x) - \min(x)} \quad (5.1)$$

$$z = 1 - \frac{1 - \min(x)}{\max(x) - \min(x)} \quad (5.2)$$

where z is normalized value and x is a value on original scale.

Comparison to mean value can be computed by dividing original scale value by mean of all values on the scale in dataset. Again as the $z \in \langle 0; 1 \rangle$ we can multiply result z by 100 to transform the result into percent. This approach does not have specific form for ascending and descending scales. Equation is available in (5.3).

$$z = \frac{x}{\bar{x}} \quad (5.3)$$

Logarithmic normalization can be used for nonlinear relations on original scale. Normalized value is result of ration of the natural logarithm of original value divided by natural logarithm of the product of all values in dataset for scale. The equation is available for ascending scales in (5.4) and descending scales in (5.5).

$$z = \frac{\ln(x)}{\ln(\prod_{i=1}^m x_i)} \quad (5.4)$$

$$z = \frac{1 - \frac{\ln(x)}{\ln(\prod_{i=1}^m x_i)}}{m - 1} \quad (5.5)$$

Where m is number of values in dataset for the scale (criterion).

Vector normalization

We can also approach the normalization by geometric means. In this case we divide original value x by square root of the sums of the squares of the individual components, see (5.6) for ascending scales and (5.7) for descending scales.

$$z = \frac{x}{\sqrt{\sum_{i=1}^m x_i^2}} \quad (5.6)$$

$$z = \frac{\frac{1}{x}}{\sqrt{\sum_{i=1}^m \frac{1}{x_i^2}}} \quad (5.7)$$

Linear normalization

Another type of normalization is so called *linear normalization*. This form of normalization operates such, that the value on original scale is divided by sum of all values on that scale. Equation for ascending (5.8) and descending (5.9) scales are available.

$$z = \frac{x}{\sum x} \quad (5.8)$$

$$z = \frac{\frac{1}{x}}{\sum \frac{1}{x}} \quad (5.9)$$

We already mentioned this method when talking about normalization of weight coefficients in previous chapters.

All above mentioned methods allow us to re-scale values to new scale, but all of them will lead to different values. Let's demonstrate this on small dataset of {1, 2, 3}. Results of various transformations is available in tab. 5.1. Please note that since these transformations lead to different values, choice of normalization method will influence the results of MCA, which use these re-scaled values to formulate recommendations.

Table 5.1: Normalization of values 1, 2, 3 using various normalization methods

method	1	2	3
min-max	0	0,5	1
z-score	-1	0	1
to average	50	100	150
logarithmic	0	0,1155	0,1831
vector	0,2673	0,5345	0,8018
linear	0,1667	0,3333	0,5

Since for explanation purposes we are using R language, we provide code listing used to compute these values. Normalization methods are available for easiness of use in package MCDASupport, which is available either in course on [Learning Management System \(LMS\)](#) or my homepage.

Výpis 5.1: Normalization of values in R using package MCDASupport

```

1  install.packages("MCDASupport_0.21.tar.gz", repos=NULL, type="source")
2  library("MCDASupport")
3  tonorm <- c(1,2,3)
4  norm_minmax(tonorm)
5  norm_zscore(tonorm)
6  norm_toaverage(tonorm)
7  norm_logarithm(tonorm)
8  norm_vector(tonorm)
9  norm_linearagreg(tonorm)

```

Listing presumes, that the MCDASupport package is downloaded in same folder as the script the installation is called from and that it is also set as working folder in R environment. Also please note, that the name of package may be slightly different, for example in version number.

We are installing local package, that not so common. The package itself is fine, it has been created for purposes of this course. It is open source licensed, but we decided not to upload it on CRAN as to make it much more available. Reason for this decision is that packages available for whole community require regular support, which is commitment we didn't want to make. That decision has one undesirable side effect. CRAN packages are able to download and install other packages automatically to resolve missing dependencies. When installing local packages, these dependencies must be resolved manually.

We provide script to install such packages, but note, that newer versions might have different dependencies. If that is the case, R will complain, so you should be able to quickly identify, what is missing and install it.

Výpis 5.2: Installing packages MCDASupport 0.21 package depends on

```
1 install.packages("mathjax")
2 install.packages("graphics")
3 install.packages("igraph")
4 install.packages("diagram")
5 install.packages("stats")
6 install.packages("dplyr")
```

Documentation of the available functions of the package is available in PDF format [72]. Package itself is available in accompanying course in LMS or from my homepage [71]¹.

5.2 Weight Coefficient Estimation Methods

There are many methods we can use to estimate weight coefficient. All of them are based on some kind of either preference mapping or using some underlying presumption on how these preferences look like.

Apart from pairwise comparison (i.e. Fuller's triangle) we can use also philosophically very different methods, allowing us to be much more precise and also describe more complex relations between the criteria. In that regard the weight are measure of these relations.

It may be feasible to use externally stated weight for certain problems. If information exist (in sufficient precision and quality) to solve our problem, we can estimated the weight *directly*. We perceive as direct also cases, when the weights are stated in problem formulation (directly) by decision maker.

Alternative methods use scoring to evaluate the criteria based on perceived importance. We can think about two basic scenarios for weight estimation. In first we score the criterium using predefined scale. The evaluation will be performed on every single criterium independently on evaluation of other criteria. In this case the sum of scores is not set in advance. We call this method as *direct scoring*.

Second variant presumes, that we have set amount of point we can allocate to the criteria to estimate weight coefficients. Logically sum of the scores across the criteria will be constant in this case. In this case we can talk about the score budget to be allocated to criteria - hence the name *budget allocation*. This approach gives the analyst high flexibility as (s-)he can allocate whichever part of the budget to the criteria as necessary and bonify or penalize some criteria. In extreme case the analyst can allocate the budget to single criterion, transforming the multiple criteria decision problem to mono-criteria one as all other criteria would be effectively excluded from decision making.

From assigned scores we can easily derive weight coefficient by using linear normalization (5.8). We divide assigned number of points by sum of all points (scores) to compute the weight.

5.3 Weighted Sum and Product Methods

We discussed to certain degree *weighted sum method* (WSM). We used it for utility and risk preference matrices (see tabs. 4.5 and 4.9). Formally we can formulate the computation in following way (5.10).

$$U = \sum_{i=1}^n w_i \cdot u_i \quad (5.10)$$

¹<https://fbiweb.vsb.cz/~sen76/index.php?id=publikace>

Where U is utility of the alternative, u_i is the performance of criterium i , n is number of criteria used and finally w_i is the weight attributed to criterium c .

In equation (5.10) the utilities are used, but we can adapt it easily to compute risk $R = \sum_{i=1}^m w_i \cdot r_i$. We transformed U and u to overall risk R and risk r , with m being number of risks being evaluated. Significant difference is in interpretation of the results as for utility we maximize the result, while for risk we minimize it.

The WSM method is very easy method both from point of view of usage and results interpretation. Unfortunately there are also some limitations of the method. For example we need to normalized inputs for the method, before we use it. That brings burden of responsibility of these scale transformations to analyst. As we noted in previous chapters choice of normalization method will impact the results.

Another significant limitation, perhaps a most important one, is a presumption that all criteria do have from point of view of the contribution to the optimal can be substituted for each other. That means that bad performance of the alternative in one criterium can be compensated by better performance of the alternative in another criterium. If the presumption does not hold for decision problem, we can't use WSM method as the results would be misleading.

In such case **Weighted Product Method (WPM)** may be better option. Computation is very similar to WSM (5.10) but instead of sum we compute product of the weighted utility (5.11).

$$U = \prod_{i=1}^n w_i \cdot u_i \quad (5.11)$$

The result in this case will be product of weighted performances of alternative across the criteria. Practically, if the performance of the alternative in criterium is unacceptable \rightarrow equal to 0, than the result itself will also be 0 as anything multiplied by zero equals to zero. That property of the method can be useful for evaluation of utility, but this property makes it unsuitable for risk evaluation. Since we are minimizing risk and work with positive values only, minimal value will be always 0. So be careful when using the method that your results actually make sense.

It is also clear, that the result will no longer be interpretable as a percentage. So the interpretation will be much less intuitive.

Requirement on normalizing inputs of the method before usage are same as for WSM. And again this burden is entirely on analyst.

Main strength of the method is in that it gives analyst a tool to effectively transform performance measure in natural units to utility by specifying threshold over/under which the alternative performance will be evaluated as unacceptable regardless of alternative's performance in all other criteria. That may be useful especially when computing using computer.

In this case we can leave all the alternatives inside the processed matrices. The method will clearly identify "excluded" alternatives by making their utility $U = 0$. That might not seem like much, but without it, the decision to exclude alternative from decision analysis must be made separately and must be well described. Another benefit is, that it is much easier for analyst to experiment with cut-off levels and thus better understand properties of decision.

In listing below there is demonstrated usage of MCDASupport package in R to solve our use case using **WSM**. This method is available using `mca_wsm()` function.

Výpis 5.3: E-book reader purchase using WSM in R

```

1 library("MCDASupport") #attach package
2 #define decision problem
3 #choosing best e-book reader - evaluating utility only
4 KoboGlo <- c(80, 97, 50, 100, 100, 94)
5 SonyPRST3 <- c(80, 88, 100, 20, 70, 70)
6 KindlePaperwhite2 <- c(100, 85, 50, 100, 70, 100)
7 PBTouchLux <- c(80, 90, 100, 100, 70, 94)
8 BookeenCybookOdyssey <- c(80, 100, 100, 100, 85, 50)
9 criteria <- c("Display", "weight", "HWButtons", "FrontLight", "battery", "price")
10 PM <- as.data.frame(
11   rbind(KoboGlo,
12         SonyPRST3,
13         KindlePaperwhite2,
14         PBTouchLux,
15         BookeenCybookOdyssey))

```

```

16 colnames(PM) <- criteria
17 w <- c(5, 3, 4, 5, 2, 1)
18 preferences <- mcda_wsm(PM, w, 'max') #use WSM method
19 preferences #print results
20 plot.scoreM(preferences$weightedPM) #visualize score of alternatives
21 dom <- mcda_get_dominated(PM) #adjancancy matrix
22 plot.prefM(dom) #network graph of alternative domination
23 mcda_del_dominated(PM) #simplify the problem by removing dominated alternatives
24 t(preferences$weightedPM)

```

The listing is self describing, so only short remarks on the process. First we define alternatives as vectors of its performance in the criteria, using vector creation function `c()`. Every vector member represents performance in single criterium. Order of criteria is clear from vector defining it on row 9. Order of criteria must be same for all alternative vectors. Next we construct performance matrix using alternative vectors as rows using `rbind()` function. The matrix will have alternatives in rows and criteria performances as columns.

Alternatively we could construct the matrix per columns using `cbind()` function. That would require the vectors to be representative of criteria performance in alternatives instead of alternative performance in criteria.

In both cases the resulting matrix will lack row and cols names. We can remedy the problem by assigning them using `colnames()` and `rownames()` functions. Please note, that we did not set row names in the listing.

The utility computation is performed by `mcda_wsm()` function. Which takes performance matrix PM as input, together with vector of weights `w` and information of whether the criteria are ascending (maximizing) or descending (minimizing). If all criteria have same direction of scale, we can use 'max' (like in our case) or 'min'. If we work with mixed scales we need to provide vector with information on the criteria direction. Logically such vector must have number of elements equal to number of criteria. That is same for vector of weights as all criteria need to have some weight.

Method also provides graphical outputs. First of these is a network diagram showing domination of alternatives over other, see fig. 5.1. Second one is a bar chart, which shows contribution of the criteria to overall score of the alternative see fig. 5.2.

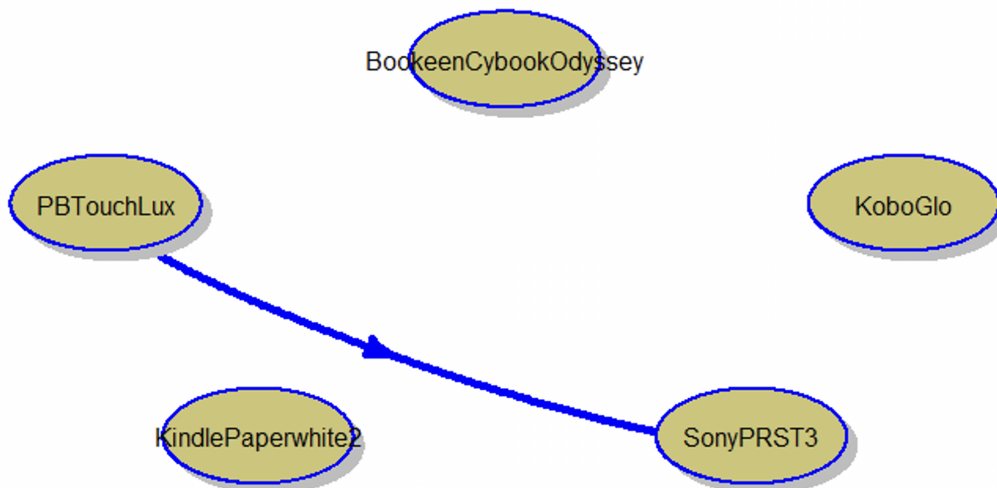


Figure 5.1: Network diagram for visualization of dominance between the alternatives

As you can see from fig. 5.1 the only clearly dominant relation exists between PocketBook Touch Lux and Sony PRS T3, where PocketBook dominates Sony PRST3. For remaining devices such dominance does not exist, which means that the ranking of these alternatives must be derived from overall utility score.

WSM uses concept of *simple dominance*. Such dominance means, that the alternative has better performance than other alternative in *all* criteria. In such case we can say, that such variant dominates inferior variant. If our goal is to identify best alternative (as opposed to problems of

ordering or scoring of the alternatives), we can clearly see, that the dominated variant cannot be best. We may even decide to exclude such alternatives from further evaluation.

This exclusion can also be performed in R using `mcdad_del_dominated()` function from MCDA-Support package.

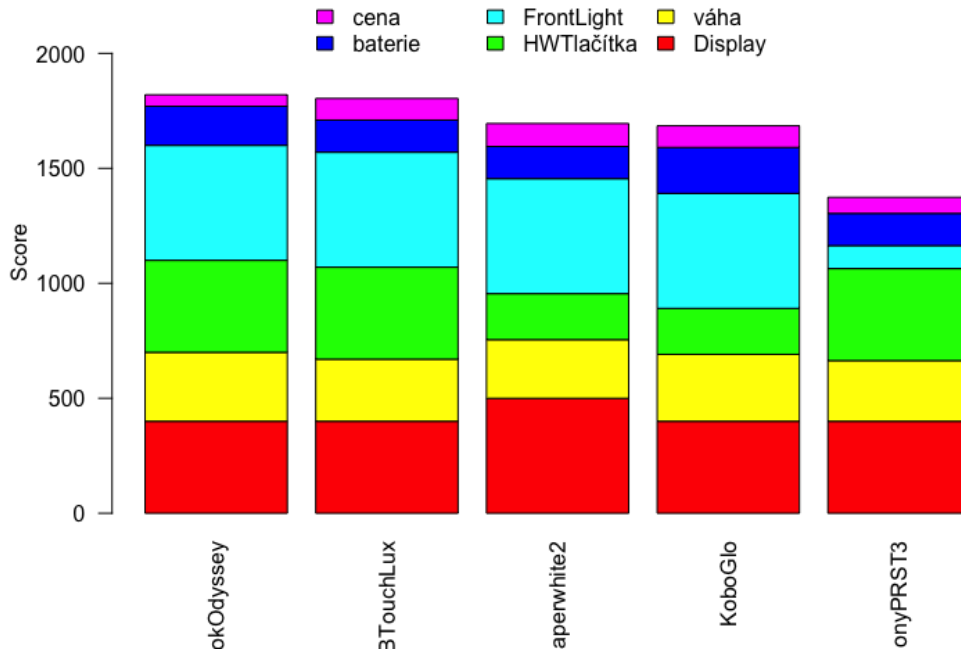


Figure 5.2: Contribution of criteria performance in alternatives to overall score of the alternative using WSM

We can also print results in tabular form, similarly as we used table of weighted preferences in previous chapter.

Listing of matrix of weighted preferences

	Display	weight	HWButtons	FrontLight	baterly	price
KoboGlo	400	291	200	500	200	94
SonyPRST3	400	264	400	100	140	70
KindlePaperwhite2	500	255	200	500	140	100
PBTouchLux	400	270	400	500	140	94
BookeenCybookOdyssey	400	300	400	500	170	50

Our results in overall score:

KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
1685	1374	1695	1804	1820

Method also provides percentages for these results for better interpretability.

BookeenCybookOdyssey	PBTouchLux	KindlePaperwhite2	KoboGlo	SonyPRST3
100.00000	99.12088	93.13187	92.58242	75.49451

As you can see, even simple method can provide pretty and interesting results. From interpretation perspective, we always need to consider what is the core of the decision making and what kind of results are expected from us. By expectations we mean format of the decision analysis, not confirming some kind of expected result.

5.4 ELECTRE

WSM method was very simple, on the other hand there were a lot of limitations and requirement on input preparation for analyst connected to it. There are lot of other interesting methods, which might make our analyst lives more easier. There are actually whole families of methods with member methods focusing on different types of problems. For example we may want to use some measure for uncertainty of the criteria performance. If we talk about battery, and its duration say in hours, then WSM always takes a single value, which it considers being precise, i. e. 6 hours. But what if we are not entirely sure about that and estimate lifespan of battery to 6 hours \mp 1 hour.

In real world situations we often can't come to precise estimations, so it would be beneficial if the method itself was at least somewhat aware of this uncertainty and was capable to work with it. As WSM, does not provide any support for it, we need to search for other methods. Out of all available methods we decided to present ELECTRE family of methods. Methods in the family are relatively old, well understood and described in literature including use cases and limitations. Methods are also capable to avoid or limit lot of disadvantages of WSM.

MCDASupport package supports currently following ELECTRE methods:

- ELECTRE I
- ELECTRE 1S (experimentální implementace)
- ELECTRE II
- ELECTRE III
- ELECTRE IV
- ELECTRE TRI

Even as we listed six separate methods, it is not exhaustive list, just more popular methods in the family.

The methods differ significantly in its goals and parameters, whit they require for computation. For example ELECTRE I, 1S and II allow to identify co called *kernel solution*, but each of these approaches the solution differently. All methods have some common properties, like ability specify limits for alternatives performance in criteria, which need to be exceeded to consider performance of other alternative in this criterium as beet or worse. Methods differ in implementation of these thresholds. ELECTRE II and 1S define it using fuzzy logic, ELECTRE I defines it as single number.

ELECTRE II, III, IV and TRI then can be used to solve ranking problem (we search for order of the alternatives considering its performance in decision criteria). Even for this methods use different approaches. ELECTRE III allows us to specify limist (preference and veto) separately for every criterium. That is hugely beneficial as the performance can remain expressed in its natural units. Analyst then does not need to prepare input data so thoroughly. Also we know that the transformations used for normalization do influence the results.

ELECTRE IV then for computation does not require formal weight specification. ELECTRE TRI allows the criteria to have categorical values, so we deal with situation when continuous scale is divided into in advance defined number of intervals, with exactly specified limits.

Let's try at least shortly look at process of the computation for these methods.

5.4.1 ELECTRE I

Oldest and also simplest method in the family is *ELECTRE I* method. As already specified in previous section of text, it allows us to identify kernel of the solution as set of alternatives which can be the optimum. That doesn't mean that all of them are the optimum, just that we failed to prove, that these alternatives cannot be optimum and as such cannot be removed from analysis. Method doesn't serve for ranking.

Required inputs are very similar to those required by WSM method. In addition to that we also can specify *concordance threshold* (implicitly 1) and *discordance threshold* (implicitly 0).

These values directly related to the need to compute so called *concordance* and *discordance matrices*.

We start with creation of *concordance index* (5.12) as direct comparison of two alternatives a and b. Index's values are written into matrix $m : m$, where m is number of alternatives. To be clear comparing alternatives a, b is concordance index. All comparisons written into the matrix will form *concordance matrix*.

$$C(a, b) = \frac{\sum_{\forall j: g_j(a) \geq g_j(b)} w_j}{\sum_{i=1}^n w_i} \quad (5.12)$$

Where n is number of criteria, j criteria, in which the alternative a is better than b , $g_j(a)$ is then performance of alternative a in criterium j .

If we look again on equation (5.12), we find out that the values of concordance index can only be in the interval of $C(a, b) \in < 0; 1 >$. The index is greater the greater the proportion of the weights of the criteria in which the alternative a performance was better than alternative b . Considering nature of comparison it can be proven, that values in concordance matrix must be always 1 as we will technically compute $C(a, a)$, so $\forall j : g_j(a) = g_j(a)$. In other words, in both the numerator and denominator of the formula will be the sum of all the weights, which means that $C(a, a) = 1$.

Discordance matrix is complementary matrix to concordance matrix. It uses different angle for alternative's comparison. We are interested in criteria, for which the alternative b performed better than alternative a . Computation at level of pairs of alternatives comparison is realize using *discordance index*, see equation (5.13).

$$D(a, b) = \begin{cases} 0 & \forall j : g_j(a) \geq g_j(b) \\ \max_j \frac{g_j(b) - g_j(a)}{\delta_j} & \text{for all other cases} \end{cases} \quad (5.13)$$

Where δ_j is maximum of difference in criterium j (max. - min. value in criterium: $\max_j - \min_j$).

Evaluating whether alternative a has generally better performance than alternative b needs to use both matrixes. We can say that matrix a is superior to b (aSb) if concordance index describing evaluated pair is higher than concordance threshold c : $C(a, b) \geq c$ and at same time its value of discordance index is lower than discordance threshold d : $D(a, b) \leq d$.

These conditions will help us construct *adjancancy matrix*. The fields representing alternative pairs comparisons will be equal to 1 for comparisons complying to these conditions and 0 otherwise. We can use the matrix to construct graph such as one on fig. 5.1. In this case for our example with decision on e-book readers, the result will be exactly same as for WSM method, so we do not provide it.

Simply said part of solution kernel are alternatives, which were not dominated, so the sum of values in the row of adjancancy matrix is equal to 0.

Using R, the script will be very similar to the one using **WSM**. If we use default thresholds $c = 1$ and $d = 0$, then all we need to do is change name of the used function to `Electre_1()` and run the script. Thou for convenience we provide full script.

Výpis 5.4: Using ELECTRE I method to solve choosing e-book reader problem in R

```

1 library("MCDASupport") #attach package
2 #define decision problem
3 #describe e-book readers
4 KoboGlo <- c(80, 97, 50, 100, 100, 94)
5 SonyPRST3 <- c(80, 88, 100, 20, 70, 70)
6 KindlePaperwhite2 <- c(100, 85, 50, 100, 70, 100)
7 PBTouchLux <- c(80, 90, 100, 100, 70, 94)
8 BookeenCybookOdyssey <- c(80, 100, 100, 100, 85, 50)
9 criteria <- c("Display", "weight", "HWButtons", "FrontLight", "batery", "price")
10 PM <- as.data.frame(
11   rbind(KoboGlo,
12         SonyPRST3,
13         KindlePaperwhite2,
14         PBTouchLux,
15         BookeenCybookOdyssey))
16 colnames(PM) <- criteria
17 w <- c(5, 3, 4, 5, 2, 1)
18 preferences <- Electre_1(PM, w, 'max') #use the method
19 preferences #print the results

```

The results in tabular form:

"Concordance Matrix"

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	1.00	0.80	0.70	0.80	0.65
SonyPRST3	0.45	1.00	0.45	0.55	0.50
KindlePaperwhite2	0.75	0.65	1.00	0.65	0.55
PBTouchLux	0.75	1.00	0.70	1.00	0.75
BookeenCybookOdyssey	0.85	0.95	0.70	0.95	1.00

"Discordance Matrix"

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0.00	1.0	1	1.00	1.00
SonyPRST3	1.00	0.0	1	1.00	1.00
KindlePaperwhite2	1.00	1.0	0	1.00	1.00
PBTouchLux	1.00	0.0	1	0.00	0.67
BookeenCybookOdyssey	0.88	0.4	1	0.88	0.00

"Preference Matrix"

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0	0	0	0	0
SonyPRST3	0	0	0	0	0
KindlePaperwhite2	0	0	0	0	0
PBTouchLux	0	1	0	0	0
BookeenCybookOdyssey	0	0	0	0	0

"Kernel"

KoboGlo	SonyPRST3	KindlePaperwhite2	BookeenCybookOdyssey
1	2	3	5

5.4.2 ELECTRE II

ELECTRE II method is extension of ELECTRE I to add capability to rank the alternatives. Both methods share computation of concordance, discordance matrices and kernel. ELECTRE II further works with information to estimate order of the alternatives.

In comparison to ELECTRE I method also defines differently concordance and discordance limits. ELECTRE II uses fuzzy number for the purpose. Concordance threshold is defined by three parameters c^- , c_0 a c^+ , where $c^- \leq c_0 \leq c^+$, for discordance threshold two parameters are used d^- a d^+ .

Using fuzzy parameters allows the method to distinguish between strong (S^F) and weak (S^f) dominance when comparing the alternatives. The difference between the two is intensity of our believe that alternative a performs better then b .

We say, that aS^Fb , if and only if:

$$C(a, b) \geq c^+ \quad (5.14)$$

$$g_j(a) - g_j(b) \leq d^- \text{ for } \forall j \quad (5.15)$$

$$\frac{P^+(a, b)}{P^-(a, b)} \geq 1 \quad (5.16)$$

$P^+(a, b)$ corresponds to sum of weights in criteria, where a performs better then b (aPb), analogically $P^-(a, b)$ correspond to sum of weights where b performs better then a (bPa).

These condition can be used for construction of matrices describing strong outranking (aS^Fb). If above conditions are met we set the comparison field to 1, otherwise 0.

To compute weak outranking (aS^fb) we use conditions (5.15, 5.16), but instead of condition (5.14) we use following condition:

$$C(a, b) \geq c^- \quad (5.17)$$

Since $c^- \leq c^+$ we need to ease up the rate we we are able to say that really aSb .

Formally in both cases we work with adjancancy matrices, which we can visualize and further work with.

ELECTRE II uses integration procedure, allowing to use computed information for purposes of estimating alternatives order. Method computes separately two so calls *pre-orders* and then derives final order from them.

- if aPb in both pre-orders, then aPb also in final order
- if $a = b$ in one pre-order, but aPb in second pre-order, then a precedes b in final order
- if aPb in one pre-order, but bPa in second, then the alternatives are not directly comparable

Again we can demonstrate method's usage easily in R using MCDASupport package for e-book reader example. And again if we use preset default values for the thresholds $c^- = 0,65$, $c_0 = 0,75$, $c^+ = 0,85$, $d^- = 0,25$, $d^+ = 0,5$ the computation only needs to change name of the used function to `Electre_2()`.

Výpis 5.5: Using ELECTRE II to compute alternative orders of e-book reader recommendation in R

```

1 library("MCDASupport") #attach package
2 #define decision problem
3 KoboGlo <- c(80, 97, 50, 100, 100, 94)
4 SonyPRST3 <- c(80, 88, 100, 20, 70, 70)
5 KindlePaperwhite2 <- c(100, 85, 50, 100, 70, 100)
6 PBTouchLux <- c(80, 90, 100, 100, 70, 94)
7 BookeenCybookOdyssey <- c(80, 100, 100, 100, 85, 50)
8 criteria <- c("Display", "weight", "HWButtons", "FrontLight", "bateriy", "price")
9 PM <- as.data.frame(
10   rbind(KoboGlo,
11     SonyPRST3,
12     KindlePaperwhite2,
13     PBTouchLux,
14     BookeenCybookOdyssey))
15 colnames(PM) <- criteria
16 w <- c(5, 3, 4, 5, 2, 1)
17 preferences <- Electre_2(PM, w, 'max') #use method
18 preferences #print results

```

We provide results also in tabular form, without concordance and discordance matrices, as these are same as in ELECTRE I.

\$StrongOutranking

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0	0	0	0	0
SonyPRST3	0	0	0	0	0
KindlePaperwhite2	0	0	0	0	0
PBTouchLux	0	1	0	0	0
BookeenCybookOdyssey	1	1	0	1	0

\$WeakOutranking

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0	0	0	0	0
SonyPRST3	0	0	0	0	0
KindlePaperwhite2	0	0	0	0	0
PBTouchLux	0	1	0	0	0
BookeenCybookOdyssey	1	1	0	1	0

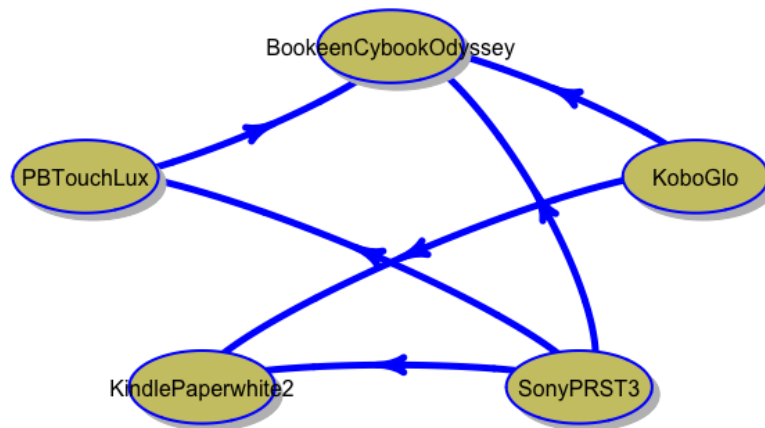


Figure 5.3: Network diagram of outranking of alternatives using ELECTRE II

```
$firstTotalPreorder #simplified and shortened
```

```
[1] "KindlePaperwhite2"
[2] "BookeenCybookOdyssey"
[3] "KoboGlo"
[4] "PBTouchLux"
[5] "SonyPRST3"
```

```
$secondTotalPreorder #simplified and shortened
```

```
[1] "BookeenCybookOdyssey"
[2] "PBTouchLux"
[3] "KindlePaperwhite2"
[4] "SonyPRST3"
[5] "KoboGlo"
```

```
$finalPreorderMatrix
```

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0	0	1	0	1
SonyPRST3	0	0	1	1	1
KindlePaperwhite2	0	0	0	0	0
PBTouchLux	0	0	0	0	1
BookeenCybookOdyssey	0	0	0	0	0

```
$incomparableAlternatives
```

	KoboGlo	SonyPRST3	KindlePaperwhite2	PBTouchLux	BookeenCybookOdyssey
KoboGlo	0	1	0	1	0
SonyPRST3	1	0	0	0	0
KindlePaperwhite2	0	0	0	1	1
PBTouchLux	1	0	1	0	0

BookeenCybookOdyssey	0	1	0	0
\$finalPreorder				
KindlePaperwhite2	BookeenCybookOdyssey	PBTouchLux	KoboGlo	SonyPRST3
1	1	2	3	4

5.4.3 ELECTRE III

ELECTRE III method approaches decision situation in different way. Similarly to ELECTRE II it uses fuzzy definition of the thresholds, but it uses three, and works differently with them:

- preference threshold - p - after going over this threshold we can say, that a is preferred to b : aPb
- indifference threshold - q - after going over this threshold, either of the alternatives cannot be preferred over one another, from comparison point of view we are indifferent to then: aIb
- veto threshold - v - after going over this threshold we **veto** thesis, that: aPb .

Thresholds p, q, v are set separately for each criterium. That allows us to leave criteria performance in its natural units, thus simplifying preparation phase of the analysis.

Apart of that it uses parameters α and β to compute so called *cut-off criteria*. Method uses these to estimate alternative's order.

As the thresholds are set separately for each criterium, we can't compute concordance matrix in single step. First we need to compute one concordance matrix for each criterium and then integrate matrices into overall concordance matrix.

$$c_j(a, b) = \begin{cases} 1 & \text{iff } g_j(a) + q_j \geq g_j(b) \\ 0 & \text{iff } g_j(a) + p_j < g_j(b) \\ \frac{g_j(a) - g_j(b) + p_j}{p_j - q_j} & \text{for all other cases} \end{cases} \quad (5.18)$$

$$C(a, b) = \frac{\sum_{j=1}^n w_j c_j(a, b)}{\sum_{j=1}^n w_j} \quad (5.19)$$

Discordance matrix is also computed separately for every criterium. Unlike concordance matrices, we do not perform integration step.

$$D_j(a, b) = \begin{cases} 1 & \text{iff } g_j(b) < g_j(a) + q_j \\ 0 & \text{iff } g_j(b) \leq g_j(a) + p_j \\ \frac{g_j(b) - g_j(a) - p_j}{v_j - p_j} & \text{for all other cases} \end{cases} \quad (5.20)$$

Information gathered in concordance matrix and discordance matrices are then used to compute *credibility index*, from which we can construct *credibility matrix*.

$$S(a, b) = \begin{cases} C(a, b) & \text{iff } D_j(a, b) \leq C(a, b) \text{ for } \forall j \\ C(a, b) \prod_{j \in J(a, b)} \frac{1 - D_j(a, b)}{1 - C(a, b)} & \text{else} \end{cases} \quad (5.21)$$

Where $J(a, b)$ is criteria set for which holds condition $D_j(a, b) > C(a, b)$.

Credibility index is then used as base for ordering of alternatives.

Order is computed using two distillation procedures, which is then combined into final order. Ranking is derived from cut-off criterium $s(\lambda_k)$. Its initial value when distilling downward is equal to $s(\lambda_1) = \max S$. In following iterations then $s(\lambda_{k+1}) = \alpha \cdot \lambda_k + \beta$. Based on value of cut-off criterium alternatives are selected which are conforming to the criterium. Ordering is constructed iteratively.

Distillation upward work analogically, provided we use min instead of max function.

Usage of method is demonstrated in next listing. This time we compare five projects using five criteria.

Výpis 5.6: Ranking of projects using ELECTRE III in R

```

1 # the performance table
2 PM <- cbind(
3   c(-14,129,-10,44,-14),
4   c(90,100,50,90,100),
5   c(0,0,0,0,0),
6   c(40,0,10,5,20),
7   c(100,0,100,20,40))
8 rownames(PM) <- c("Project1","Project2","Project3","Project4","Project5")
9 colnames(PM) <- c("CR1","CR2","CR3","CR4","CR5")
10 minmaxcriteria <- 'max'
11 Q <- c(25,16,0,12,10) #Indifference thresholds
12 P <- c(50,24,1,24,20) #Preference thresholds
13 V <- c(100,60,2,48,90) #Veto thresholds
14 w <- c(1,1,1,1,1) #weights
15 Electre_3(PM, w, P, Q, V, minmaxcriteria)

```

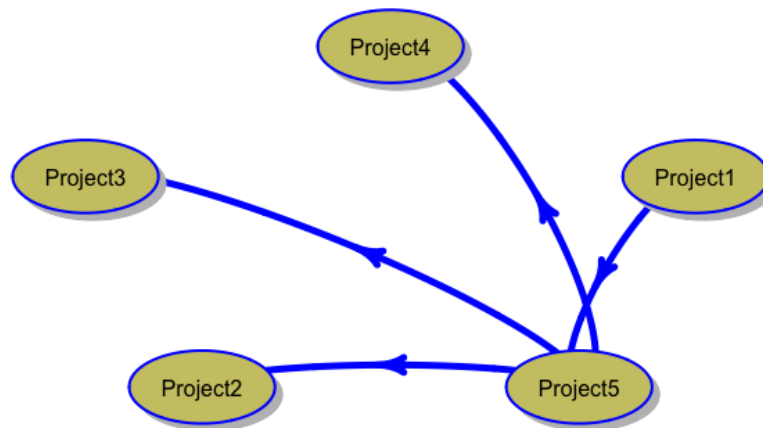


Figure 5.4: Network diagram of alternatives outranking using ELECTRE III method (project ranking example)

```

[1] "Concordance matrix"
      Project1 Project2 Project3 Project4 Project5
Project1  1.0 0.6000000    0.6    0.60 0.6666667
Project2  0.8 1.0000000    0.6    0.80 0.8000000
Project3  1.0 0.8000000    1.0    0.80 0.8000000
Project4  0.8 0.8000000    0.6    1.00 0.8000000
Project5  1.0 0.6666667    0.8    0.75 1.0000000
[1] "Discordance matrixes of criteria"
[[1]]
      Project1 Project2 Project3 Project4 Project5
Project1  0    1.0    0    0.16    0
Project2  0    0.0    0    0.00    0
Project3  0    1.0    0    0.08    0
Project4  0    0.7    0    0.00    0

```

```
Project5      0      1.0      0      0.16      0
```

```
[[2]]
```

```
      Project1 Project2 Project3 Project4 Project5
Project1 0.000000 0.000000      0 0.000000 0.000000
Project2 0.000000 0.000000      0 0.000000 0.000000
Project3 0.444444 0.722222      0 0.444444 0.722222
Project4 0.000000 0.000000      0 0.000000 0.000000
Project5 0.000000 0.000000      0 0.000000 0.000000
```

```
[[3]], [[4]] # shortened (all 0)
```

```
[[5]]
```

```
      Project1 Project2 Project3 Project4 Project5
Project1 0.000000      0 0.000000      0 0.000000
Project2 1.000000      0 1.000000      0 0.2857143
Project3 0.000000      0 0.000000      0 0.000000
Project4 0.8571429      0 0.8571429      0 0.000000
Project5 0.5714286      0 0.5714286      0 0.000000
```

```
[1] "Credibility matrix"
```

```
      Project1 Project2 Project3 Project4 Project5
Project1 1.000000      0.0 1.000000      0.8 1.000000
Project2 0.000000      1.0 0.000000      0.8 0.6666667
Project3 0.600000      0.0 1.000000      0.6 0.800000
Project4 0.2142857      0.8 0.5714286      1.0 0.750000
Project5 0.6666667      0.0 0.800000      0.8 1.000000
```

```
[1] "First pre-order" #shortened to single table
```

```
      action rank      rank
(down) (up)
1 Project1      1      1
2 Project5      2      3
3 Project2      3      1
4 Project3      3      3
5 Project4      3      2
```

```
[1] "total"
```

```
      Project1 Project2 Project3 Project4 Project5
Project1 "-"      "p+"      "p+"      "p+"      "p+"
Project2 "p-"      "-"      "I"      "I"      "p_"
Project3 "p-"      "I"      "-"      "I"      "p_"
Project4 "p-"      "I"      "I"      "-"      "p_"
Project5 "p-"      "p+"      "p+"      "p+"      "-"
```

```
[1] "Adjancancy matrix"
```

```
      Project1 Project5 Project2 Project3 Project4
Project1      0      1      0      0      0
Project5      0      0      1      1      1
Project2      0      0      0      0      0
Project3      0      0      0      0      0
Project4      0      0      0      0      0
```

```
[1] "Final ranking"
```

```
      alternative sum_outrank ranking
1 Project1      4      1
5 Project5      3      2
```

2	Project2	0	3
3	Project3	0	3
4	Project4	0	3

5.4.4 ELECTRE IV a TRI

For sake of completeness we also provide information on methods ELECTRE IV and TRI, in shortened form and without example. The MCDASupport package has tested examples available, so you can easily experiment on your own.

ELECTRE IV doesn't need weights for recommendations - that is probably it largest benefit. Method uses more complex system of preference mapping:

- $m_p(b, a)$ - number of criteria, for which b is strongly preferred to a
- $m_q(b, a)$ - number of criteria, for which b is weakly preferred to a
- $m_j(b, a)$ - number of criteria, for which there is no difference between a and b - indifference
- $m_o(b, a) = mo(a, b)$ - number of criteria, for which performance of alternatives is same

Above mentioned variables are kvazi-criteria, which are then used to construct credibility matrix, using following approach:

- Kvazi-dominance: $S(a, b) = 1$, if $m_p(a, b) + m_q(a, b) = 0$ and $m_i(a, b) < m_i(b, a) + m_q(b, a) + m_p(b, a)$
- Canonical dominance: $S(a, b) = 0, 8$, if $m_p(a, b) = 0$ and $m_q(a, b) \leq m_q(b, a)$ and $m_q(a, b) + m_i(a, b) \leq m_i(b, a) + m_q(b, a) + m_p(b, a) + 1$
- Pseudo dominance: $S(a, b) = 0, 6$, if $m_p(a, b) = 0$ and $m_q(a, b) \leq m_q(b, a) + m_p(b, a)$
- Sub dominance: $S(a, b) = 0, 4$, if $m_p(a, b) = 0$
- Veto dominance: $S(a, b) = 0, 2$, if $mp(a, b) = 1$ or if $(m_p(b, a) \geq \frac{m}{2})$ and $g_j(b) + v_j \geq g_j(a)$
- else $S(a, b) = 0$

We establish alternatives order from credibility matrix by using same procedure as in ELECTRE III (distillation downward, upward and final order derivation).

Last method in ELECTRE family, we are going to discuss in this chapter is ELECTRE TRI method. Method is specific by using categorical variables. By categorical variable we mean situation in which criterium us specified on continuous numeric scale, which si divided to intervals. We assign label to each interval. Uncertainty in this case is in question of how much sure are we, that performance of the criterium truly belongs to given category and not for example lower or higher category?

Concordance index is computed separately for every criterium.

$$c_j(a, b_h) = \begin{cases} 0 & \text{if } g_j(b_h) - g_j(a) \geq p_j \\ 1 & \text{if } g_j(b_h) - g_j(a) \leq q_j \\ \frac{p_j + g_j(a) - g_j(b_h)}{p_j - q_j} & \text{else} \end{cases} \quad (5.22)$$

Where $g_j(b_h)$ is the profile assigned to category of criterium v variant b

Then we compute c_j and also in inverse form: $c_j(b_h, a)$. We use equation (5.22) for this purpose, inly with switched a and b_h parameters.

Discordance index is also computed separately for each criterium, but the computation is simpler:

$$d_j(a, b_h) = \min(1, \max(0, \frac{g_j(b_h) - g_j(a) - p_j}{v_j - p_j})) \quad (5.23)$$

In this case we also compute inverse discordance index $d_j(bg, a)$.

Credibility index:

$$s(a, bh) = \begin{cases} c_j(a, b_h) \prod_{j \in V} \frac{1 - d_j(a, b_h)}{1 - c_j(a, b_h)} & \text{if } V \neq \emptyset \\ c_j(a, bh) & \text{else} \end{cases} \quad (5.24)$$

Where $V = \{j \in J : d_j(a, b_h) > c_j(a, b_h)\}$.

For dealing with fuzzy logic we use credibility index. We search for lowest value of credibility index, for which we can say, that a has better performance than b_h (aSb_h).

$$s(a, b_h) \geq \lambda \Rightarrow aSb_h \quad (5.25)$$

$$s(a, b_h) < \lambda \Rightarrow \neg aSb_h \quad (5.26)$$

$$s(b_h, a) \geq \lambda \Rightarrow b_hSa \quad (5.27)$$

$$s(b_h, a) < \lambda \Rightarrow \neg b_hSa \quad (5.28)$$

Comparing aSb_h and b_hSa we can distinguish relations between a and b_h :

- $aIb_h \iff aSb_h \wedge b_hSa$
- $a > b_h \iff aSb_h \wedge \neg b_hSa$
- $b_h > a \iff \neg aSb_h \wedge b_hSa$
- $aRb_h \iff \neg aSb_h \wedge \neg b_hSa$

Final order (ranking) is then derived using optimistic and pessimistic procedure.

When using *optimistic* procedure, the alternative a is compared with categories b_k, b_{k+1}, \dots , one after another until $b_h > a$. In such case the a is assigned to worst category C_h , for which condition $b_h > a$ holds.

When using *pessimistic* procedure the alternative a is compared with b_k, b_{k+1}, \dots , one after another until aSb_{k-1} . In such case the a is assigned to best category C_h , for which aSb_{h-1} holds..

In both cases the alternatives are ordered by categories, they were assigned to, from which we derive rank. Since we have two methods for ranking, we get two orders. Unlike previous ranking ELECTRE methods ELECTRE TRI does not try to consolidate result to single order.

Summary

In this chapter we discussed in detail some aspects of advanced decision making. In section **WSM** and **WPM** we discussed problem of performance measurement additivity of the alternatives. We also discussed problem of compensation mechanism deeply integrated into WSM method. By compensation we mean using good performance in one criterium to compensate bad performance in other.

ELECTRE methods allow to certain degree compensate uncertainties in value estimation in performance matrices. Using fuzzy parameters we can lower imprecision in the computation process.

The methods are intended to solve variety of problem. From point of view of decision making we distinguish between two basic goals:

1. identifying solution kernel
2. ranking



Questions

1. What is normalization and for what purpose do we use it in decision making
2. Describe budget allocation method for weight coefficient estimation.
3. What does substitution effect mean (i.e. for WSM method)? When does such substitution present a problem?
4. What is a solution kernel? How different are alternatives in it compared to those which didn't make the cut?
5. What is ranking?
6. What is the difference between concordance and discordance index?
7. What do we mean by credibility (in ELECTRE methods)?
8. How are the thresholds (preferential, indifference and veto) in ELECTRE methods used?



Example

In this chapter we discussed a lot of methods. We also presented a variety of listings demonstrating them. Install R and RStudio. In it install MCDASupport package in the latest version. You can download it from my homepage <https://fbweb.vsb.cz/~sen76/index.php?id=publikace> or from LMS. Installation is described in chapter 5.1. Note that newer versions of the package (if there are newer versions) may require fulfillment of different dependencies. So examine closely the error message the R produces. You will need to install missing dependencies.

Chapter 6

Analytic Hierarchy Process (AHP)



Study guide

In this chapter we will finish problematic of multiple criteria decision making. From portfolio of problems we have been facing, only one remains, and that would be dealing with hierarchies of criteria. **AHP** provides us a tool to deal with this problem in effective way.

After reading this chapter you will know

- how to use AHP method for
 - weight coefficient estimation
 - check weights for randomness
 - score alternatives to find best alternative or rank all of them



Time required for study

To go through the chapter you will need approximately two hours, but we recommend to invest more time to practically try the method and experiment with the examples using for example MS Excel template available in LMS or R.

6.1 Introduction to AHP

Firs let's consider a question: what the problem is and why should we even consider existence of hierarchies? When talking about **WSM** method we formulated the requirement on criteria used for decision making to be independent. Almost immediately we found out, that it is not easy task to fulfill it. So we presumed to be sufficient for criteria to be only weakly dependent.

Problem that if such dependencies exist, regardless of whether weak or strong, then when using WSM, the results will be impacted by it, possibly providing misleading results. Please note that by assuming weak dependence we basically believe that this impact will be so low, that it will not alter the result significantly. Thus making it possible for us to use the method. Please note that distinguishing between weak and strong dependencies is not necessary easy task.

This problem is actually typical for basically all of multiple criteria decision method including ELECTRE family of methods, even thou we didn't explicitly specify this requirement.

Under these conditions, it might be beneficial to use a method, which is actually capable of working with hierarchies and somewhat alleviate this problem. Most popular method for multiple criteria decision support using criteria hierarchies is **AHP** method.

AHP was developed in 80-tees of last century by Thomas L. Saaty [59] to solve this kind of problem. Method formulates decision problem if for of the criteria hierarchy. Hierarchical organization gives us

certain control on influence of clusters of criteria, which would otherwise present significant problem for WSM and similar methods¹.

AHP also provides tools to evaluate basic quality of stated preferences that are being used for weights estimation. AHP is capable to identify possible randomness in these preferences using *consistency index*, but unfortunately the quality control does not answer question of precision of the weights. So the answer AHP provides is that weights are not random, but not that the weights are correct. AHP thus does not solve problems with precision of the expert opinions, or more correctly notorious lack of it.

Because AHP method is very popular, there are hundreds or even thousands of studies for various problems, utilizing this method. Usage and limitations of the method is well described and documented, which gives us a solid base for using it. Also there are software suites and libraries for various computational environments (such as R), which makes the method suitable to be solved using computers.

One example of such software is SuperDecisions software [38] developed directly by Saaty and a team around him.

In literature the method is also sometimes being refereed to somewhat incorrectly as Saaty's method. But it is necessary to state, that it is misleading as Saaty develop multiple method being widely used, among others ANP. So technically name Saaty's method may refer to any of these. So we recommend the practitioners of the method to be specific.

ANP in comparison with AHP does not see the decision problem as criteria hierarchy, but a network of it. ANP is thus more general method capable to cover broader portfolio of problems. It could be also shown, that the AHP is actually special case of ANP. Since the space in this textbook and time for lectures are limited, we decided to focus on AHP.

Some software supports only one of these methods, but for example SuperDecisions supports both methods.

6.2 AHP Process

Process of AHP has three basic steps:

1. model decision problem as a hierarchy
2. pairwise comparison of the criteria at the various parts of hierarchy to estimate their contribution to problem solution
3. weight estimation

We can use the results either to estimate weights only or to rank the alternatives.

Let's examine simple example of hierarchy, see fig. 6.1. We decide on best **Chief Executive Officer (CEO)** candidate.

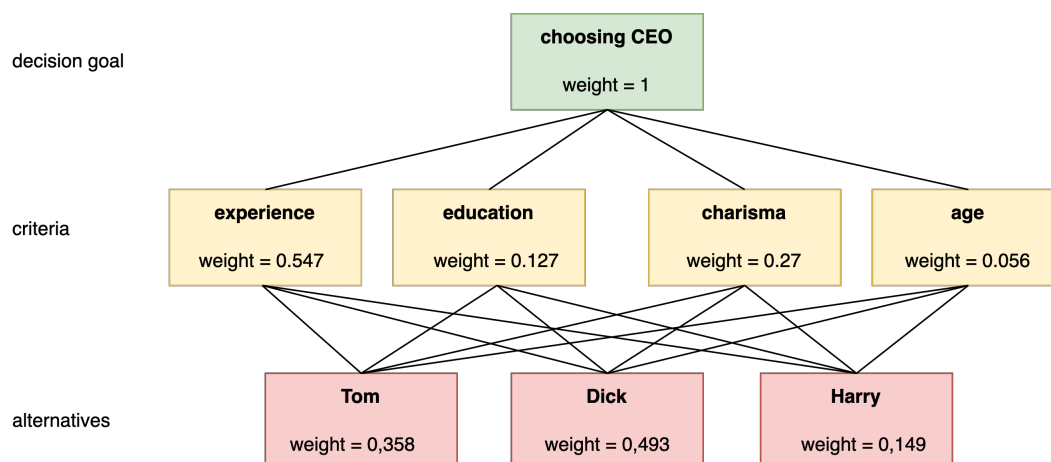


Figure 6.1: Simple hierarchy of best CEO candidate selection (adapted from [60])

¹other methods which do not use hierarchies

Hierarchy in this case is only two levels. In first (root) level we specify decision goal, manager choice in our case. Into this goal, all criteria are consolidated. To that corresponds overall weight 1 (100 %). This weight is being distributed in level 2 to the criteria on this level. Assigned weights must comfort to condition, that $\sum w$ of the criteria is equal to weight of criterium from which these are derived.

Hierarchy on fig. 6.1 is very simple. In fact it is so simple, that its usage brings almost no benefits in comparison to WSM (and similar methods). But from point of view of hierarchy's complexity, we are not limited. The hierarchy can have three, four or even more levels of hierarchy, as needed.

That is a situation, when use of AHP will bring significant benefits.

If our goal was only to compute weight coefficients, we could limit the hierarchy only to goals and criteria and omit the alternatives and its performance in them. We need to consider the alternatives only in case we use AHP for scoring them - finding the best or rank them. Weight coefficients then correspond to overall performance of alternative across all "leaf" criteria in hierarchy.

In this case we also perform pairwise comparison. We compare alternatives performance in the criteria. Each pair is consequently being compared in all criteria, to estimate the result.

This approach is beneficial in that the performance of alternatives in criteria can stay in its natural units, even in cases, when the criterium is specified in categorical form using for example ordinal scale or even text description. Compare that to ELECTRE methods family, where we had to choose different method based on how our inputs look like.

In case that the alternative performance in criteria is normalized and the used scale correspond to our understanding of utility we do not need to perform alternatives pairwise comparison. Formally we can simplify such situation into single level formed by leaf nodes of the hierarchy and use WSM to compute overall score of the alternatives.

To finish the computation of information from fig. 6.1, from nodes values it is apparent that best candidate to CEO position is Dick, followed by Tom and Harry on last place.

To estimate weight coefficients we need to perform pairwise comparison on every level and every group. For our easy hierarchy on fig. 6.1 we will need single matrix to compare the criteria and four matrices to compare alternatives performance in the criteria.

Compare that with solution of hierarchy presented later in chapter. For its solution we will need 3 matrices to compare the criteria and 8 matrices to compare alternative performance in them, see fig. 6.2.

In each matrix we compare the criteria using following scale:

- 1 - equally important
- 3 - moderately more important
- 5 - strongly more important
- 7 - very strongly (demonstrably) more important
- 9 - extremely strongly more important
- even numbers can be used to improve granularity of comparison
- decimals can be used to further improve comparison resolution

if we compare criteria a and b , with a being strongly more important than b , then $aSb = 5$. From this we can derive value of inverse comparison $bSa = 1/5$. This approach allows us to limit required preferences, which needed collected from problem domain experts to realize the comparison.

So the preference values will conform to following conditions. If a is preferred to b , then the preference value will be $\in < 1; 9 >$. In case b is preferred to a , the the preference will be in interval $\in < 0.11; 1 >$, or $1/9(0.11)$ to $1/1$.

Furthermore, considering that comparison matrix is always square matrix: $n:n$, where n is number of criteria we compare. We can derive, that the diagonal must be equal to 1 as we compare the criterion with itself.

We perform pairwise comparison, but of other type then in case of Fuller's triangle, which is *binary*. Since our preference resolution is much finer then just saying that a is more important then b or vice versa, we are capable to also gain some insight in intensity of this preference.

Preference matrix P (6.1) is basis for estimation of weight coefficients.

$$P = \begin{bmatrix} 1 & p_1 & \dots & p_{1n} \\ \frac{1}{p_1} & 1 & \dots & p_{\dots n} \\ \dots & \dots & 1 & \dots \\ \frac{1}{p_{1n}} & \dots & \dots & 1 \end{bmatrix} \quad (6.1)$$

We presume, that the preferences correspond to real ratios between criteria's weights. That is strong requirement, which can be problematic, if we fail to meet it, than the results provided by the method could be misleading. This is a property of the method, we can perceive as a weakness and or limitation. Method also does not provide any mean to evaluate the quality of mapped preferences (measure of how much it corresponds to reality). On the other hand method has a measure which helps us identify situation, when the preferences has been estimated randomly.

Problem of weights coefficient estimation can be seen as optimization problem, see (6.2).

$$F = \sum_{i=1}^k \sum_{j=1}^k \left(p_{ij} - \frac{w_i}{w_j} \right)^2 \rightarrow \min \quad (6.2)$$

Where k is number of evaluated criteria.

The solution is possible using *quadratic programming* method, but the solution is usually considered as relatively computationally intensive and very complicated for computing in hand. Because of that the computation is often approximated using geometric mean (6.3). This approximation introduces some level of error into the result and is considered as undesirable for high value decisions.

$$w_i = \frac{\left(\prod_{j=1}^k p_{ij} \right)^{\frac{1}{k}}}{\sum_{i=1}^k \left(\prod_{j=1}^k p_{ij} \right)^{\frac{1}{k}}} \quad (6.3)$$

We can evaluate weight consistency using *consistency index* CI , computed by (6.4).

$$CI = \frac{w_{max} - k}{k - 1} \quad (6.4)$$

We can compare computed CI against *random consistency index* RCI to compute *consistence ratio* CR , see (6.5).

$$CR = \frac{CI}{RCI_k} \quad (6.5)$$

So what exactly is RCI ? It is average CI for preference matrices generated randomly. Computation itself is easy if you use computational environments like R, MathLab, SciLab or any other. Start with eye matrix $k : k$, then randomly generate number in interval $\in \langle 0; 9 \rangle$ as randomly generated preference value. Note that only half of matrix's fields need to be estimated in this way, the other half presents inverse preference and thus must be equal to $1/\text{random preference}$. Then compute CI and add it to the dataset of CIs. Then repeat, until you get robust enough dataset of CIs, for which you compute mean, which we call RCI . Original Saaty's RCI values were computed on few thousand elements large datasets. In literature there are RCI computed on dataset order of magnitude larger.

Conventional wisdom says, that larger the k is the larger CIs dataset is needed to estimate robust mean. But there is also law of marginal utility in the play, so increasing number of CIs in dataset over certain point does not significantly improve precision. For $k < 11$ adding CIs over 500 000 will not induce change on first 5 decimal points.

Original Saaty's values for RCI are provided in tab. 6.1 and are considered good enough for most use cases.

Table 6.1: AHP - random consistency index RCI (courtesy of [59])

k	1	2	3	4	5	6	7	8	9	10
RCI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

As statistically significant value of $CR = 0.1$ is considered. If $|CR| < 0.1$ we presume, that mapped preferences we used for weights estimation are not random.

Please note that value $CR = 0.1$ should be considered more as rule of thumb than strong statistical test. Then estimating weights we should try to achieve lowest possible value. For complex problems with large k -number of criteria, it is very hard to get lower than 0.1 using preferences stated by experts as expert opinions on their own are large source of random errors introduced into computation. There is also a name for this problem - *cognitive bias*. All human made decisions/opinions are influenced by it. If we understand the nature of bias, we can somewhat minimize its influence, but it is always present. We will discuss it in more detail in next chapter.

Another source of errors is created if we use group of experts as preference source instead of single person. There are many strategies we can use to integrate varying preferences. Easiest approach is to use the average (mean) value. But if there is significant variance in these opinions large CI will be probably observed for the weights.

6.3 Case study - Choosing Best Car for Johnson's Family

This case study has been published originally on Wikipedia [26], we will use slightly modified version for demonstration purposes.

Goal of the decision is to find best car for Johnson's family. The family after thinking really hard came with criteria hierarchy on fig. 6.2 with criteria relevant for utility consideration. Family also identified 6 cars, it is choosing from.

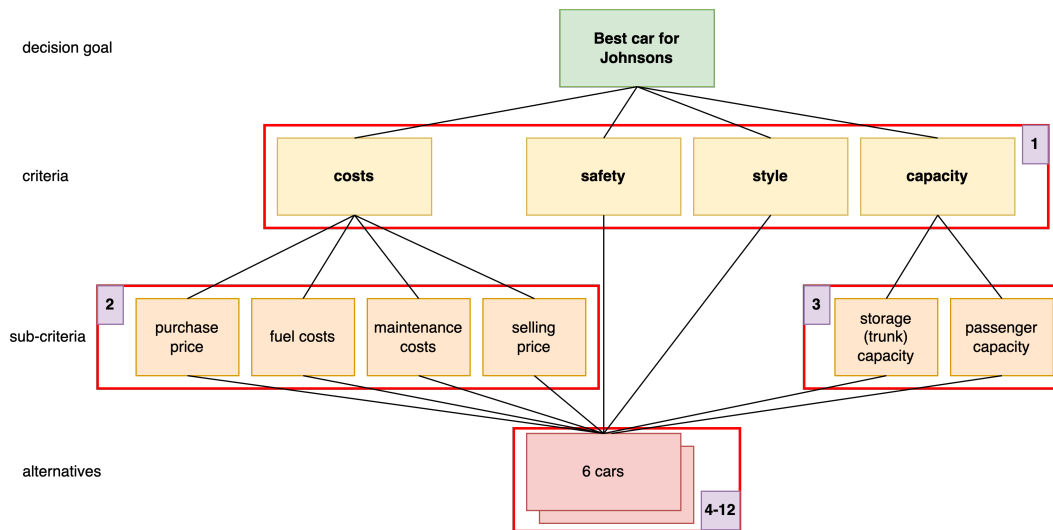


Figure 6.2: Criteria hierarchy for choosing Johnson's new car, comparison groups are visualized (courtesy of [26])

Johnson's choose from following cars:

1. Accord (sedan)
2. Accord (hybrid)
3. Pilot SUV
4. CR-V SUV
5. Element SUV
6. Odyssey minivan

Pairwise comparison is available in tab. 6.2.

For subcriteria of costs pairwise comparison is available in tab. 6.3.

Last group of sub-criteria is formed by capacities, see tab. 6.4.

Whole decision hierarchy has 8 leaf nodes. Which means we will need to create 8 comparison matrices, where we deal with these criteria. We will fully explore the first of these - purchase price, for

Table 6.2: Choosing car - criteria comparison (courtesy of [26])

	costs	safety	style	capacity
costs	1	3	7	3
safety	1/3	1	9	1
style	1/7	1/9	1	1/7
capacity	1/3	1	7	1

Table 6.3: Choosing car - comparing costs sub-criteria (courtesy of [26])

	purchase price	fuel costs	maintenance costs	selling price
purchase price	1	3	3	5
fuel costs	1/2	1	2	2
maintenance costs	1/5	1/2	1	1/2
selling price	1/3	1/2	2	1

Table 6.4: Choosing car - comparing capacity sub-criteria (courtesy of [26])

	storage capacity	passenger capacity
storage capacity	1	1/5
passenger capacity	5	1

rest we only specify matrix with comparison. If you are interested in more details and explanations (rationale behind there numbers) consult original Wikipedia article [26]².

When family considers *purchase price*, it actually uses variety of other supportive information connected to this price. First the family can invest only 25 000 USD into purchase and its extremely important that the family remains in budget for the purchase. As supportive measure price difference and price ration of compared cars can be used.

Basic evaluation is available in tab. 6.5, weight coefficient estimation is in tab. 6.6 and finally the matrix comparing them is in tab. 6.7.

Table 6.5: Choosing car - compare price of cars (courtesy of [26])

#	comparison		purchase price		better price	price better difference ratio		under/(over) budget	
	A	B	A	B		A	B	A	B
1	Accord s.	Accord h.	20 360	31 090	A	10 730	1,53	4 640	(6 090)
2	Accord s.	Pilot	20 360	27 595	A	7 235	1,36	4 640	(2 595)
3	Accord s.	CR-V	20 360	20 700	A	340	1,02	4 640	4 300
4	Accord s.	Element	20 360	18 980	B	1 380	1,07	4 640	6 020
5	Accord s.	Odyssey	20 360	25 645	A	5 285	1,26	4 640	(645)
6	Accord h.	Pilot	31 090	27 595	B	3 495	1,13	(6 090)	(2 595)
7	Accord h.	CR-V	31 090	20 700	B	10 390	1,5	(6 090)	4 300
8	Accord h.	Element	31 090	18 980	B	12 110	1,64	(6 090)	6 020
9	Accord h.	Odyssey	31 090	25 645	B	5 445	1,21	(6 090)	(645)
10	Pilot	CR-V	27 595	20 700	B	6 895	1,33	(2 595)	4 300
11	Pilot	Element	27 595	18 980	B	8 615	1,45	(2 595)	6 020
12	Pilot	Odyssey	27 595	25 645	B	1 950	1,08	(2 595)	(645)
13	CR-V	Element	20 700	18 980	B	1 720	1,09	4 300	6 020
14	CR-V	Odyssey	20 700	25 645	A	4 945	1,24	4 300	(645)
15	Element	Odyssey	18 980	25 645	A	6 665	1,35	6 020	(645)

For other leaf criteria the comparison is available in tab. 6.8 - 6.14.

²Johnson's new car on Wikipedia: https://en.wikipedia.org/wiki/Analytic_hierarchy_process_%E2%80%93_car_example

Table 6.6: Choosing car - price comparison - preference estimation (courtesy of [26])

#	Comparison		better car	pref.	justification
	A	B			
1	Accord s.	Accord h.	A	9	B over budget
2	Accord s.	Pilot	A	9	B over budget
3	Accord s.	CR-V	A	1	price almost same
4	Accord s.	Element	B	2	price of B better by 1 000 USD
5	Accord s.	Odyssey	A	5	price of A better by 5 000 USD
6	Accord h.	Pilot	A	1	both cars over budget
7	Accord h.	CR-V	B	9	A over budget
8	Accord h.	Element	B	9	A over budget
9	Accord h.	Odyssey	B	7	A over budget, B slightly over budget
10	Pilot	CR-V	B	9	A over budget
11	Pilot	Element	B	9	A over budget
12	Pilot	Odyssey	B	7	A over budget, B slightly over budget
13	CR-V	Element	B	2	price of B better by 1 000 USD
14	CR-V	Odyssey	A	5	price of A better by 5 000 USD
15	Element	Odyssey	A	6	price of A better by 6 000 USD

Table 6.7: Choosing car - comparison matrix for purchase price (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	9	9	1	1/2	5
Accord h.	1/9	1	1	1/9	1/9	1/7
Pilot	1/9	1	1	1/9	1/9	1/9
CR-V	1	9	9	1	5	1/2
Element	2	9	9	1/5	1	6
Odyssey	1/5	7	9	2	1/6	1

Table 6.8: Choosing car - comparison matrix for fuel costs (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1/3	5	3	4	3
Accord h.	3	1	9	5	7	6
Pilot	1/5	1/9	1	1/4	1/3	1/4
CR-V	1/3	1/5	4	1	2	1
Element	1/4	1/7	3	1/2	1	1
Odyssey	1/3	1/6	4	1	1	1

Table 6.9: Choosing car - comparison matrix for maintenance costs (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	2	4	4	4	5
Accord h.	1/2	1	4	4	4	5
Pilot	1/4	1/4	1	1	2	1
CR-V	1/4	1/4	1	1	1	3
Element	1/4	1/4	1/2	1	1	2
Odyssey	1/5	1/5	1	1/3	1/2	1

We will demonstrate computation in R environment using AHP package [44]. This package is not on CRAN anymore, but can still be downloaded from the CRAN archives and installed manually in same way as we installed MCDASupport package in last chapter.

You can download the package from <https://cran.r-project.org/src/contrib/Archive/ahp/>.

Also the package is not being actively developed anymore, but what is in there is still fully sufficient for our scholarly experiments. Also the package supports only "full" regime of usage, so it requires also alternatives for evaluation. Simplified computation of weights only for the hierarchy is not supported

Table 6.10: Choosing car - comparison matrix for selling price (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	3	4	1/2	2	2
Accord h.	1/3	1	2	1/5	1	1
Pilot	1/4	1/2	1	1	1/6	1/2
CR-V	2	5	1	1	4	4
Element	1/2	1	6	1/4	1	1
Odyssey	1/2	1	2	1/4	1	1

Table 6.11: Choosing car - comparison matrix for safety (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1	5	7	9	1/3
Accord h.	1	1	5	7	9	1/3
Pilot	1/5	1/5	1	2	9	1/8
CR-V	1/7	1/7	1/2	1	2	1/8
Element	1/9	1/9	1/9	1/2	1	1/9
Odyssey	3	3	8	8	9	1

Table 6.12: Choosing car - comparison matrix for style (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1	7	5	9	6
Accord h.	1	1	7	5	9	6
Pilot	1/7	1/7	1	1/6	3	1/3
CR-V	1/5	1/5	6	1	7	5
Element	1/9	1/9	1/3	1/7	1	1/5
Odyssey	1/6	1/6	3	1/5	5	1

Table 6.13: Choosing car - comparison matrix for storage (trunk) capacity (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1	1/2	1/2	1/2	1/3
Accord h.	1	1	1/2	1/2	1/2	1/3
Pilot	2	2	1	1	1	1/2
CR-V	2	2	1	1	1	1/2
Element	2	2	1	1	1	1/2
Odyssey	3	3	2	2	2	1

Table 6.14: Choosing car - comparison matrix passenger capacity (courtesy of [26])

	Accord s.	Accord h.	Pilot	CR-V	Element	Odyssey
Accord s.	1	1	1/2	1	3	1/2
Accord h.	1	1	1/2	1	3	1/2
Pilot	2	2	1	2	6	1
CR-V	1	1	1/2	1	3	1/2
Element	1/3	1/3	1/6	1/3	1	1/6
Odyssey	2	2	1	2	6	1

in this package.

The usage could look like this:

Výpis 6.1: Usage of AHP method for choosing new car for Johnson's family

```

1 library(ahp)
2 library(data.tree)
3 cars <- Load("c:/path/cars.ahp")
4 Calculate(cars)

```

```

5 print(cars, filterFun = isNotLeaf)
6 Analyze(cars)
7 AnalyzeTable(cars)

```

That is not too bad. We load data file cars.ahp, which has defined decision hierarchy including preference matrices and we put it through *Calculate()* function. This function will compute weight coefficient and also scores alternatives.

Remaining lines of code only visualize the results.

Print() lists decision hierarchy to the scree. For file cars.ahp³ the hierarchy looks like:

```

levelName
1 Root
2 |--Cost
3 | |--Purchase Price
4 | |--Fuel Costs
5 | |--Maintenance Costs
6 | °--Resale Value
7 |--Safety
8 |--Style
9 °--Capacity
10 |--Cargo Capacity
11 °--Passenger Capacity

```

Graphical representation of solution can be visualized as on fig. 6.3:

	Weight	Accord Sedan	Odyssey Minivan	Accord Hybrid	CR-V SUV	Element SUV	Pilot SUV	Inconsistency
Root	100.0%	22.0%	21.2%	18.3%	15.8%	13.3%	9.3%	7.5%
Cost	51.0%	12.8%	4.7%	9.1%	11.3%	11.2%	1.9%	1.5%
Purchase Price	24.9%	6.1%	2.4%	0.6%	6.1%	9.1%	0.6%	8.0%
Fuel Costs	12.8%	2.9%	1.1%	6.2%	1.3%	0.9%	0.4%	3.1%
Resale Value	8.2%	1.8%	0.9%	0.8%	3.4%	0.9%	0.5%	0.5%
Maintenance Costs	5.1%	1.9%	0.3%	1.5%	0.5%	0.4%	0.5%	4.0%
Safety	23.4%	5.1%	10.2%	5.1%	0.8%	0.5%	1.8%	8.0%
Capacity	21.5%	2.8%	6.0%	2.8%	3.1%	1.4%	5.5%	0.0%
Passenger Capacity	17.9%	2.4%	4.9%	2.4%	2.4%	0.8%	4.9%	0.0%
Cargo Capacity	3.6%	0.3%	1.1%	0.3%	0.6%	0.6%	0.6%	0.2%
Style	4.1%	1.5%	0.3%	1.5%	0.6%	0.1%	0.2%	10.1%

Figure 6.3: AHP - solution of choosing new car for Johnson's

First column, *weights*, shown estimated weight coefficients. As least important criteria of trunk capacity and style have been identified with weights of 3,6 % a 4,1 %. Also note, that the style in column *inconsistency* has value 10.1 %, which can be interpreted in our context as $CR = 0.101$. Since value $CR = 0.1$ is perceived as limit, than we can't exclude possibility, that the stated preferences leading to the weight estimation are actually random. Since we are over the limi just by 0.001 and our decision problem is purely demonstrative, we can let it slide this time.

From point of view of formulation of the recommendation to Johnson's family, best car was Accord sedan followed by Obyssy minivan. On opposite side of evaluation is Pilot SUV.

As you can see usage of the method is relatively simple provided, that you have well prepared inputs for the computation. And that is the main obstruction to adoption. For our case that would be preparation of file cars.ahp.

This file is in **YAML Ain't Markup Language (YAML)** format, and it ain't pretty. Bellow short excerpt is presented to demonstrate basic structure of the file.

³the file is available for download in LMS (<https://lms.vsb.cz> in course *Modeling of Decision Processes*)

Výpis 6.2: YAML structure - excerpt from cars.ahp

```

1  Version: 2.0
2
3  Alternatives: &alternatives
4    Accord Sedan:
5    Purchase Price: 20360
6    MPG: 31
7    Residual Value: 0,52
8    Safety class: midsize car
9    Cargo Capacity: 14
10   Passenger Capacity: 5
11   Surb Weight: 3289
12   crash rating: 4* in side impact front
13   60K tire cost: 700
14   Brakes Cost: 1x
15   Consumer Report: +++
16   ...
17  Goal:
18    preferences:
19    pairwise:
20      - [Cost, Safety, 3]
21      - [Cost, Style, 7]
22      - [Cost, Capacity, 3]
23      - [Safety, Style, 9]
24      - [Safety, Capacity, 1]
25      - [Style, Capacity, 1/7]
26  children:
27    Cost:
28      preferences:
29      pairwise:
30        - [Purchase Price, Fuel Costs, 2]
31        - [Purchase Price, Maintenance Costs, 5]
32        - [Purchase Price, Resale Value, 3]
33        - [Fuel Costs, Maintenance Costs, 2]
34        - [Fuel Costs, Resale Value, 2]
35        - [Maintenance Costs, Resale Value, 1/2]
36  ...

```

Structure of file is very rigid. On first line, there is always identification of the version. Definition of the alternatives follows, see lines 3 - 15 in the listing. Remainder of the file is then formed by definitions of decision hierarchy and pairwise comparisons. We need to compare each pair only once, the inverse comparison is computed automatically in the package's functions, but still, it is lengthy process.

Without this feature the number of rows of the input file would have literally doubled. Even with this simplification the file has around 250 lines of text. Full listing of the car.ahp file is Available in Annex 3 of this textbook.

Manual creation of the file is complicated. We can expect, that longer the file is, more error will be made during manual input. To streamline the process I prepared small editor, allowing its user to click and edit the file, with possibility to export it into YAML format. The program is called AHP YAML Editor [69] and is available either in LMS and or on my homepage <https://fbiweb.vsb.cz/~sen76/data/uploads/programy/AHPeditor%20v0.1.7z>. Exporting should limit number of errors, which will be present in the YAML file.

Graphical User Interface (GUI) of the program is presented on fig. 6.4 and fig. 6.5.

Program has two perspectives - alternatives and criteria. Since AHP package works only in "full" mode, we must input data into both of these. We start with alternatives as alternatives will be also used in criteria perspective form comparison in leaf nodes. In alternative perspective we define the alternatives and possibly its properties. You can speed up the process using cloning functionality. That is effective especially when you deal with similar structure of properties and values. Just clone and edit to adjust for different name and property values of various alternatives.

You can clone the alternative by selecting it and clicking *Clone* button.

Defining properties is not mandatory. AHP package supports properties for purposed of preference derivation using mathematical functions, but AHP YAML Editor does not support that. One option you have is to define properties in editor, after finishing whole process of defining also criteria and all the comparisons, export results into YAML and manually add missing features. While not ideal, it is

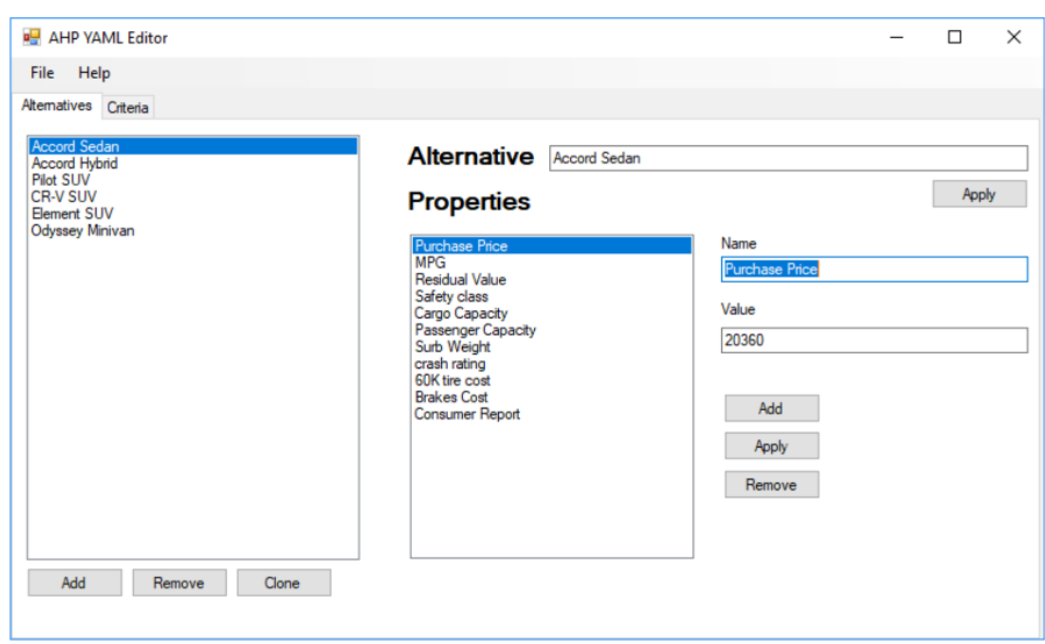


Figure 6.4: Setting up alternatives for decision problem in AHP YAML editor

still better than writing the whole file by hand. Of course, if you do not plan to take advantage of this functionality you can safely ignore this functionality.

In criteria perspective we create the hierarchy from root down to leaf nodes.

Before starting of preference mapping I recommend to define whole decision hierarchy. Technically it is not required, but such approach is logical, because you usually do not need to change this hierarchy, at least not structurally - most changes will be probably in preference statements. So it is logical to finish the structure and then focus on preferences and computation.

For leaf nodes, do not forget to click on check box *leaf node*. It changes way how the editor works with the node. If the node is not leaf editor provides criteria down in hierarchy for comparison. But if leaf node check box is checked, it provides alternatives of comparison instead.

After preparing everything, export the file into YAML. Export is necessary as the editor uses its own internal file format to store the data. The format is JSON based, it is humanly readable, but AHP package in R does not support this format.

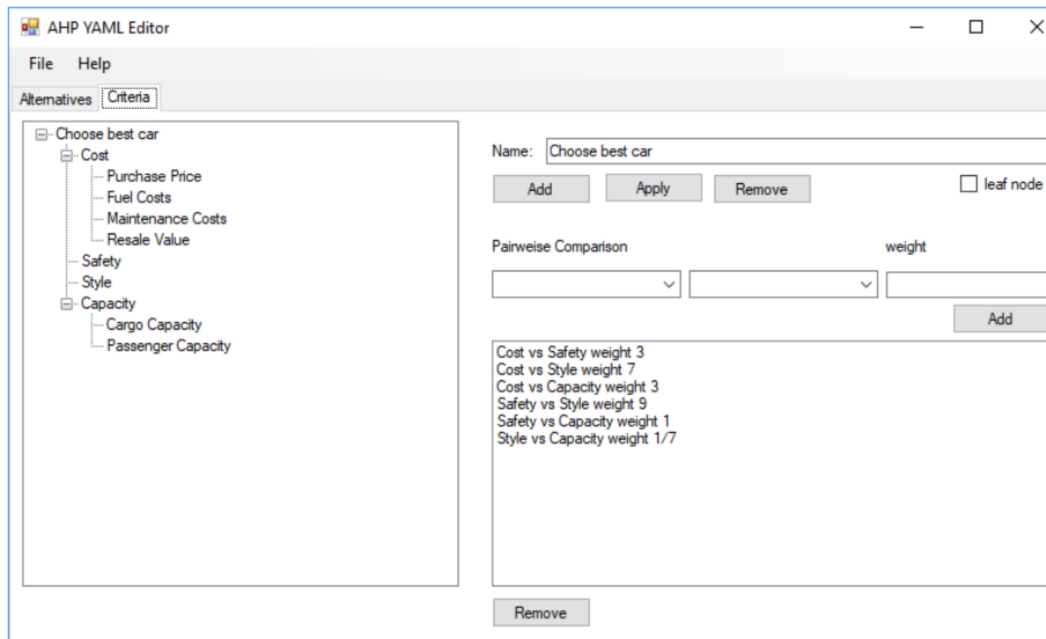
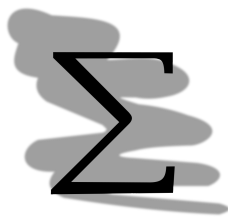


Figure 6.5: Setting up decision hierarchy for criteria in AHP YAML editor



Experiment

Try to implement the examples on your own in the editor a then compute it using R. Explore the results.



Summary

AHP method allows us to formulate the decision problem into decision hierarchy. The weights are estimated based on preference evaluation. Preferences are gathered for both criteria (for weight estimation) and alternatives (for alternative ranking). Method uses pairwise comparison, but at scale of $\in \langle 0; 9 \rangle$ and allows to compute consistency index we can use to evaluate randomness of stated preferences.



Self-evaluation questions

1. What differences are between AHP and ANP methods?
2. What differences are between AHP and WSM methods?
3. Explain how AHP evaluates randomness of stated preferences.
4. Explain system of preferential evaluation in AHP.

Chapter 7

Other methods usable in general decision situation



Study guide

In this chapter you will learn about other supporting methods, which can help in almost any decision situation.

After reading this chapter you will understand how to

- use mind maps
- use Delphi oracle method

know

- what problems are connected to data gathered by questionnaires and how to minimize them



Time required to study chapter

For getting through this chapter you will need at least one hours, perhaps more if you try to solve the tasks practically with software support.

7.1 Delphi method

Delphi method has been developed by RAND in fifties of last century as side product of research financed by US Air Force - Project Delphi. Project's goal was to develop methodology to identify possible targets of Soviet nuclear strikes on US soil. Basic problem was incomplete information. If we look at the problem logically we can form some assumptions. If nuclear war took place SSSR would intend to do as much damage to USA as possible. USA, since it has in depth knowledge on its infrastructure possesses the knowledge of which targets would induce such damages. But this knowledge is not public. Target choice by SSSR is limited from this point of view on what it knows about infrastructure of USA. That means that infrastructure with most damaging potential is not necessary most probable target of attack.

So the problem can be reduced to question on what exactly does SSSR know about the infrastructure of USA. The answer to that question could be partially on hard data. Since we are talking here about nuclear weapons, then we can presume the espionage and intelligence gathering could be good source of information, but with low probability to get actual target list, as such list would be highly privileged, and accessing it would be very hard.

Other than that there is always possibility to use expert opinion on capabilities of the adversary, his technical capabilities and derive possible answer from that.

In this environment RAND developed general method to bring answers in situation with limited or no hard data to support the decision. Since it is based on expert opinions the name Delphi method has been chosen after famous Oracle of Delphi from ancient Greece. Method tries to formalize intuitive understanding of decision situation by the experts. For its ability to work with minimal formal inputs and to always provide some kind of answers the method become very popular.

On the other hand, since the method works with minimum hard data and with expert opinions it is also less precise then methods based on mathematical models. Results are also prone to various errors and influences introduced during preparation phase by the decision maker (i. e. wrong questions), during the information gathering phase by decision maker (improperly leading process of gathering information) and experts (cognitive bias - wrong answers) and in evaluation phase again by decision maker (wrong interpretation of information).

Since the method has been so popular, it evolved in time and today there are plenty of variants of the method to choose from. We are going to discuss only basics of the method. For advanced information on the method we can recommend *The Delphi Method: Techniques and Applications* [51] by Lindstow and Turoff¹.

Since the method focuses on gathering information from experts. We need a way collect information from them, set some kind of process:

1. problem specification
2. expert selection
3. creating questionnaire (to collect the information from experts)
4. create documentation the the questionnaire
5. send the questionnaire to experts
6. evaluate answers
7. in case of identified problems, repeat from step 3. In additional iteration we often send the questionnaire together with its evaluation from previous iterations
8. we can perform as many iterations as needed - we repeat until we get the results we need (quality wise! not the answers we want!)
9. summarization of the results

While there is relatively high number of steps in the method's process, it doesn't appear, that the method is complex. Unfortunately practical experience with the method prove, that its usage is very resource intensive in both financial requirements and time required to realize it. As bonus to that, the results provided by method can be easily misleading if we do not ensure impartiality of experts, by improperly prepared questions or by improper choice of experts.

There are also numerous benefits of method usage. It performs even if we have only very limited information, or in situation that we have extremely (prohibitively) lot of information and are not capable to process them using for example common statistical methods. Method is good at eliminating extreme opinions of experts, is expert's pool is large enough based on following broader experts opinion consensus.

Elimination of extreme opinions can under certain circumstances be considered as a limitation of the method, because we can argue, that extremeness of the opinion does not necessary correlate with correctness of it (there could be correct extreme opinions).

If we were to choose one most critical factor regarding our chance on succeeding in using the method, it would be *choosing of experts* step. That is why we have relatively strict requirements on them:

- they must be experts on certain aspect of the problem or generalist with knowledge of the problem as whole (expert pool must have both types of experts)
- experts need to be independent
- experts need to be capable and willing to answer our questions

Requirements on expertise of the experts is logical and intuitive. In simple term we try to avoid the situation when we for all the trees do not see the fores (too much focus on detail) and opposite pro problem when we see the forest but missing the trees (our view lack necessary details). As

¹the book is provided on-line free of charge by its authors from <https://web.njit.edu/~turoff/pubs/delphibook/index.html>

a rule of thumb we need to model/gather information as generally as possible and as detailed as necessary. Choosing right "resolution"/detail level for the problem may not be apparent, especially in the beginning phases of the problem solution.

More complex is the question of *experts independence*. If we say independence, then we may refer to many very different things. Dependence can be for example financial. If it exist the expert is motivated to follow line of thought of the institution which is paying him/her. If the expert is being paid by us to answer our questions, he may be inclined to say what (s-)he think that we expect, or avoid pieces of information, which may be perceived by us as controversial or opposite to our believe system.

Dependencies can be much more sophisticated. The dependency in general can be on certain result or on usage of same information, which leads to same results. Neither of these is easily identifiable.

Good example of dependence on results are **Willingness to Pay (WTP)** and **Willingness to Accept (WTA)** studies when respondents may be trying to steer our decision outcome by certain answers, see separate Study guide in this chapter on these methods (marginal utility measurement).

Second type of problems is closely connected to imperfect information available. We know that for decision making rarely we have available all information we would like to have. If there is acute lack of information available the chance of interpreting available information wrongly significantly increases. If multiple experts all use exactly same data source to form their opinions, then while they are technically independent, in fact they may be even unaware of each other's existence, the will probably come to same or similar conclusion. User of these opinions may be swayed by "support" of such opinion by multiple experts. That support may not be grounded in reality. What if underlying information is false? Conclusions based on it will also be false regardless of how many experts say it.

Since the experts are technically independent, they have no way of identifying the problem. Only way of how to do it is by the decision maker, who organizes the information gathering to ask right questions to identify such problems, like for example examining on detail on what the expert's opinion has been based.

All answers collected from the experts need to be processed. The way we do it will significantly differ depending od type of the answer. Basic typology of the answers can be following:

- numeric
- ordinal (choice from several possibilities, number can be attached, but has no numeric meaning)
- free form answer

There are variety of problems that are common to all types of answers. These can grouped into two basic groups: 1) missing values and 2) dealing with outliers.

By missing values we mean experts answers which are either empty (expert for example forgot to write answer) or refused to answer. Reasons for refusal can be multiple from identifying his/hers own bias, or lack of knowledge to ethical reasons. Since in Delphi method the surveys are not anonymized, we can (and should) find exact reason of missing data. In case of simple forgetting, we can even allow the expert to amend the survey, thus solving the problem. But if there are other reasons - refusal to answer, we will actually need to deal with missing value.

Simplest way of how to deal with it is to omit such answer. Not always such approach is possible. For example the survey may be constructed in such way, that it gives proper answer only if it has been filled in whole, or some subset of answers is required. Missing value in such case may invalidate these answers too. By removing the answers, or whole surveys we also diminish to certain degree robustness of the dataset we are going to use to derive information from. If the number of respondents is low, then by removal of some answers may lead to devaluation of the explanatory value of the whole statistical population.

For numeric answers various imputation method exist, which may be feasible to use. Such methods always come hand in hand with some level of added noise to derived information. Analytic needs to decide, how to approach the problem considering all pros and cons. Further discussion of imputation problematic goes beyond scale of this course, as it requires also basics of statistics in addition to understanding imputation methods.

Another problem are so called *outliers*. By outliers we mean values which can be considered as extreme in comparison to other values for the answer. Say you let experts predict GDP grow a year from now. Most of the answers will fall for example into interval $\in < -1; 3 > \%$. What if one of experts predicts 15 %? Such value can be considered as outlying. To understand why it can be problem we

will need to address a question what do we we actually do with these values? How do we process them?

For numeric value we may be interested some statistical properties of the dataset for the answer, such as mean, median or distribution of the answers. Consider two dataset of GDP predictions $x_1 = -1, 0, 1, 2, 3$ and for x_2 we add extreme prediction 15 %, so $x_2 = -1, 0, 1, 2, 3, 15$. If we compute mean for the variables we will find that $\bar{x}_1 = 1$ but $\bar{x}_2 = 3.3$, which is significantly more. The single value did significantly influence the result. Consensus of economic experts in this case was prediction of weak economical growth, x_2 dataset with extreme value predicts very healthy economic grow, if we use the average as a measure.

Please note that median value is not so sensitive to these extremes. $median(x_1) = 1$, while $median(x_2) = 1.5$. This is a classic example, but one that we see every day around us. Difference between average pay and median of pay. Differences in mortality due to covid infection - average for whole dataset, compared averages for age stratified dataset. Similarly in field of security - average damages vs peak damages or expected yearly loss. We could continue on.

With extreme values, removal is an option, with same impact on robustness of the dataset as in case of missing values. First step is usually identification of such values. Good graphical approach is Box Plot, we could also use histograms to see value distribution and "eye ball" its type. With this knowledge we can identify the extremes and start thinking on what to do with them.

Some inspiration for the extremes identification can be found in existing literature on Delphi method. For example relatively popular is the upper and lower 25th percentile as limit for identification. As this value can be too low or high for the purpose, we need to decide on these limits on case per case basis.

When evaluation choices from predetermined variants (*ordinal choice*), we can't compute averages etc. Or more specifically we can, but the results are meaningless. When evaluating such type of answers we should focus on number of occurrences of the opinion. Is some choice preferred significantly more than the other? Using percents for the purpose simplifies the process. Results then are relatively easy to understand. We can also use pie or donut charts to visualize answer distribution. Bar charts are also usable for the purpose, thou they are less popular.

Free text answers (open questions) are most problematic to process. There are no statistical tools, charts, etc. available to kick start our analysis. We usually need to process the answers by hand and derive additional information from the answers. This information can be processable by statistical methods, or not, depending on the information provided by experts.

Evaluation often tries to find common properties in the answers. Does some idea keep appearing in the answers of the experts? This can be interpreted as a consensus among the experts (something the experts agree on). Question of consensus is one of more problematic areas of the method, we will address it also separately later in chapter.

We can also be interested in some different/interesting answers. Expert may communicate some piece of information or angle, we originally did not think about. Maybe we can identify the need to review the survey and add or change the questions. In other worlds we may have identified a gap in our understanding of the problem.

Since the free text answer is very flexible, open, the surveys always do have some questions available in this form. Minimally as a place to facilitate the communication between the expert and analyst.

Disadvantage of the open questions is the time and effort required on side of both analyst and respondent to deal with them. Will the respondent have time to answer it properly or will (s-)he be forced to cut the corners? While these questions are important and to certain degree always present in the questionnaires, they should form only minority of the questions.

The basic message we are trying to is that you need to truly understand the nature of the data you are processing. Are you using proper measures? What do the results tell you? How can we interpret them?

Last question we are going to discuss around the Delphi method is problem of experts consensus. We actually saw symptoms of consensus or lack of it in the data in paragraphs above. If there is no clear consensus, blindly using average, mean or any other measure will probably be not helpful as experts either understand the question differently, or there is not one "right" decision.

To effectively process data, some kind of consensus must be reached. To increase chances for consensus one can prepare:

- the survey in such way that it is clear what kind of information is needed
- in depth documentation to the survey, so that the expert has a reference point to lean on

- indoctrination seminary where the experts are taught what is expected of them in survey (rationale of the survey)
- multiple rounds of survey. Each round includes all information gathered in previous round. Previous round serves as limiter to answers.
- if nothing else helps prepare seminary with intend to find a consensus - experts will discuss the problem and decide on answer.

As you can see, reaching consensus is not an easy task and is one of reasons why Delphi method is so resource intensive.

7.2 Cognitive bias

One of reasons why Delphi method is so hard, is a keen ability of the human brain to come almost instantly to conclusions, with almost no requirements on input data. Modern science sees this ability as a result of adjustment of our ancestors to survive in hostile environment. Closely connected to it is *fight or flight mechanic*. This refers to decision whether to stand up and fight or try to flee in presence of possible danger. This decision needed to be instant to be maximally effective and it doesn't need to be that precise as most signals of danger have been false alarms.

In case of false alarm, the decision does not matter as a danger does not exist. But in case of real danger immediate reaction has potential to save life further strengthening the mechanism. Today this ability is not so useful for normal life but it is part of us and we use it everyday.

From point of view of thinking there are two basic types: 1) fast (immediate) decision making and 2) slower analysis based decision making. Daniel Kahneman described this in his bestseller *Thinking, Fast and Slow* [48], where he summarizes 40 years of research with his colleague Amos Tversky in the field of what we today call *cognitive bias*.

Main idea of the book is that we all are primed to behave in certain way. Our split second decision may seem random, but they are determined a lot of things which basically force the decision. That all is part of "fast" thinking. In that sense we all are biased. Since results of fast thinking are wrong lot of time, we need to actively pursuit "slow" type of thinking, which is analytical, makes lot of effort but also has much better results.

There are many types of cognitive bias. From the better described ones we can for example name *central bias*, which explains problems with ordinal choices. Central bias states that people tend to eliminate extreme values. If we have five point scale $< 1; 5 >$, and we expect experts to equally distribute their opinions in this scale, it will not happen as 1 and 5 options will have significantly lower preference rate. If we were not aware of this our logical conclusion would be that a true value is somewhere in $\in < 2; 4 >$ instead of correct conclusion that the value may be in whole interval and that in fact there might be no "true" value (random selection).

Decision support methods were designed to support this slow, analytic thinking and minimize effect fast thinking. Kahneman was awarded for his research by Nobel price for economy in 2002. There are also broad applications of the cognitive bias in other research fields. In economy Richard Thaler developed whole sub-field of economy called *behavioral economy* to explain certain behavior of markets and societies, which were previously unexplainable by traditional economy and perceived as irrational and thus to be considered being an anomaly and ignored. Thaler proved that not only this is not a case, but by adjusting for example policies we can use these to influence the society.

Since influencing can have negative connotation related to all oppressive regimes in their everlasting pursuit of perfect societies supporting the regime, he uses different name for these influences: *nudge*. Nudge is small, unobtrusive measure which motivates the people to make a good decision. Nudge is also a name of a book [62], where Thaler describes his research and onset of behavioral economy.

For his research Thaler was awarded in 2017 also by Nobel price for economy.

As you can see, the problematic of cognitive bias is very broad topic. Present state of the research on topic in form of cognitive bias typology map is available on fig. 7.1.

Before we move to another topic, let's explore some more how the cognitive bias influences Delphi method and survey construction and evaluation in general. There are two notable biases:

- hypothetical and
- strategic

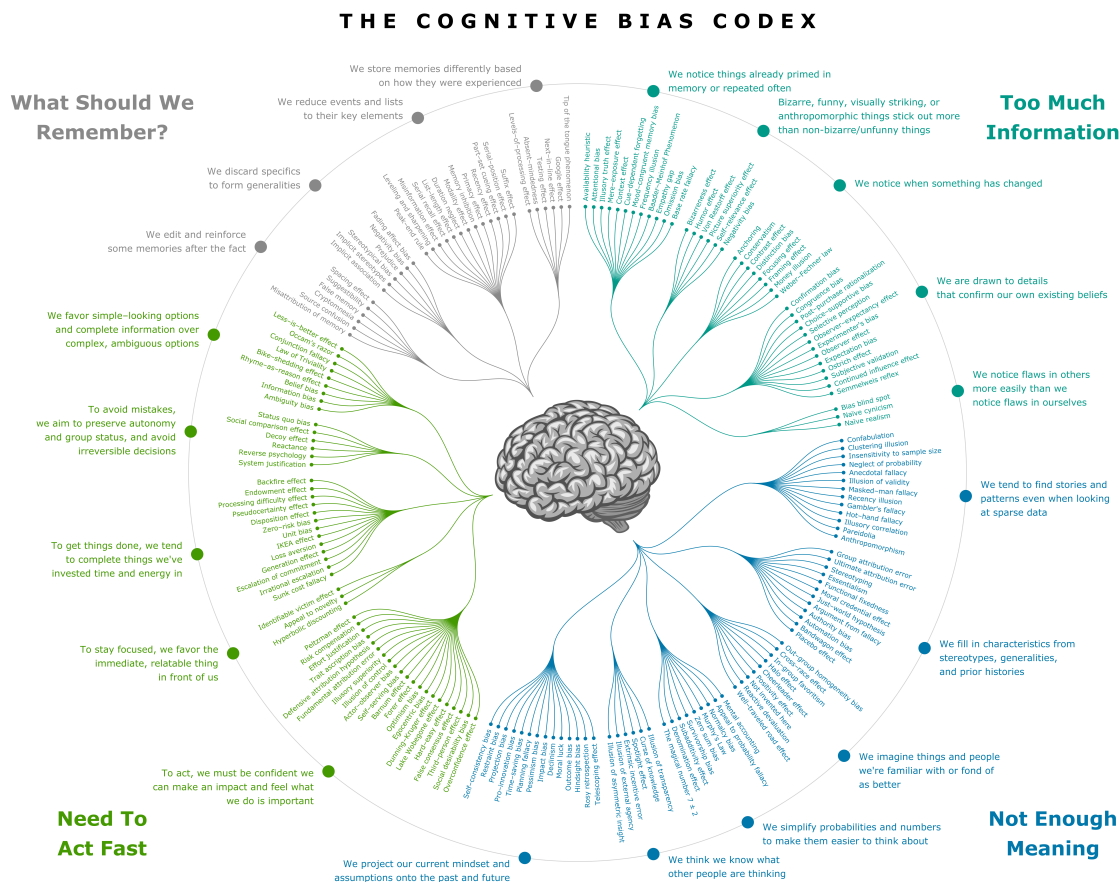


Figure 7.1: "Codex" of more than 180 forms of cognitive bias (courtesy of [52])

We in fact touched the topic briefly when talking about the answer evaluation, though we had no name for effect, until now. *Hypothetical bias* happens when the respondent sees the question as purely hypothetical, cannot happen in real life situation. In such case the respondents do have a tendency to underestimate the answer or not put much effort into it. This may lead to heavily flawed answers.

We can defend against this bias by specific formulation of our questions. Especially important is proper framing of the question and supporting it by examples which happened recently. Goal is to introduce expert to the situation and possible gravity of the answer. This should motivate him or her to put necessary effort to the answer.

Supporting documentation with in-depth rationale behind the questions together with indoctrination seminars can also help. The seminary is recommended also in case the questions are complex and we anticipate problems. Under such conditions investment into seminary would be a rational decision as it has potential to prevent these anticipated problems or at least minimize them. Early investment may limit number of Delphi method iterations analyst will have to do, thus decreasing overall time and money required.

The respondent (expert) will need to study the documentation and participate of seminary. That may limit number of experts available, especially if we do not pay them.

Strategic bias is almost exact opposite to hypothetical bias. In strategic bias the respondent not only believes, that the situation presented in survey is real, but (s-)he also believes that something can be gained by certain type of answers. As to what motivates certain types of answers. It may be the effort to make him/her-self more attractive to potential employer or benefactor. In this case the expert may be motivated to give opinions which conform to image of the analyst organization, or simply what (s-)he believes the analyst wants to hear.

Another example can be studies conducted on endangered regions to map resident's sentiment toward possible protective measures against floods. If respondents believe that certain types of answers will lead to change in policies and public investment in protective measures. Since side effect of

such investment would be increase of utility (value) of all property present on former flood plain, respondents would be heavily motivated to give answers which improve chance of such investment.

Preparing survey is very hard. In addition to the wording of the questions themselves, it is also necessary to address whether there will be a need to create supporting, training materials, organize training for respondents, etc. The form in which the survey will be conducted is also an important decision. Will the forms be sent out in paper form or electronically? One can also opt for face-to-face consultations, either by telephone or face-to-face. Will it be one-on-one or organized more as discussion with goal to find consensus.



Methods based on marginal utility measurement

In the text we have been discussing Delphi method and survey realization. There is variety of other methods based on similar principles, we can use for decision support. Methods like **Contingent Value Method (CVM)**, **Travel Cost Method (TCM)** and others are based on measurement of marginal utility and use surveys to collect required data. Difference to Delphi method is that we are not asking experts, but general public in certain area of interest. It may be city, region or even relatively small area defined by polygon of expected impacts of some kind of accident.

These methods are also based on measurement of **WTP** or **WTA**. WTP measures how much are the residents willing to pay to avoid or minimize the risk. We can safely presume, that higher level of endangerment will be, the more the residents will be willing to pay. WTA is based on similar principle, but it measures how much needs to be paid to residents so that they would be willing to accept risk existence. Such payment may have different form, for example rent discount, tax credit, subsidies to realize protective measures, etc.

CVM is being used for measurement of so called non-tradable goods. It can help us when dealing with questions of environment, or in public goods in general, where there is no clear monetary value for such goods (value of existence of certain endangered species, ...). **TCM** can be used to establish "price" for recreational areas, nature reserve, etc. We estimate the price based on willingness of the tourists to pay for visit into analyzed area. The more the visitor is willing to pay, the higher value area has.

Unfortunately we have no room to discuss these methods in detail, there is lot of studies, which describe use cases for these methods and can be recommended as further study material, see for example [46, 58].



Questions

1. How Delphi method works?
2. What kind of answers can Delphi method support and how do we process it (are there differences in processing between the types of the answers)?
3. What is cognitive bias?
4. What is difference between hypothetical and strategic bias?

7.3 Brainstorming and mind maps

Brainstorming of one of most used methods for analyzing (not only) decision situation. This method works best for smaller teams trying to find all aspects of the given situation. As such it is especially useful in beginning stages of decision making.

The method itself is not usable to identifying optimal solution of the problem, but rather to describe it or identify all important aspects of it. The solution itself must be reached using different analytical method.

When brainstorming, there is always one person in charge, who set the stage for brainstorming session, defines the problem and writes down opinions and ideas of other team members. On black board or in the computer then gradually grows network of subject and ideas with main topic in the

center of it. Brainstorming team should be broad enough to cover whole problem domain of the topic on question.

During the session all ideas are being written down without prior evaluation. If we immediately performed evaluation, it would have negative impact on level of participation (number of generated ideas). Remember, that it is easy to filter out bad ideas afterward. Afterward is also safe point of view number of generated ideas.

Some of more exotic ideas, which look weird or even stupid at first sight, may just be misunderstood or may function as a source of inspiration that can be elaborated.

All participant have same rights, nobody is preferred. Leader and speed (s)he writes down the ideas is limiting factor. Because of that the leader must also function as moderator and specify who will say his idea.

Since there is connection between number of participants of brainstorming session and number of ideas they generate, it is not possible to increase number of participants without limits. For complex problems multiple brainstorming sessions with different set of participants may be necessary, leading to multiple networks. Separate integration brainstorming session may be good idea.

There were some studies on correlation between number of participants and number of ideas generated, see fig. 7.2 and 7.3.

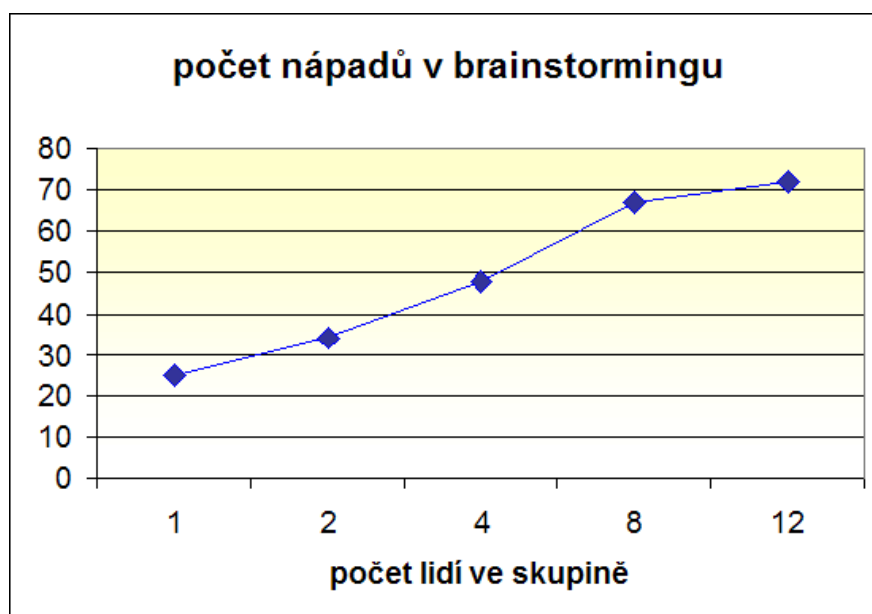


Figure 7.2: Dependence of number of ideas generated on size of group in brainstorming (courtesy of Osuský and Fajmontová [1])

We can generally say, that to certain point number of ideas grows with increasing number of participants. Beyond that point the time each participant has to voice his/hers opinions diminishes significantly and the yield of ideas stagnates or even shrinks. With limited time only part of knowledge of the present participants is mapped, which would limit usefulness of the method as starting point of decision making.

We can conclude that there is no single number we can use as optimal size of the group, or number of ideas we absolutely need to generate in brainstorming session. It is responsibility of the leader to decide on all organizational aspects of the session. It's the experience and ability to detect subtle nuances of "energy flow" during the session to conclude whether the session was successful, its length, or whether to organize subsequent sessions with some subgroup. Did all participants have possibility to express fully their opinions?

We can also use computers as support tool for brainstorming. Modern conference room have either large displays or allow to project screen using data projector. Writing down ideas in the computer introduces flexibility to the session, which was not available before. Is some part of the diagram getting out of hand size wise? Reorganize the diagram by dragging and dropping its parts, or even create sub-diagram to separate more complex parts of it. Use links, icons, free form notes ... None of it is

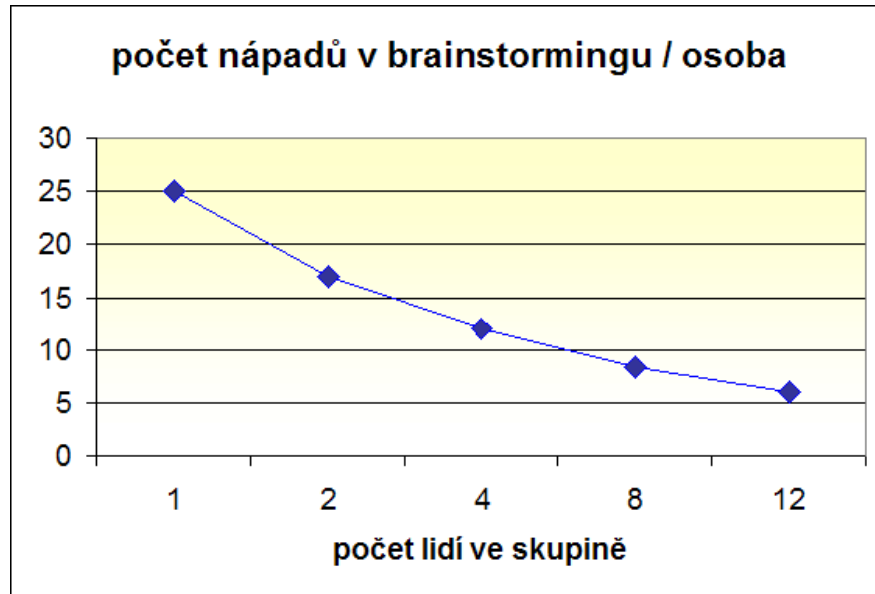


Figure 7.3: Dependence of number of ideas generated per participant on size of group performing brainstorming (courtesy of Osuský and Fajmontová [1])

available when using blackboard.

While we can use general purpose graphical program for that, it is more efficient to use *mind mapping software*. Mind maps (as a technique) have originally developed by Tony Buzan [36] as a tool to better understand problems, make notes, learn new things, etc. His original idea was to use mind maps more as self-help tool. But with proper software packages, we can successfully use the approach to support our brainstorming sessions.

While it is possible to draw mind maps in hand, see fig. 7.4 for artistic take on it, software is usually preferred.

There are many software packages available, some are paid like MindManager [19], some are available free of charge FreeMind [9]. In comparison with hand made mind maps, the software generates more visually unified maps, maybe even a little bit boring. But we are able to generate them fast, which is huge benefit of using software.

There are significant differences between creating mind map and brainstorming session. Usually only one person works with mind map at time - the person sitting before the computer. In comparison brainstorming is group effort.

Software also allows to create interactive mind maps, link them together, or link them with other resources - be it documents, images, videos.

As you can see on fig. 7.5, we are starting with single noun describing our problem. From it we derive various aspects of the problem and describe it again by noun. The number of the nodes in the map is virtually unlimited. Even large mind map is easy to work with, because the nodes can be folded do hide large parts of the map, when you do not need it.

We can link different mind maps together to form huge, self describing structures. Because we can also link leaf nodes to the documents on local computer or on network (LAN or Internet), the mind map is not limited only to interconnected brainstorming nodes. Better orientation is also possible by using icons visually describing the node.

FreeMind as a tool written in programming language JAVA is capable to function as JAVA applet and allows for diagrams to be viewed inside web browser over the Internet. Of course, that on the other hand also means, that you will need the **Java Runtime Environment (JRE)** installed. Fortunately Oracle provides JRE free of charge for many platforms at its dedicated web page [14].

There are available many more free or proprietary software for this tasks.

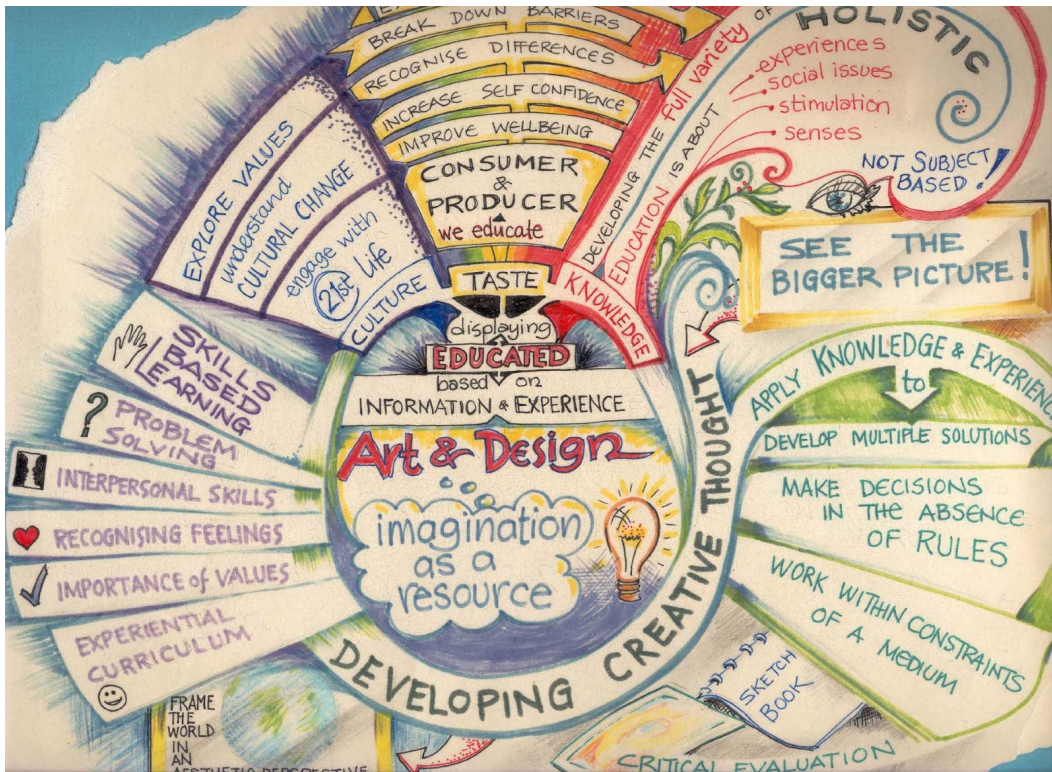


Figure 7.4: Mind map of Art and Design (Courtesy of [18])



Mind map example

Let's try to construct short mind map around the theme – modelling of decision processes, see fig. 7.5.

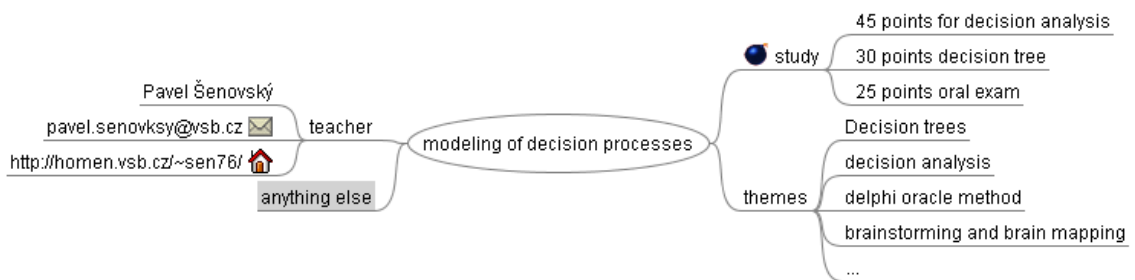


Figure 7.5: Mind map example



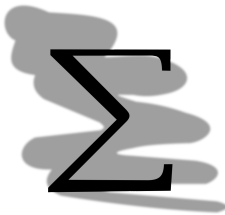
Create mind map

Download FreeMind (or any other freely available tool for the task) and create mind map of emergency: accident of tank transporting methane under pressure.



Questions

1. What is mind map good for?
2. When in decision process should brainstorming session take place?
3. What is the role of team leader during brainstorming session?



Summary

Delphi method is one of methods we can use in case no or not enough hard data are available for our decision making. The method is based on problem assessment by experts. Analytic prepares the questionnaire to guide the experts through the problem. The questionnaire is sent to selected experts in problem's domain, who will answer the questions. Analytic then collects and processes answers. In case no consensus is reached by the experts, another iteration of the method is needed. In this case the existing answers are provided to guide experts in their thought process. Since the method is expert based, it takes a lot of time, and has large requirements on quality of supporting documentation and abilities of analytic to process the answers and interpret them correctly.

Brainstorming is well known methods usable for variety of problems, especially in beginning phase of finding solution to them. We can use it to better specify or understand the problem, its various aspects and summary the information in one place. Brainstorming is a team method, during which leader writes down ideas or opinions of brainstorming session participants on the topic.

Mind maps are usable for organization of notes, ideas, information, etc. Each map has a main topic of the map in its center. Map then works on this topic and develops it to the sides in form of trees of ideas/notes describing given aspect of the topic. We can also use mind maps as a support tool for brainstorming. Mind map is used as replacement to blackboard.

To improve speed of mind map creation, the maps are usually visually standardized, when using software to create them. Most maps today are created using such software.

Chapter 8

Surveys



Chapter outline

In this chapter we will discuss the surveys preparation and its evaluation using statistical methods. Surveys are one of most often used methods for information gathering on almost any topic across research fields. That also means, that you should know a thing or two about them too.

After finishing this chapter you should know

- what are the surveys and how to make them (at least the basics of it)
- what kind of problems can we meet when evaluation results of such surveys
- what methods do we have available for the evaluation



Time required for study

To study topic presented in this chapter you will need approximately 3 hours. Thou if you are not confident in basics of statistic you may have problems requiring some more time and studying additional statistical related study materials. Only basics of statistic are required so any textbook on the topic will suffice.

Surveys are being used extensively almost every day, for variety of problems. We can see its results in the news, Statistical offices of the states use them and they are also often being utilized as an input for theses, dissertations, research and many other things. So, since we realize these surveys so often, it is an easy thing to do, right? Well, not really, while creation and evaluation of the survey is not a rocket science, it is easy to do mistakes which can lead to errors in interpretation of the results, provide misleading or strain wrong answer.

Also if organization and processing of surveys was so easy, there would be no need for specialized firms on the problematic to exist. Since such firms not exists, but they strive today more then ever, we can presume that surveys can be complicated. So let's start from beginning - by creation of the survey.

8.1 Introduction to survey creation

Basic obstacle in surveys, both creation and evaluation, that for uninitiated lot of problems is not obvious - intuitively derivable, apparent on first sight. To understand them we will need to define some basic terms first.

First term is *population*. In statistics the term has specific meaning which is different then the population definition in for example course *Civil protection*. In civil protection we see population as residents and persons working (or being) at given time at given place, be it state (state population), continent or region. In context of surveys we define term differently as part general population, we are interested in from point of view of the properties being explored by the survey. So we are never speaking about everybody (entire population). We are always working with some subset of it.

The survey itself is also usually a "*sample survey*" - that is, data are not collected on the population of interest as a whole, but only on a sample of it, which we refer to as a *representative sample*. It is not that the full sample of the population cannot be measured (using a questionnaire), but it is done only in limited cases, e.g. when the population of interest is small to make the survey financially manageable (e. g. a survey of regional directors of the Fire Service). A full survey of the population of a country is carried out, for example, by the **Český statistický úřad (Czech Statistical Office) (CSU)** as part of the Population census [66]¹. However, this is a process so demanding, even for state, that it can only be carried out once every 10 years.

For purposes of this textbook, we will see representative sample as such sample of the population, whose characteristics, we are interested in, are statistically similar with population as whole. Meaning, that we can use results of the survey we can describe some characteristic of the whole population.

We can describe the situation in graphical form, see fig. 8.1.

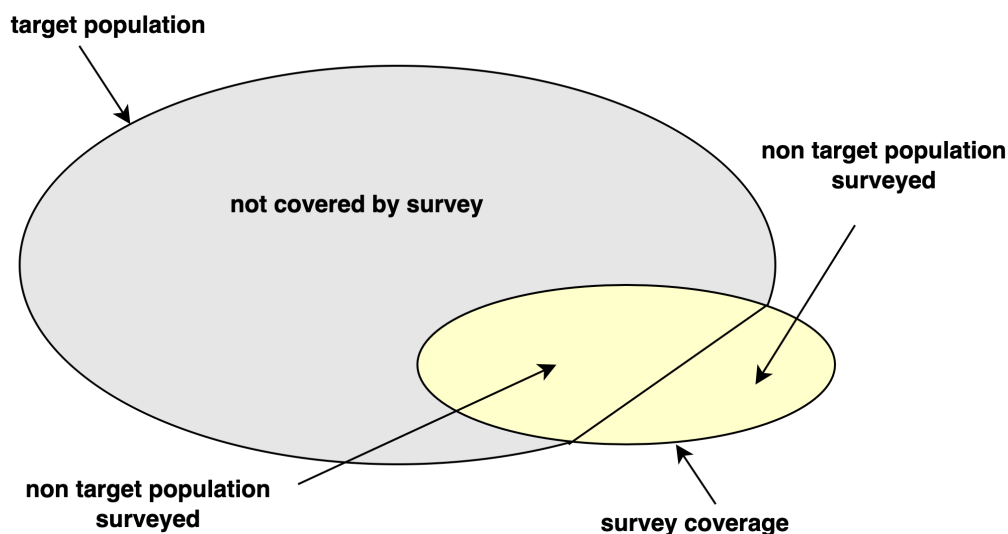


Figure 8.1: Sampling frame for the survey (adapted from [34])

From fig 8.1 we can deduce some problems. The survey must be prepared in such way to target target population **only**. So for example if we conduct survey on awareness of public safety in selected part of the city we are interested only in respondents who either live there or work there (generally spend significant time in there). But if we randomly ask people on the street, we can also ask people who do not belong to this target population. For example they can be tourists, or they randomly go through the area for shopping. If we were not able to identify such respondent, using such answers would influence results of the survey as whole. If there was a lot of such respondents, it could even invalidate the results.

Opposite scenario, when our sample size population is not large enough to be reasonably representative of true population, is also possible. One of first question, we will need to answer would be, how large the sample size should be to be representative. This question needs to be answered ideally in advance. We can also compute it afterward, but there will be no guarantee, that the collected sample size is large enough. Question then would be, what to do if we compute that our sample size is too small? Are we going to throw out results, accept larger then anticipated error, try to collect information from more respondents?

Preparation is a key in this case. Computing required sample size will allow us to plan our survey campaign including required resources, and will also tell us when we can stop.

The survey itself is realized in following steps:

1. develop survey
2. data collection (realize the survey)
3. data edit
4. data correction (dealing with missing values)
5. data evaluation

¹Census of People, houses and housing

6. publish results

Survey design

The survey always tries to achieve some kind of goal. So the first thing we need to do is to answer the question *what we need to find and why*. Answer to this question will serve as basis for *variable* selection, whose value we want to find using survey. We formulate our questions to find them.

Some survey prefer to also formulate working hypothesis before realizing the survey. Survey then has a potential to confirm or disprove it. Reason for this is that without such hypothesis, we will see pattern in data anyway. Intuitive approach would be to describe the pattern and formulate hypothesis explaining it. So, we would be using inverse approach. Such approach is not recommended. Problem is that it is very hard to prove, that observed patterns are not a random "noise".

On the other hand if we specify hypothesis, that we will see certain pattern before realizing the survey, then realize it and see such pattern, chances that it is result of random selection (noise) is much smaller than in previous scenario.

Not all surveys require such hypothesis. The goal of survey might be to find previously unknown distribution of some variable. For example we may be interested in some preferences mapping in population, or how many people have certain knowledge, or equipment, etc. In such a case, however, even greater demands are placed on the analytical part of the survey, to ensure that the data collected are appropriately "mined out".

There are variety methods to use for such purpose with entire sub-field of informatics called *data mining*. Discussing data mining goes beyond scope of this course and textbook, but you can get acquainted with some of these methods in courses like Statistics, Informatics in Security, Artificial Intelligence and others.

There are two basic types of variables:

- target and
- supplementary

Target variables are directly related to survey's goal. Quantifying (finding for non numerical) these is basic requirement to consider the survey to be successful. It is a must, without these we will not be able to meet stated goal of the survey.

Supplementary variables are not directly related to survey's goal, but these variables can help to stratify the dataset compiled from survey, or to identify respondents who are not part of target population. Logical next step would be to remove such respondents from the dataset.

Typical examples of supplementary variables are age, sex, marital status, relation to place (resident, going through, ...), employment, specialization, etc.

The questions need to be formulated in such way, that they are short and easy to interpret for the respondent (amateur). This requirement is especially important if the respondent fills in the survey unassisted. Typically that is the case for on-line surveys, but sometimes it is also case for in attendance surveys. Clerk only serves to give out and collect survey forms.

The respondent does not have anybody to ask if (s-)he has some question. Respondent will deal with the question somehow in the end, but there is no guarantee, that question's interpretation will be same across the respondent. Which is undesirable property, especially if we are not aware of it.



Cognitive bias

When formulating the questions for survey, we need to also consider the possibility, that the respondent will not be impartial to the question, or more specifically answer to this question. We are already aware of the problem as we discussed it in *Delphi method* section of previous chapter. So if the term *cognitive bias* seem unfamiliar to you, now its time to revisit the topic.

To properly fill in the survey even such minor things like the order of the question may be significant. The survey as whole must function in wholesome/consistent manner. That means that we need to group the question based on its topic and order them to form certain "story". We need to remember, that previous questions will influence somewhat the respondent. That may be desirable, if we constructed the survey to utilize it, but it may become major source of errors, if we didn't.

When designing survey we need to also understand that our respondents are usually unpaid volunteers, who value their own time. That means that with increased number of question and needed

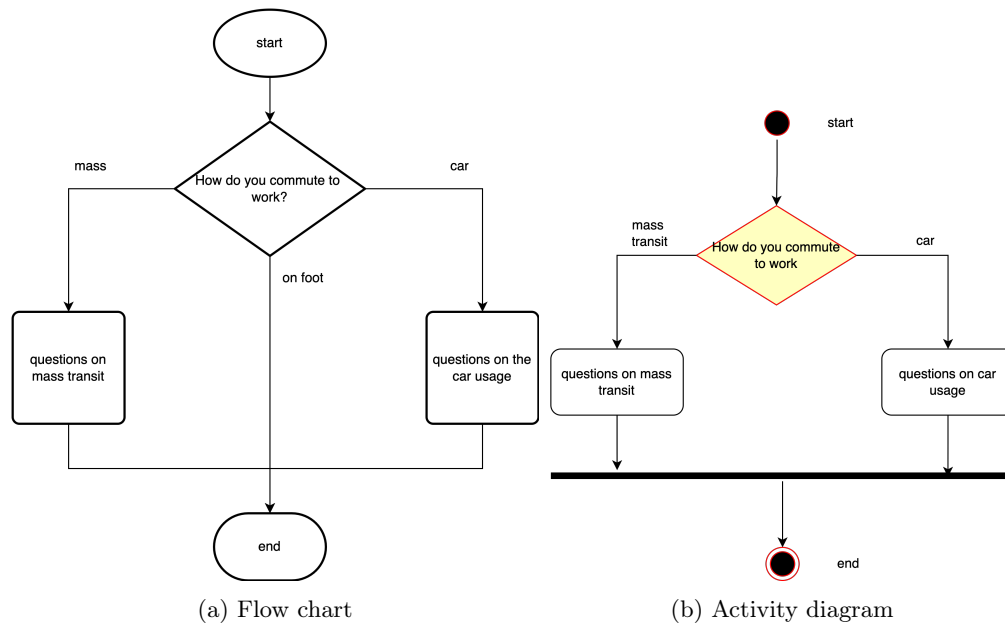


Figure 8.2: Flow chart vs UML's activity diagram

to fill them, willingness to participate will decrease. We can also expect the respondents to be less inclined to answer questions perceived as personal or sensitive. Examples of such could be age, health, income. That is a case event if the survey is anonymous.

We can somewhat alleviate negative perception of such question by putting them in the end of the survey and marking them as voluntary. Adding option "I prefer not to answer" may also help. It will also allow us to easily filter out such answers when evaluating them. If we do not present escape option to the respondent, (s-)he may be more inclined to choose random or not true answer. Which we would not be able to easily identify, thus introducing further error into the dataset.

Data collection

In this phase we have prepared the survey and start with its deployment. The preparation may be short, for example we may need to print the survey and publish it on Internet. But in case we are planning to perform assisted survey, the preparation will be more complicated. We will need to select the future assistants and educate them on rationale of the questions asked and the way they should deal with expected problems.

Apart from explaining the meaning of the questions (and answers), the training can also focus on the way of filling in the questions. For example some answers may narrow down number of questions - in other words some questions in context of our knowledge about the participant gained answering the questions may make some questions meaningless. For example if we learn in one question that the respondent commutes to work using his/hers car, that it would be pointless to ask on quality of city's mass transit system.

Finding possible walkthroughs through the survey has also one positive feat in that it limits number of question the respondent needs to answer. But is the walkthrough is set in wrong way it may lead to state when we will force meaningless questions on our respondent and possibly invalidate data gained that way.

To better understand various paths through the survey, it may be good idea to visualize it. If the survey is simple, such visualization will bring only small or no benefits. But for complex surveys with interconnected question visualization can be invaluable tool for both design and validation of survey implementation if we are using software or on-line form.

We can use flow chart [13] or activity diagram from **Universal Modeling Language (UML)** [54]. Simplified example of possible survey traversal are available on fig. 8.2. Resulting diagrams from flow chart and activity diagram are similar, but not the same.

As you can see on fig. 8.2 we can use UML's activity diagram as a logical extension of the flow charts. UML is also much more modern standard supporting variety of other diagrams for various aspects of system analysis.



UML

You can learn basics of **UML** language in course *Artificial intelligence in security*, you can choose during your study.

Data editing

By *data editing* we can mean a lot of things. We can mean a process in which we transcript data from paper forms into electrical for for further processing. Editing may also mean correction of detected errors.

There are three basic types of errors in surveys:

1. range errors
2. consistency errors
3. errors in traversal of the survey

We can at least somewhat check validity of the variables against in advance set limits for the variable. If the variable's value is outside these limits, we talk about the *range error*. For example age over 100 is improbable for respondent, but possible, age over 130 is unfortunately not possible at this time. If the responded was expected to choose from preset list of options, we can check, if (s-)he really did so and didn't create another type of answer on his/her own.

By *errors in consistence* we mean the situation when the survey has been filled in, but the answers contradict each other. For example age: 8, marital status: married is in Czech Republic basically unheard of. One or both answers are probably wrong. These contradictions need to be detected and dealt with.

In fact, we have only limited number of options available, if something like that happens. If the survey was not anonymous and we have contact information, we can do follow up on survey and amend it. If the survey was anonymous, than we can only decide to throw out contradicting answers or all answers by the respondent.

Errors in traversal happen when the survey is wrongly implemented, or the assistant makes a mistake in guiding the respondent. This result in situation, when some questions which should have been answered were not asked, while others we were not interested in were answered.

We deal with these errors in similar way as with consistence errors - so either amend it if possible or throw out part or all answers for respondent.

For this type of error it is also important to identify reason for error. Was the question sequence wrongly encoded? If the survey continues we might want to correct on-line form and rules for questioning themselves. For assisted surveys human error is possible. If we see that the assistant has higher then usual level of errors in surveys it may be good idea to train him/her further or to fire him/her if the errors are presumably result of negligence for example.

8.2 Basics of survey evaluation



Please note - examples used

The examples used in this section are only illustrative, they are intended for demonstration purposes only of the evaluation process.

The moment we get the task to evaluate the survey, most of the people images something similar to graphs on fig. 8.3 [41].

There is nothing inherently wrong on using the charts, on the contrary - we recommend it wholeheartedly. Graphs on fig. 8.3 are taken from thesis of Ing. Dratva [41]. The survey has been realized

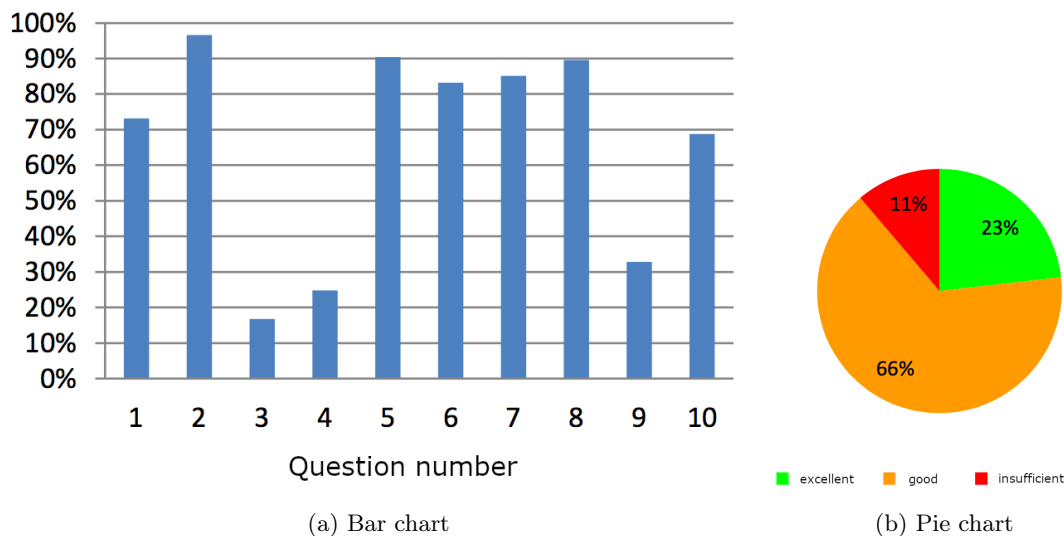


Figure 8.3: Bar and pie charts (courtesy of [41])

to estimate level of knowledge in area of civil protection and crisis management among the secondary school students and among normal population.

Bar chart represent percentage of correct answers in the questions. So what is the problem, if there is a problem? Well, the graph itself should be at least somewhat self-sufficient. By self-sufficiency we mean, that if we look at the chart we should be able interpret results it presents without need to study accompanying text extensively. It should communicate clear message.

We have only limited amount of tools available for the purpose:

- graph label (i.e. *Fig. XY: Results of civil protection and crisis management evaluation among secondary school students in 2014*)
- main heading of the graph (i.e. *Knowledge test: civil protection and crisis management - secondary schools*)
- labels graph's axes (i.e. X axis: *question number*, Y axis: *correct answers [%]*)
- legend

In context of what we know about the survey carried out we could replace bar chart from fig. 8.3 by different type of bar chart, see fig. 8.4. We can use this type of graph also in case we need to split the result into more target classes then 2, as in our illustrative example.

The graph should not be only way we present the information as it doesn't contain all the information for reader (report user) to form adequately his/hers opinion. Data should be presented also in tabular form. Survey results in graphical and tabular form are usually the most interesting part of the report and also most useful. Graphical form is often more connected to the intend of the survey, its goal. But it is not useful to think, that we know in advance full value of collected data. Perhaps it can be used in the way we did not think of. Or they may present additional values when integrated with other datasets.

So the reader might want to use the data in different way. If done properly, part of original data can be reconstructed from the graph. But it is lot of work and there is large potential for errors. If the data are also available in tabular form, such reconstruction is not necessary.

Therefore we recommend using both forms, thou data tables may be stored separately from graphs in report annexes or even entirely separate from report itself using document repository. In this case report must contain information on how to reach the underlying data.

We can demonstrate the problem on question 1 from fig. 8.4, try to estimate the percentage correct answers. Was it 72 % or maybe 73 % of respondents? Without underlying data we will never know for sure.

If we process the results separately for secondary schools and adult population, similarly to the fig. 8.3 or 8.4) we can estimate level of knowledge prevalent in area of civil protection and crisis management in these target populations. Requirement for such stratification is existence of variable

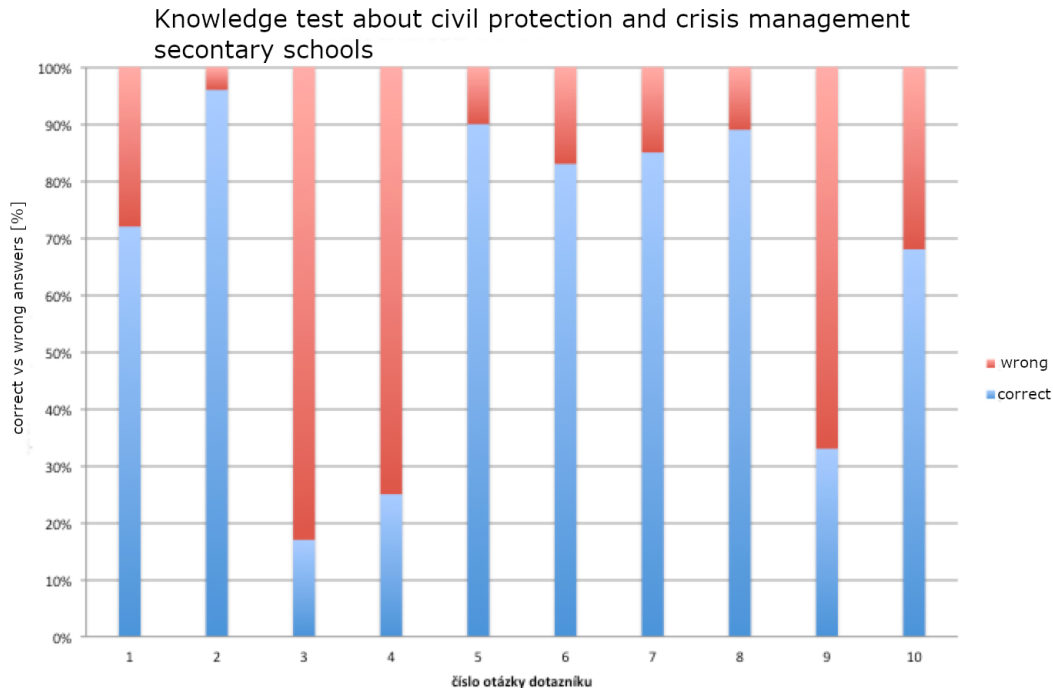


Figure 8.4: Results of Knowledge evaluation in civil protection and crisis management among secondary school students in 2014 (data: Dratva [41])

in dataset specifying whether the respondent is student or adult. That was a case of the study we are using for demonstration purposes.

What if we need to estimate properties of the population as whole? Such population has both students and adults. Can we just use the whole dataset without stratification prepare graphs and be done with it? Well we can do that, see tab. 8.1, but what does it tell us exactly?

Table 8.1: Students vs adults (data: Dratva [41])

evaluation	students [-]	students [%]	adults [-]	adults [%]	total [-]	total [%]
excellent	29	23	8	31	37	24,6
good	82	66	13	52	95	63
sufficient	14	11	4	17	18	12

The *total* value is very problematic in tab. 8.1, because statistically it does not make any sense. To derive correct conclusions from data and integrate these values to form meaningful total, we also need to consider:

1. size of population sample - in our case 125 students and 25 adults
2. size of real population (we are interested only in the target population, to be specific) - students on secondary schools in ČR in school year 2013/2014 313 130 [53], number of adults in ČR is 8 665 578 according ČSÚ [65]

So according to this information we covered by survey 0.04 % of secondary schools students and 0.0003 % of adult population. In other words secondary school students form 3.5 %, while adults form 96.5 % of target population. Presuming that the respondents of our survey form representative sample of its part of population, we can adjust the sample data using eq. (8.1).

$$V = \frac{\sum_{i=1}^N w_i v_i}{\sum_{i=1}^N w_i} \quad (8.1)$$

where V - is the value of the variable for whole population, N - number of samples, w - weight of sample in target population.

We can demonstrate the computation in our case to recompute size of population with excellent knowledge in civil protection and crisis management:

$$V = \frac{3,5 \cdot 23 + 96,5 \cdot 31}{100} = 30,6 \quad (8.2)$$

Logically the overall result must be very close to the results for the adults as they represent majority of the population.

In the text we several times presumed, that the sample of the population is representative, but we did not prove it. We even didn't explore how large the sample size should be, for it to be representative. Related question would be, if I have a sample of certain size, what level of accuracy regarding derived information from it can I expect? These questions are very important, so let's try to find answers for these, shall we?

First we need to understand, that whatever we "measure" in the survey will be only an estimate of true value in target population be it average, mean or anything else. Difference between the estimate and true value is given by fact that we deal with much smaller sample of population then the target population is. So statistically the sample population, if representative, is similar, but not same as target population.

As a guideline we can use for example confidence interval estimation of the error of the mean, see eq. (8.3).

$$E = z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \quad (8.3)$$

where E - error, z - critical value of normal distribution for chosen confidence level α , σ - standard deviation, n - number of values measured.

We can derive easily sample size n from this equation for chosen confidence level, see eq. (8.4). In other words, if our sample size will be $> n$, then we can expect the results in be reliable at chosen confidence level α .

$$n = \left(\frac{z_{\frac{\alpha}{2}} \sigma}{E} \right)^2 \quad (8.4)$$

Equation (8.4) is usable for numeric values with known or estimated standard deviation.

If our data are of ordinal type, we were using such data in our illustrative example, we need to approach the analysis in slightly different way, see eq. (8.5).

$$E = z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \quad (8.5)$$

where p is the estimate of the variable, other mathematical symbols are same as in eq. (8.3). From that we can easily derive necessary sample size n , see eq. (8.6).

$$n = \frac{z_{\frac{\alpha}{2}}^2 p(1-p)}{E^2} \quad (8.6)$$

Let's apply the approach on estimate of number of adults with excellent knowledge in civil protection and crisis management. We will work with the data used in our illustrative example. Usual requirement would be for estimate to be valid at confidence level 95 % and expected error, let's say 4 %. The 95 % level is standard, the 4 % error is more our choice. For variable estimate p we will use value 31 % as measured in original survey.

$$n = \frac{1,96^2 \cdot 0,31(1-0,31)}{0,04^2} \doteq 514 \quad (8.7)$$

We computed, that to evaluate the property present in 31 % of population, in confidence interval of 95 % a expected error 4 %, we need to use sample size of 514 respondents, presuming that all of them will respond and we will not be required to exclude some of these due to uncorrectable errors in their answers.

Since our sample size was 25 adults and 125 students ... the results suddenly do not look so good. Sample size of the survey is simply too low. We have unrepresentative results and try to derive some knowledge from it and form some recommendation based on the survey. Validity of such

recommendation would be very questionable at best. To prevent such questioning, we plan the survey to improve our chance on success!

Yes, its lot of work, but we will get results we can trust and we can safely build our recommendations on them.



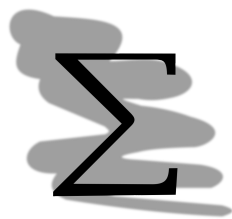
So error and confidence level for sample size

Considering our illustrative example, sample is clearly not representative for chosen error and confidence interval. Try to experiment with the computation and find confidence levels and error for which the sample size would provide valid answers.



Statistics

If the previous paragraphs in this chapter sound like gibberish to you and you cannot understand of how we come to our conclusions, now its time to return to your study materials in *Statistics*.



Summary

Survey organization and evaluating its results is not an easy task. In preparation phase it is necessary to focus especially on setting appropriate goal for the survey and prepare the question to allow us collect the data we will be able to use to reach this goal. Questions should be short, to the point and clearly interpretable.

When we release the survey, we need to realize, that we will always measure only subsample of target population, that is why we need it to be large enough for the results to be considered representative for this target population. (The results to be statistically significant.) When choosing the sample size, we need to consider target confidence interval and error and also the prevalence of searched for measure in population.

We often visualize the results using graphs, such as bar or pie charts. Survey results should be also available for reader in tabular form, either directly in text accompanying the graphs or in annex. If the resulting dataset is stratified using some variable (i.e. education level, sex, age, etc.) we need to consider true size of these sub-groups in target population and transform measured values to com-form it.

Numeric values need to be further processed, meaning we need to compute standard deviation and use it to interpret the data.



Self-test questions

1. What do we mean by target population?
2. Define term representative sample.
3. Explain the process of adapting in survey measured values to size of target population.
4. Explain what impact will have choice of confidence interval and error on sample size.
5. What is survey traversal error and how to solve it?

Chapter 9

Network models



Study guide

In this chapter we will acquaint ourselves with application of the network models for description of the activities as part of the project realization as a basic tool of manager for identification of problematic activities inside of project – allows him to redevelop them.

After reading this chapter you will

Know

- what the network graph is
- what the progress chart is
- how is used CPM method



Time required to study chapter

For getting through the theory in this chapter you will need at least an hour, but if you try network models practically, reserve few hours for your experiments.

9.1 CPM method in project management

Network models are commonly used models for analysis of so called *Critical Path Method (CPM)*. Network models are also used for visualization of interconnected project activities, for analysis of distribution networks etc.

When talking about distribution network, the configuration of network is derived from physical configuration of modeled network. When modeling project, the configuration of model is derived from activities necessary to finish the project and time connection between them.

Before we dwell into network model construction, let's define several terms first, we will use on next few pages. First of them is management. The term management is derived from French word *manège* - as circular arena for horses training or French word *managere* – in English management. There are many definitions of management; one of definitions used more often is that management is completing the goals using others. Another definition of management is that management is mobilization of all company's resources to accomplish planed goals.

Management as science branch is considered to be arts, that's why it is not completely possible to for "hard" mathematic methods to hold all the answers we need. Such methods are usable as important support tools. Its usage will not enable the manager to magically succeed whatever he is doing, but if used properly it can increase his chances for success.

Another term we will discuss in this chapter is *project*. Project means chain of activities, which lead to the goal of the project. For each project we have clearly stated start and finish dates, we also have only limited resources to achieve the project goals.

The activities of the project have to take place in certain order. It is possible to express such order in tabular form. Let's demonstrate the approach on example of simplified building construction (see table 9.1).

Table 9.1: Activities of the project

	activities	followers	Predecessors	Length [days]
A	Foundation excavation	B	-	2
B	Concreting of excavations	C	A	10
C	Fabric of the building	D	B	17
D	Roof	E,F,G,H	C	5
E	Electro installation	I	D	5
F	Piping (water, gas)	I	D	5
G	heating	I	D	5
H	windows	I	D	14
I	Facade rendering, flooring	J	E,F,G,H	21
J	Building acceptance	-	I	1

We can describe the sequence by specification of followers (activities which have to follow after finishing present activity) and predecessors (activities which have to be finished before present activity can start).

The both approaches are completely equivalent by its meaning; they differ by the direction of construction. When considering followers we are going from start to finish, while when considering predecessors we do opposite – going from finish back to start.

Some software tools for project management such as Microsoft Project [17], open source Libre Project [22] or Open Workbench [21] allow the user to choose follower or predecessors approach as needed or as preferred by user and even to change then between each other – followers and predecessors will convert accordingly.



Software support

Creating the diagrams by hand is tedious task, which takes time and you can easily make a mistake. That's why there are many software tools to support us in project management.

The leader in the branch of project management is Microsoft with its MS Project Suite [17]. It is proprietary software and is usually considered as standard for such type of the software. Proprietary nature allows MS Project to cooperate with other softwares in MS Office family and with Exchange to take project management one step closer to the project members by planning their work task and propagating them into their own calendars in MS Outlook.

There are also available various other softwares to support us which are open source. Example of such could be ProjectLibre (formerly known as OpenProj, [22]). ProjectLibre is available on practically any platform (MS Windows, Linux or Mac OS). Alternative could be Open WorkBench [21], but only MS Windows operating system is supported and it has far less features than ProjectLibre.

You can find practical, hands on example in ProjectLibre in next chapter.

List of the activities in tabular form is not entirely usable for operative control of activities progress, resources planning etc. That's why the graphical interpretation of this information is actually chosen, in the form of *progress charts*, sometimes also called *Gantt chart* and *network graphs*.

As you can see on figure 9.1, the red road here is so called *critical path*. By critical part we mean sequence of the activities, for which any increase in the time over the planned time would automatically lead to increase the time needed for the project as whole. Because of this property it is necessary for manager to especially thoughtfully manage these activities.

On network graph the activities are represented by the edges. Edges E, F, G, H may occur at the same time, so they have to be finished before activity I takes place. To demonstrate it we add so



Figure 9.1: Progress chart of the project

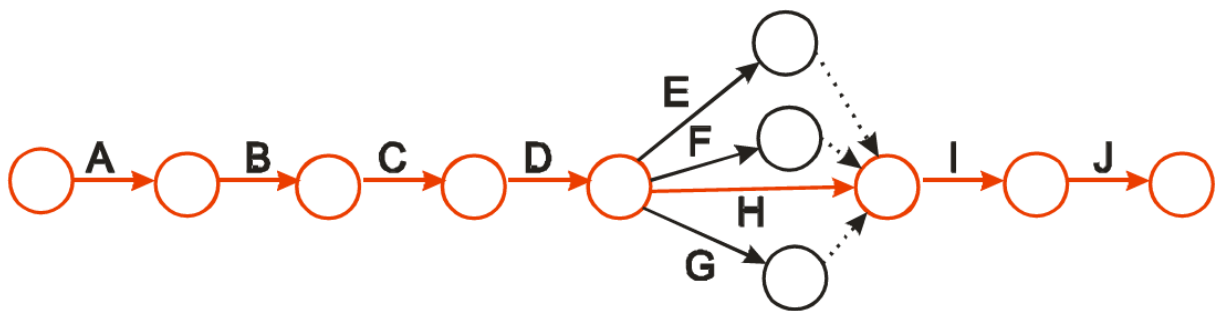


Figure 9.2: Network graph of the project

called virtual activities (look for dotted arrows on figure 9.2).

These virtual actions do not represent real activities; we add them to graph only to improve its readability.

We can also note critical path in the graph. On figure 9.1, the critical path is noted in red color.

Maybe you ask why we use two types of graphs for single task (identification of critical path). The answer is that we use them both because identification of critical path is only one task the graphs are meant to solve. Primary goal of the network graph is to visualize interconnection between the activities of the project. Primary goal of the progress chart is to visualize activities from point of view of the time to allow its optimization. For this task all non-critical actions may be used, since we can increase the time needed for such non-critical task by sparing some resources (as people hours) and use them somewhere else, where they are perhaps needed more.

It's obvious that the time increase for noncritical activities can't be infinite, because by increasing length of the activity it may become critical under these changed circumstances.

Advantages of progress chart usage:

- identification of the critical path
- optimization of resources usage for non-critical activities
- overview of realized activities as a good source of information for effective management

9.2 ProjectLibre – software support to project management

First we to download the ProjectLibre from its homepage on SourceForge [22]. To beginners I recommend downloading binaries of Project Libre intended directly for your operating system. But, if you are inclined so, you can also download source code for the project and compile it yourself. Source code is not required if you use binaries.

To run the software you will need either **JRE** or **Java Development Kit (JDK)** installed. Since JRE is available for Java 8 only and is considered heavily outdated, I recommend using JDK from <https://jdk.java.net/18/>. Look for GA (general availability) version. On Linux you may also use distribution's package system to install Java. On MacOS I recommend using Brew.sh (<https://brew.sh/>)



Critical Path Method

Compute length of the critical part for example we worked with in this chapter.

Critical Path Method - solution

Possible paths

$$1 - A, B, C, D, E, I, J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61$$

$$2 - A, B, C, D, F, I, J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61$$

$$3 - A, B, C, D, H, I, J = 2 + 10 + 17 + 5 + 14 + 21 + 1 = 70$$

$$4 - A, B, C, D, G, I, J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61$$

Critical path is the path 3 (A, B, C, D, H, I, J), project will last for 70 days.



Questions

1. What is CPM method?
2. Explain interconnection between the project activities from point of view of the follower and predecessors.
3. Why do we use both progress chart and network graph, wouldn't it suffice to use one of these?
4. Why do we try to find critical path?



Study guide

In this chapter we will be using ProjectLibre for project management, but most of what is written for it should apply also to any other project management software. Though **GUI** will be slightly different. So if you have installed different tool, you do not have to install ProjectLibre, if you do not want to.

`//brew.sh`) package system and while you are at it, you may also install ProjectLibre using Brew. We provide short installation script for your convenience.

Výpis 9.1: Install JDK and ProjectLibre on Mac using Brew (use in terminal)

```
1 brew install --cask oracle-jdk
2 brew install --cask projectlibre
```

Remember that after installation you will have to maintain installation of JDK. If you used package manager, it is trivial, for example brew has update and upgrade commands to do that automatically for all formulae and casks installed using Brew.

Výpis 9.2: Update formulae and casks installed using Brew (use in terminal)

```
1 brew update
2 brew upgrade
```

Please note, that if after update system doesn't find outdated packages, it is pointless to call upgrade, as there is nothing to upgrade.

On Windows both installation and maintenance is manual.

As of time of writing this text (September 2022), latest available version of Project Libre was 1.9.3 and latest GA version of Java was 18.

After downloading and installing of the software, we may start to use. After opening ProjectLibre we have to create new project as we do not have existing project yet.

When creating new project, we have usually specify the project name, project manager and possibly some notes for us, see fig. 9.3. In fact writing these informations is not required, but it is considered as a good practice. Also different software suites support different project properties. For example MS Project allows you to specify what kind of calendar will the project use (common office format – 8 hours a day, 5 days in week, or will it be 24/7).

In this chapter we will try to recreate the example from previous chapter, so lets call the project: Family house – construction. We will specify ourselves as project managers. You also may or may not want to add your own note. Clicking on OK will create the project.

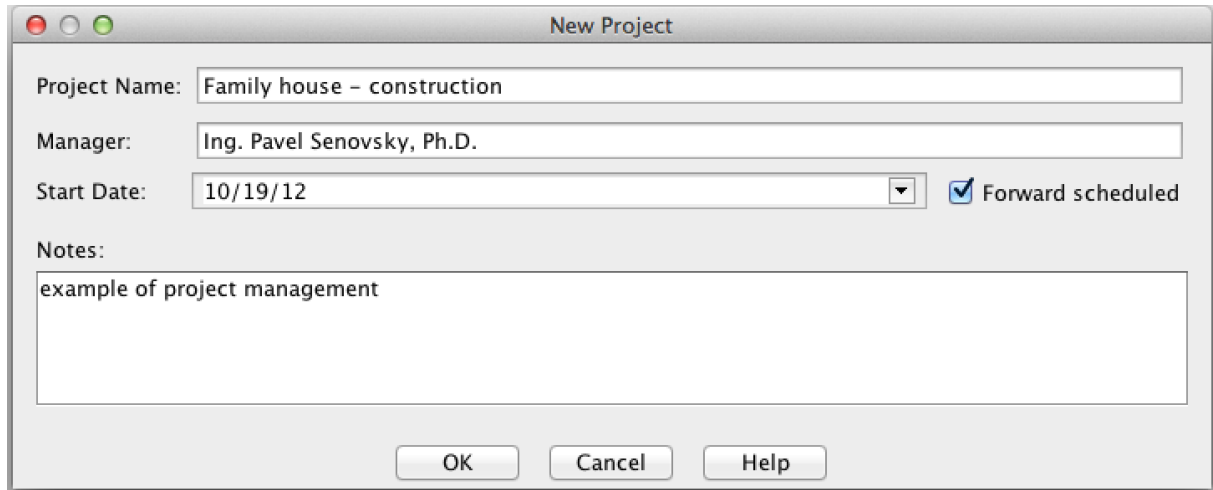


Figure 9.3: Create new project

The new project is automatically opened in Gantt chart mode, which allows us to specify the project tasks and the connection between them. Out of this information it automatically created the progress chart of the project.

We have already specified project tasks in table 9.1, with its name, connection to other tasks and its length. Lets with task definition – in Gantt chart we create the task by writing down its name into column name. When you create the task note that ProjectLibre automatically adds additional information to it. It guesses when the task will start (on the day of the beginning of the project), that its length will be approximately one day.

Activities have many properties we can use to manage the project:

- activity name
- length
- start and finish dates (software recomputes it automatically based on the relation between the activities we define)
- predecessors or follow up activities (used to define relation between activities)
- resources
- and many more - ProjectLibre even supports using custom properties

Also note that this information is automatically used for process chart on the right side of the screen. There are also other columns, which are empty so far – predecessors and resources, allowing us to specify the connection between the tasks and to assign human or material resource to the task.

Lets change length of the tasks, because right now it is one day with question mark. Question mark means uncertainty – so we are basically saying, that the task will require approximately one day. From table 16 we know that this is not the case, so lets make the corrections.

The new length can be assigned in days, weeks or months. Gantt chart presumes, that the value is in days, so writing $5 = 5d = 5 \text{ days} = 1w = 1 \text{ week}$ (this holds for office calendar). The time unit used to describe length of the task can be specified in days (d), weeks (w) or months (m). Be aware that the week may mean something different than you think. When normally speaking about the week, we usually mean monday – sunday (or sunday – saturday depending, where are you from), the meaning of the week in project management is different.

The difference is in what calendar is used – is it normal office calendar with working hours 7 - 15:30 monday to friday or is it different? Meaning that the week can have 5 days or 7 days for different tasks using different resources. In project management we also do not know exactly when we will start with the task as its predecessors may have finished earlier or later then expected, so the week in project management is described by number of working days only – with floating beginning and the end.

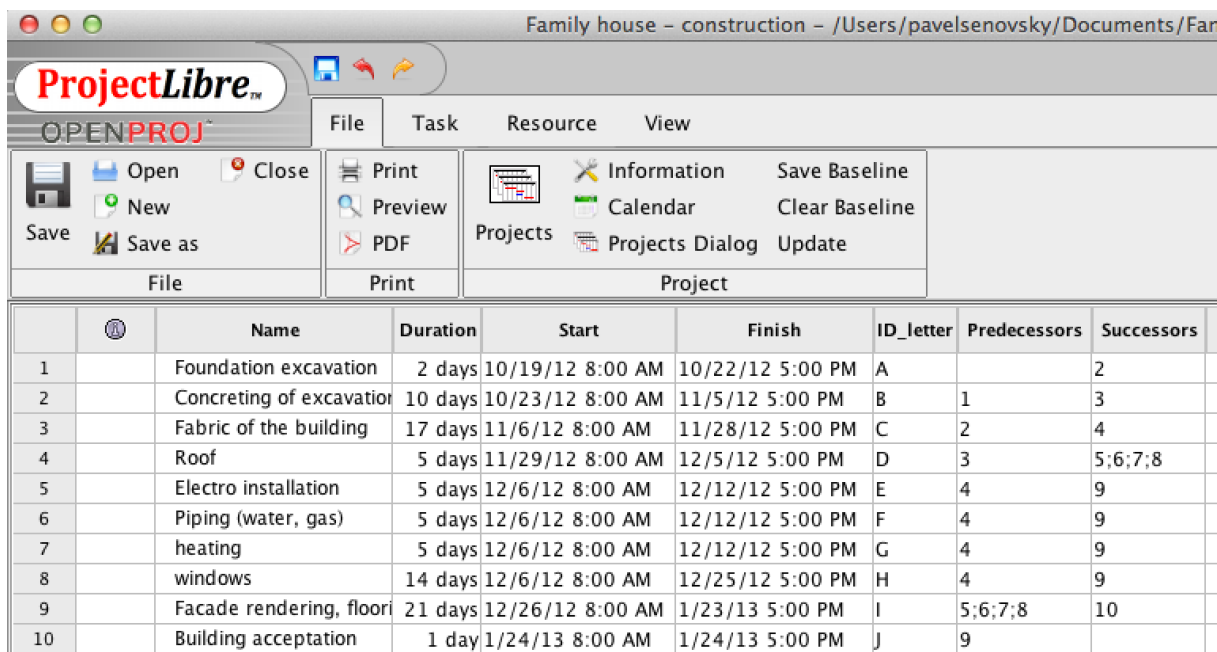
The information on tasks in table 9.1 and the way we have defined our tasks in Gantt diagram also differ by tasks identification. In table we identify the task alphabetically using capital letters A – J, in Gantt chart the tasks are identified by the number of the row they are written on (1-10). Defining

the connection between the task would be slightly harder for us to do, so we will use capabilities of the software to insert new column for us to write the letter identification in it.

Adding new column is easy. We just click on the column where we want to have new column. Then we right click on it to display context menu allowing us to insert and remove the columns. We will insert one. As you can see (well if you are trying it on the computer :-), there is plenty of possibilities to choose from as the template of the new column. We will choose Text1 – which is general template allowing us to insert any text into it. After inserting the column, we will change its name by right clicking on the column and choosing Rename. The new name will be ID Letter. Now we can write down letter IDs from table 9.1 into our new column.

Now the only information missing from table 9.1 are the connections between the tasks. In Gantt chart we already have predecessors, so we can use these, but if you rather preferred usage of the followers – you can, by defining new column with successor template. The software will provide automatic translation to the other one.

In this text, I will use predecessors. Do not forget that we will have to identify the task by number, not by the letter in the new column we created. Also we will need to assign more then one predecessor for task I – we write all IDs in predecessor column and separate them semicolons. Well, the separation by using of comma works too – ProjectLibre just automatically replaces the commas with semicolons. In the end your Gantt chart should look similar to the figure 9.4.



ID	Name	Duration	Start	Finish	ID_letter	Predecessors	Successors
1	Foundation excavation	2 days	10/19/12 8:00 AM	10/22/12 5:00 PM	A		2
2	Concreting of excavation	10 days	10/23/12 8:00 AM	11/5/12 5:00 PM	B	1	3
3	Fabric of the building	17 days	11/6/12 8:00 AM	11/28/12 5:00 PM	C	2	4
4	Roof	5 days	11/29/12 8:00 AM	12/5/12 5:00 PM	D	3	5;6;7;8
5	Electro installation	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	E	4	9
6	Piping (water, gas)	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	F	4	9
7	heating	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	G	4	9
8	windows	14 days	12/6/12 8:00 AM	12/25/12 5:00 PM	H	4	9
9	Facade rendering, floor	21 days	12/26/12 8:00 AM	1/23/13 5:00 PM	I	5;6;7;8	10
10	Building acceptance	1 day	1/24/13 8:00 AM	1/24/13 5:00 PM	J	9	

Figure 9.4: Gantt chart – task definitions

By definition of the predecessors or successors we specified the connection between the tasks and the progress chart now makes some sense, see fig. 9.5.

On the progress chart you can see that the tasks have different sizes, are of different color. The size of the task is related to the task length. So as we defined it, we have it, but once defined it doesn't mean that we stop working with it – on the contrary, our work as project managers has just started. Now we have to closely follow happenings of our project and how the tasks are finished. The software supports us by allowing us to follow percentage of completeness of the task.

To change the percentage we use the mouse. We move the mouse over the beginning of the task. The mouse pointer will change into percent symbol and the arrow pointing right. Now click and pull to right to increase percentage.

Similarly we can change the beginning or the end of the task as needed. The remainder of the tasks will adjust itself as needed without the need to micromanage it.

And lastly we have the color of the task. On our chart we have red and blue tasks. Task in blue are these which do have a time reserve. By time reserve we mean, that there is time available for us to lengthen the task without endangering whole project completion.

So the time reserve and manipulation with non-critical tasks allows us to free resources necessary to finish the critical (red) tasks on time.

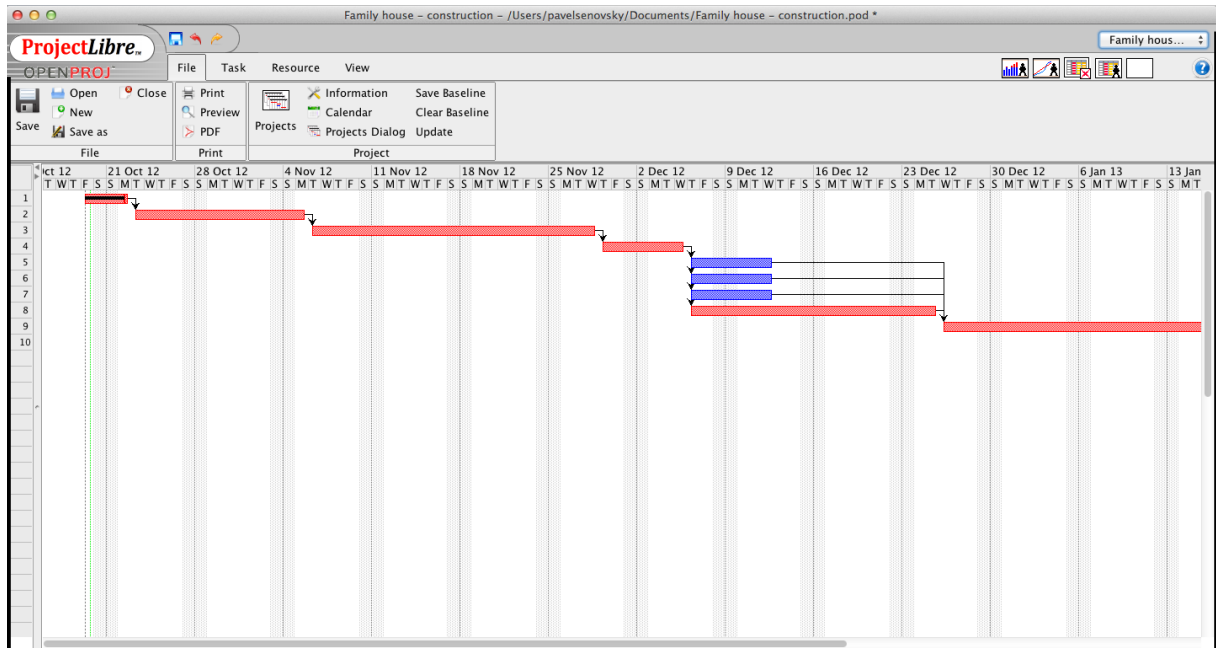


Figure 9.5: Gantt chart – progress chart

Alternative way of looking on the progress chart is by using network diagram. In network diagram we see the tasks as the nodes of the network. Connection between the tasks allow us identify which tasks must be completed before others may begin. Network diagram will be generated automatically based on the tasks definitions in Gantt chart.

Start network diagram on Task tab -> tool Network. You may see screenshot of our network on figure 9.6.

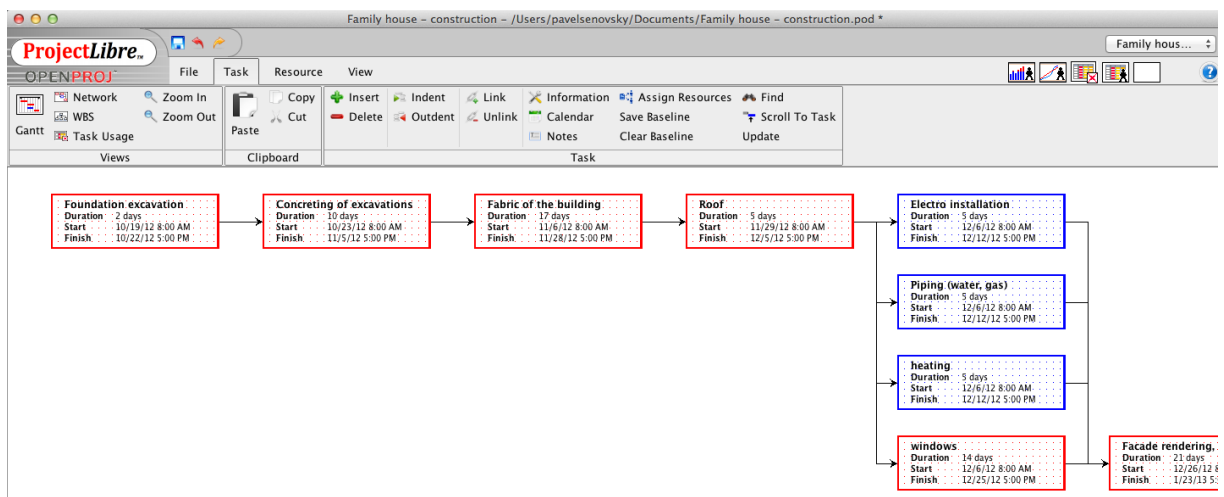


Figure 9.6: Project's network diagram

So why should we use both network diagram and progress chart when both basically show the same things? Well in fact they, but both in slightly different ways. Network diagram allows us to better see the network of the tasks structure, while progress chart allow us to better see and manipulate with reserves and manage the project when it starts.

We are talking here about the resources, can the software tools help us with these too? Yes it can. We can manage the resources available for the project on the resources tab (first tool from left on the

tab).

Lets make three resources – worker 1, 2 and sand. For the resources we specify the type of the resources. The resources can be human beings available for the work, it can be also the material. Obviously resources of the different types have also different properties.

For human resources we choose type work. To such resources we can assign contact email a group them for example by the teams they are working in. For human resources we state max. units. Automatically software assigns 100% max. unit for the resource. Usually we interpret it so, that we have available 8 hours of resource’s work. Depending on the resource usage we may have more or less available. Max. units should not exceed 150%, as we would be getting closer to the slavery work which is forbidden by law in most countries.

Each human resource has its standard pay on per hour basis and overtime pay again on per hour basis for the cases when the usage of the resource exceeds max. units. Overtime pay is usually higher then standard pay.

Cost per usage is the cost connected to the usage or the human resource, but can’t be considered income of the resource. For example I might need to transport workers on the site, which costs me money – that would be cost per usage.

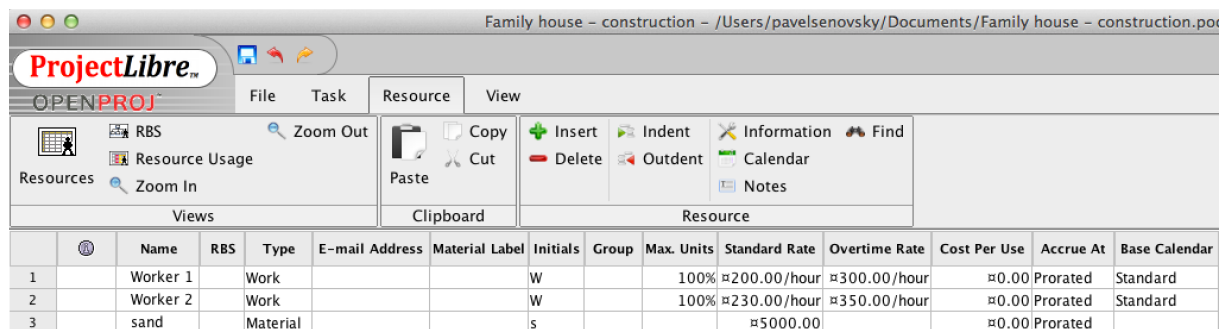
Accrue at column lets us set when we will pay the resource for his work. We may do it as prorated, at the beginning or the end. Prorated would mean, that we will pay the resource every day. In practice we may use the scenario of usage of diggers to prepare sewer network. In such case we pay them on the end of each work day (if paid on the beginning they may not come to work).

At the beginning we pay in cases when there is some kind of resources procurement necessary on side of the resource. On the end means that we will pay after we check and approbate the work.

Every human resource may have its own calendar – standard, 24 hours and night shift.

In case of the material resources many columns do not have any sense, so we do not use them. What is the meaning of the remaining columns? Standard rate should be interpreted as the rate at which we procure unit of the resource. The units may be tons, kilograms, price for the piece, square meters, . . . Cost per use is similar to its human resource’s counterpart – usually we need to move the material on site before we can use it and it costs money.

Example of the resources definition is on figure 9.7.



	Name	RBS	Type	E-mail Address	Material Label	Initials	Group	Max. Units	Standard Rate	Overtime Rate	Cost Per Use	Accrue At	Base Calendar
1	Worker 1		Work			W		100%	200.00/hour	300.00/hour	0.00	Prorated	Standard
2	Worker 2		Work			W		100%	230.00/hour	350.00/hour	0.00	Prorated	Standard
3	sand		Material			s			5000.00		0.00	Prorated	Standard

Figure 9.7: Resources definition

Now lets go back to the Gantt chart. We will assign the resources to the tasks. We can do that in two ways - we either click into resource name column or in properties of the activities. We either directly fill in the resource used by activity or double click to open dialog windows allowing us to set up the resources in more granular manner, see fig. 9.8.

Lets try to assign the worker 1 to all the tasks. You may speed the definition process by copying resource name in Resources column.

Lets see how exactly are we using the resource. First we will look at the resource’s histogram. You may start it by clicking on histogram icon on upper right side of your program’s window, or look at the figure 9.9.

The histogram is connected to the Gantt chart, to navigate through it you have to mo the Gantt chart, histogram will move automatically. As you can see the problem is in tasks starting 6th December, we have four of them and we assigned worker one to all of them, which leads to our brutal overuse of the resource – we plan to use it on 400%, which is of course not possible.

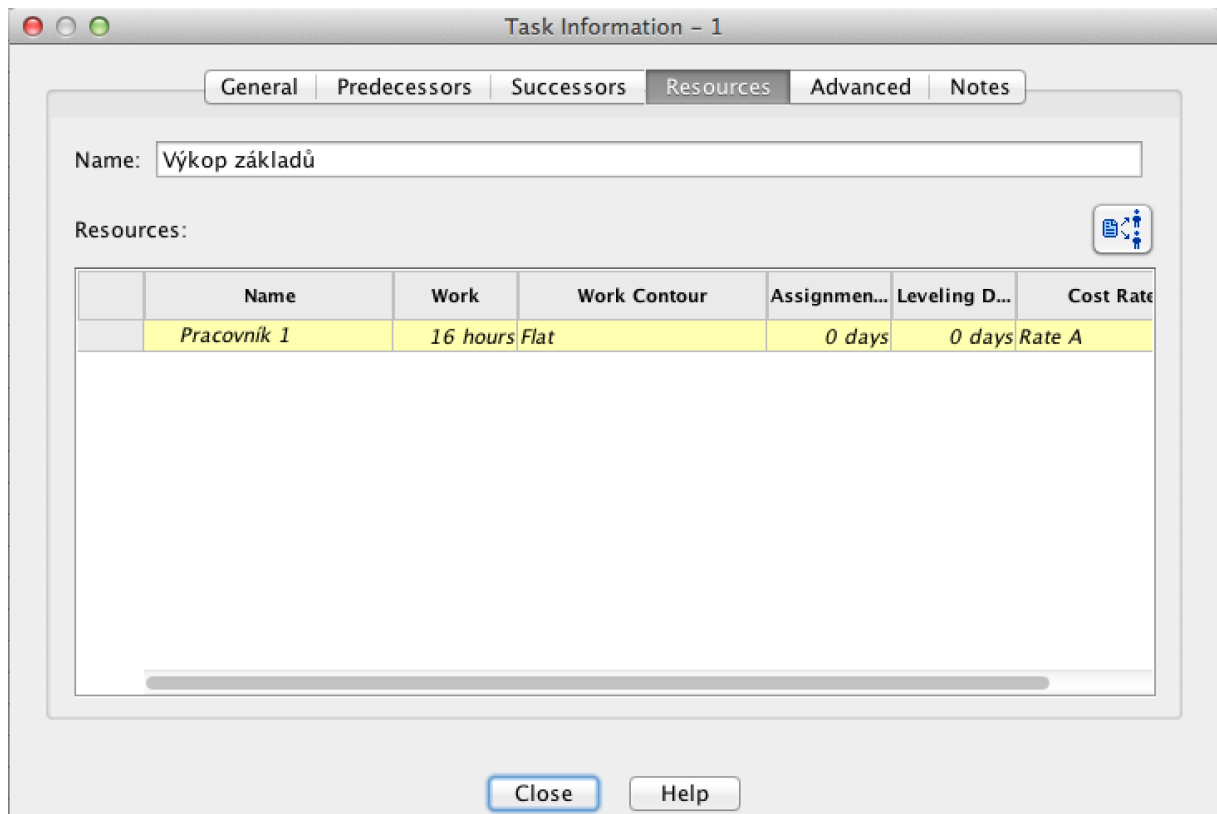


Figure 9.8: Information on activity in ProjectLibre

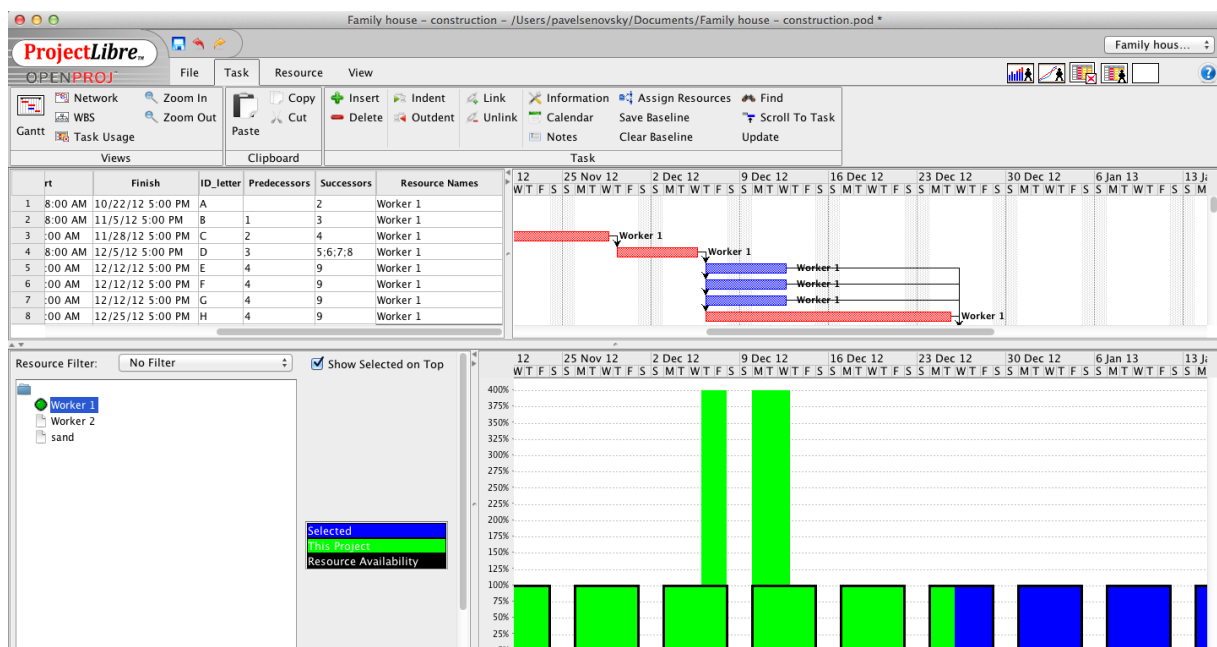


Figure 9.9: Resource's histogram

Histogram thus helps us identify the problematic tasks in our project and allows us to solve the problem before the problem manifests itself in real world.

We may look at the resources also by assigned working hours choosing Resource usage, see fig. 9.10.

And that's all, you will see in this textbook on project management. The software itself has

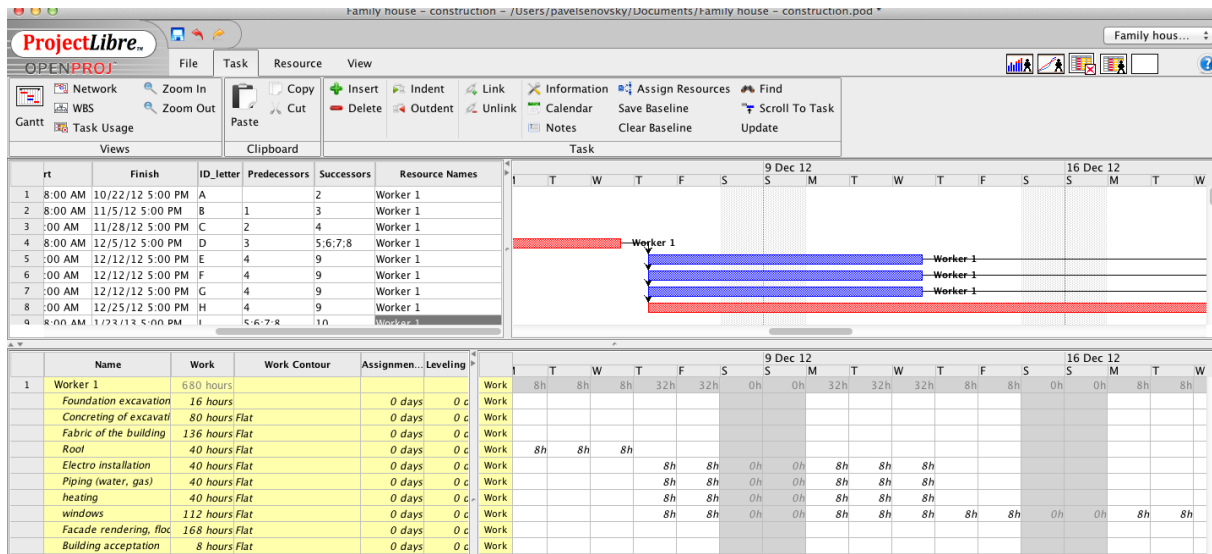


Figure 9.10: Resource usage

more tools allowing you to better track monetary resources and many more aspects of the project management.



Project management

Create new project of your choice, if you do not want to think – create project for your study program this year, look at all of the terms, semester projects, assignments...

9.3 Optimization in other network models

Project management has special needs, but the knowledge of **CPM** method may be used in different types of networks as well. For example we may want to analyze road (rail) network or some kind of pipe network. These networks need different types of answer – we may search for shortest road between two nodes or critical path from capacity point of view etc.

Such types of task are solvable if the modeled network closed – there are no outside connections to or from it. Another limitation of the model is that what comes in the first node must also go out from last node.

Let's see how we can solve some simple problems algorithmically. First type of problem is searching for **maximal flow in network between two given nodes**. Problem with this kind of problem is that the edges between the nodes usually have different capacity. And to solve this problem combinatorial would be for large networks virtually impossible as the difficulty of the problem will grow exponentially with number of nodes.

We can apply different approaches to solve it, for example **Ford-Fulkerson algorithm**, which works in following way [7]:

Inputs: Graph G with flow capacity c , source node s and the sink node t .

Output: flow f from s to t , which is maximal

1. $f(u, v) \leftarrow 0$ for all edges of the network (u, v)
2. while there is path p from s to t in G_f , such that $c_f(u, v) > 0$ for all edges $(u, v) \in p$:
 - (a) find $c_f(p) = \min\{c_f(u, v) | (u, v) \in p\}$
 - (b) for each edge $(u, v) \in p$
 - i. $f(u, v) \leftarrow f(u, v) + c_f(p)$, send flow along the path
 - ii. $f(u, v) \leftarrow f(u, v) - c_f(p)$, the flow might me “returned later”

Imaging how Ford-Fulkerson's algorithm actually works is rather hard. You can see an example of Ford-Fulkerson's algorithm in Ford-Fulkerson Demo [8].



Software support for physical network computations

Computing the physical network using Ford-Fulkerson's algorithm, or any other by hand is rather hard, even for small networks. That's why there are many software tools which employ the algorithms and virtually separate us from the algorithm itself. You can find them in modern **Global Positioning System (GPS)** and many more applications.

We will use R computational environment, we used already in previous chapters to help us.

Another type of the problem could be *searching for shortest path between nodes*. For this type of problem algorithm of Dutch scientist Dijkstra is often used (*Dijkstra algorithm*). The algorithm is as follows [3]:

1. Create a distance list, a previous vertex list, a visited list, and a current vertex.
2. All the values in the distance list are set to infinity except the starting vertex which is set to zero.
3. All values in visited list are set to false.
4. All values in the previous list are set to a special value signifying that they are undefined, such as null.
5. Current vertex is set as the starting vertex.
6. Mark the current vertex as visited.
7. Update distance and previous lists based on those vertices which can be immediately reached from the current vertex.
8. Update the current vertex to the unvisited vertex that can be reached by the shortest path from the starting vertex.
9. Repeat (from step 6) until all nodes are visited.

Both algorithms work similarly – they reduce the network into the tree graph, for which the number of computations needed to find the answer is manageable. Dijkstra algorithm is also usually applied by specialized software.

There are about dozen of various algorithms used for different types of tasks. You can review them more in depth for example on Wikipedia in article List of Algorithms [15], which has dedicated section to it.



Questions

1. What are limitations of the network models?
2. What kind of problems are we able to solve by the models?
3. What is the difference between Ford-Fulkerson's and Dijkstra's algorithms?
4. What is common for these two algorithms?

9.4 Numerical computations in R

We can use R for both computations and visualization of networks. Core environment does not support this type of computation, so we need to install proper packages, which will allow us to use this functionality.

One of more popular packages is *igraph* [12]. We can install it from CRAN repository. If you tried examples from MCA advanced models chapter, you already have installed the package. Otherwise you can use following code:

Výpis 9.3: Installing igraph in R

```
1 install.packages("igraph")
```

We can read the graph using `read.graph` function. We need to specify network structure in external file, which we pass on the function as one of its parameters. Function supports many input formats: `edgelist`, `pajek`, `graphml`, `hml`, `ncol`, `lgl`, `dimacs`, `graphdb` and `gml`. Package maintainers try to add support for additional formats in new versions of the package as they emerge. If format parameter is left empty, function presumes `edgelist` format, which is simplest format available for network definition purposes.

Before we use some of these before mentioned formats, we will defined the network using *adjacency matrix*. We actually used this approach already in advanced MCA models to visualize outranking relations between the alternatives.

This time we will define network from fig. 9.11, which you could already see on the lectures including demonstration of in hand computations using different methods.

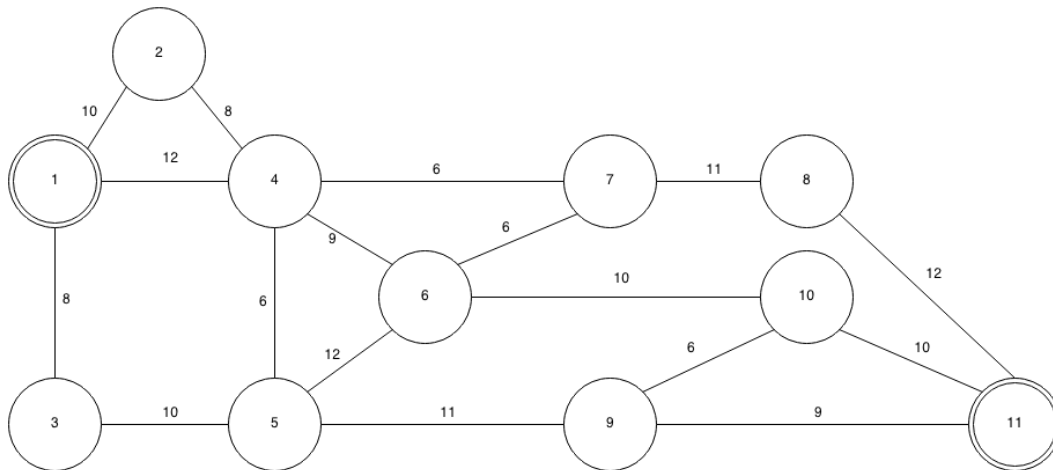


Figure 9.11: Basic network for computation realization using mark-up algorithms (courtesy of Gros [45])

Výpis 9.4: Define network in igraph using adjacency matrix

```

1  library('igraph')
2  adj_matrix <- matrix(
3    c(0, 10, 8, 12, 0, 0, 0, 0, 0, 0, 0,
4      0, 0, 0, 8, 0, 0, 0, 0, 0, 0, 0,
5      0, 0, 0, 0, 10, 0, 0, 0, 0, 0, 0,
6      0, 0, 0, 0, 6, 9, 6, 0, 0, 0, 0,
7      0, 0, 0, 0, 0, 12, 0, 0, 11, 0, 0,
8      0, 0, 0, 0, 0, 0, 6, 0, 0, 10, 0,
9      0, 0, 0, 0, 0, 0, 0, 11, 0, 0, 0,
10     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 12,
11     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 6, 9,
12     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 11,
13     0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0),
14    nrow = 11, ncol = 11, byrow = T
15  )
16  net <- graph_from_adjacency_matrix(adj_matrix, mode = "max", weighted = T)
17  plot(net, edge.label = E(net)$weight)

```

We will create matrix from vector by "pouring" the vector by rows (set parameter `byrow = T`). Our network has 11 nodes, so the adjacency matrix needs to be of 11:11 size. The graph is not oriented, so we can little but cut corners. Notice that lower left part of the matrix consist of 0 only. In case we set graph mode to directed, we would need to fill in values these too to specify from which node to which node the flow actually is.

We can interpret adjacency matrix simply as table, where the rows and columns represent network's nodes and table fields with nonzero value will define network's edges.

Since we are working with simple undirected network graph, it might be easier to define network using *edge list* format.

Result will be similar to fig. 9.12 basically regardless of what format we will use for network definition. Please note that networks on fig. 9.11 and fig. 9.12 are structurally same. Only difference

is that network in fig. 9.11 has been made in hand and as such the lengths of the edges do not exactly correspond to states length.

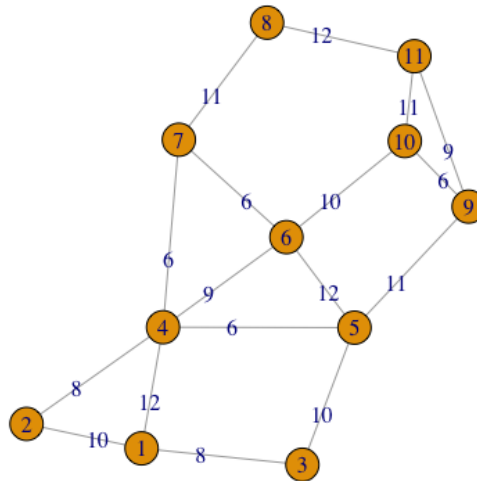


Figure 9.12: Network structure defined using adjacency matrix

Edge list is very simple format for network definition. We specify the network by defining its edges. Structure of the definition is simple: starting node, end node, weight of the edge. The format doesn't require us to put the edges definition on separate rows, but I recommend doing so for better readability.

Výpis 9.5: Defining network in edge list format: start end capacity

```

1  1 2 10
2  2 4 8
3  1 4 12
4  1 3 8
5  3 5 10
6  4 5 6
7  4 7 6
8  4 6 9
9  5 6 12
10 7 8 11
11 8 11 12
12 6 10 10
13 10 11 10
14 5 9 11
15 9 10 6
16 9 11 9

```

We do not have room in the course or in textbook to describe all supported data formats, so we will limit ourselves to studying *GraphML* only. For the rest we provide only directions to other study materials, should you require it: pajek [31], for lgl and ncol data formats consult [29], dimacs [40], graphdb [56], gml [27] and dl [28].

The GraphML format is interesting from several different perspectives. It is based on **Extensive Markup Language (XML)**, and thus is somewhat self-documenting, easy to interpret and has some very interesting properties.

Our network would look like this in GraphML:

Výpis 9.6: Using GraphML to define the network

```

1  <?xml version="1.0" encoding="UTF-8"?>
2  <graphml xmlns="http://graphml.graphdrawing.org/xmlns"
3     xmlns:xsi="http://www.w3.org/2001/XMLSchema-instance"
4     xsi:schemaLocation="http://graphml.graphdrawing.org/xmlns
5     http://graphml.graphdrawing.org/xmlns/1.0/graphml.xsd">
6  <key id="d1" for="edge" attr.name="weight" attr.type="double"/>
7  <graph id="G" edgedefault="undirected">
8     <node id="n1"/>
9     <node id="n2"/>
10    <node id="n3"/>
11    <node id="n4"/>
12    <node id="n5"/>
13    <node id="n6"/>
14    <node id="n7"/>
15    <node id="n8"/>
16    <node id="n9"/>
17    <node id="n10"/>
18    <node id="n11"/>
19    <edge source="n1" target="n2">
20        <data key="d1">10.0</data>
21    </edge>
22    <edge source="n1" target="n4">
23        <data key="d1">12.0</data>
24    </edge>
25    <edge source="n1" target="n3">
26        <data key="d1">8.0</data>
27    </edge>
28    <edge source="n2" target="n4">
29        <data key="d1">8.0</data>
30    </edge>
31    <edge source="n3" target="n5">
32        <data key="d1">10.0</data>
33    </edge>
34    <edge source="n4" target="n5">
35        <data key="d1">6.0</data>
36    </edge>
37    <edge source="n4" target="n6">
38        <data key="d1">9.0</data>
39    </edge>
40    <edge source="n5" target="n6">
41        <data key="d1">12.0</data>
42    </edge>
43    <edge source="n4" target="n7">
44        <data key="d1">6.0</data>
45    </edge>
46    <edge source="n6" target="n7">
47        <data key="d1">6.0</data>
48    </edge>
49    <edge source="n7" target="n8">
50        <data key="d1">8.0</data>
51    </edge>
52    <edge source="n8" target="n11">
53        <data key="d1">12.0</data>
54    </edge>
55    <edge source="n6" target="n10">
56        <data key="d1">10.0</data>
57    </edge>
58    <edge source="n10" target="n11">
59        <data key="d1">10.0</data>
60    </edge>
61    <edge source="n5" target="n9">
62        <data key="d1">11.0</data>
63    </edge>
64    <edge source="n9" target="n10">
65        <data key="d1">8.0</data>
66    </edge>
67    <edge source="n9" target="n11"/>
68 </graph>
69 </graphml>

```

We will use `read.graph` function to read the graph definition and then plot it using `plot` function. Generated graph image will be only similar to the one we made by hand (fig. ??), but all its properties are going to be correct (unlike the graph drawn by hand).

Structurally both networks are same, the generated one takes better into account length of the edges. we can also influence looks of the network by setting various plotting parameters.

We can compute shortest path by using function `shortest.paths`. This function has three parameters - network to be analyzed and first and last node. Apart of length, we may also be interested in the path itself. We can get this one using `get.all.shortest.paths` function, which has same parameters as `shortest.paths`, but returns the nodes on shortest path. This function does not provide information on overall length of such path.

Ford-Fulkerson algorithm is not supported by igraph, but it supports other algorithms to solve problems around the flow. For example `graph.maxflow` computes maximal capacity of the path between two nodes. To do that, network must be defines including information on capacities, which our network does not have.

Whole example could look like this:

Výpis 9.7: Shortest path computation in R

```

1 library(igraph)
2 net <- read.graph("/Users/pavelsenovsky/Documents/skripta/modelovani 3vyd/skripty/sit.graphml", "
graphml")
3 plot(net)
4 shortest.paths(net, 1, 11)
5 get.all.shortest.paths(net, 1, 11)

```

Note, that igraph doesn't allow us to specify what algorithm should be used to compute shortest path. It is by design, to shield the user against complexities of all the algorithms and their nuances. It is expected, that this type of information should posses the package developer and (s-)he needs to implement some kind of rationale into the package itself.

For sake of completeness, the shortest path computation in igraph package uses Djisktra's algorithm for small networks, but for large networks (over 100 edges) it uses *Johnson's algorithm*. Function chooses appropriate computational algorithm automatically.



Graphviz

In previous edition of this textbook, in this section program *Graphviz* [10] has been discussed. The program is intended for network graph visualizations. Considering functionality offered by R this section of text seemed to be little bit redundant. That is why this section of text has been moved do Annex 2 for network enthusiasts.

Last type of the algorithms we will use in this section of text is *Kruskal's algorithm*.

While in previous cases we begun the computation by defining the network, Kruskal's algorithm selves other type of problems. We start we node positions only (no edges exist in the graph at the beginning). Our goal will be to construct optimal network, that will interconnect all defined nodes. To express problem in more proper mathematical way, we need to *derive minimal structure of the graph interconnecting all the nodes*.

To be able to fulfill minimal set requirement, we will need some kind of optimization criterion to limit its size.

We call this minimal graph structure *spanning tree*. General outline of the algorithm is simple:

1. compute all possible/allowable connections between the nodes and compute values of its optimization criterion (often cost or length) - these will serve as potential edges of the graph
2. order potential edges by value to the optimization criterion from best to worst
3. repeat following step until all nodes are connected into network
4. from the non-selected edges of graph G , choose edge (in order), which does not create the loop with already selected edges

Computation in R is possible using *optrees* package [43]. Apart from Kruskal's algorithm it also supports other algorithms such as Prim's and in Czech Republic developed Borůvka's algorithm. We provide the computational code below:

Výpis 9.8: Example of spanning tree using Kruskal's algorithm (courtesy of [43])

```

1 library('optrees')
2 nodes <- 1:4
3 arcs <- matrix(c(1,2,2,
4   1,3,15,
5   1,4,3,
6   2,3,1,
7   2,4,9,
8   3,4,1),
9   ncol = 3, byrow = T)
10 G <- getMinimumSpanningTree(nodes, arcs, algorithm = "Kruskal")

```

In the listing there are possible relations between the nodes defined using arcs matrix in which in first columns there is starting node, in second end node and in the third length, cost or other optimization criterion of choice. The function requires the criterium to be formulated in such way that we optimize it by minimization. Resulting graph is available in fig. 9.13.

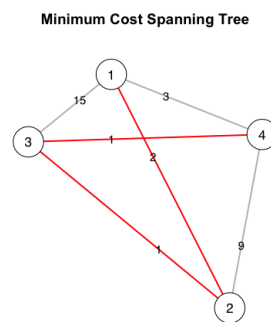


Figure 9.13: Minimal spanning tree derived using Kruskal's algorithm (in red)

Our example presumes, that the possible/allowable edges have been prepared beforehand. What if it is not so, what if we really start with nodes positions only? Presuming that the optimization criterium is (or is connected to) the distance between the nodes, we can generate easily possible combinations of the nodes and compute length of the vector they form.

Let's consider simple example with four nodes at coordinates $A = [0,0]$, $B = [1,0]$, $C = [1,1]$ a $D = [0,1]$. We will be working with the square. Optimal solution for the problem would be logically constructing graph from 3 out of 4 sides of this square to form minimal spanning tree.

Solution for this problem is provided in listing below and the result is available on fig. 9.14.

Výpis 9.9: Generating edges by combinations of the nodes for Kruskal's algorithm

```

1 library('optrees')
2 nodes <- data.frame(rbind(
3   c(0, 0),
4   c(1, 0),
5   c(1, 1),
6   c(0, 1)))
7 colnames(nodes) <- c("X", "Y")
8 a <- NULL
9 for(i in 1:(nrow(nodes)-1)){
10   for(j in (i+1):nrow(nodes)){
11     t <- sqrt((nodes[i,1]^2 - nodes[j,1]^2)^2 + (nodes[i,2]^2 - nodes[j,2]^2)^2)
12     a <- c(a, i, j, t)
13   }
14 }
15 arcs <- matrix(a, ncol = 3, nrow = choose(nrow(nodes), 2), byrow = T)
16 nodes2 <- 1:4
17 G <- getMinimumSpanningTree(nodes2, arcs, algorithm = "Kruskal")

```

Logic behind it is as follows: the costs are often connected to distance. For example if we need to connect the nodes by wire, then there is per meter price for wire. If the node symbolize cities and

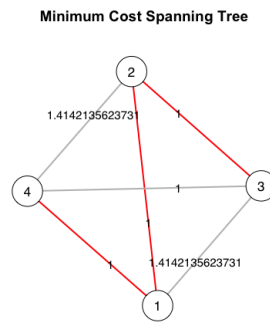


Figure 9.14: Graph generated by Kruskal's algorithm (in red) generated based on combinations of the nodes

edges some kind of pipeline, there is per kilometer price connected to it, etc. In both these examples our requirement to minimize the optimization criterion holds.

Even if the measure doesn't hold, than we can easily re scale the measure to be minimized.

Summary

Working with networks is kinda complex. That's why it is important to always understand what kind of problem do we need to solve. do we need to manage the project, or are we interested in some properties of physically realized network, such as road, railway, pipelines, etc.?

To support project managements there are many software available. Good examples are MS Project, ProjectLibre and similar. These can help us to follow activities in the projects as they are realized, plan them both time and resource wise.

In physical networks we solve typically different types of problems, so we will need to use different type of tools. There are again available many packages, but using R and its igraph package can be recommended generally as a good starting point.

There are various algorithms available for various network problems. One of most often used is Djisktra's algorithm (alternatively Johnson's algorithm) to find shortest path. Ford-Fulkerson's algorithm can be used to find maximal capacity between two points in the network.

Analyzed networks are limited to certain degree. The networks need to be closed and we presume that there is no significant loss of transported medium in the network. Principally both algorithm operate similarly, they reduce whole network into the tree, which is much more easily manageable.

Kruskal's algorithm allows us to design new network using known nodes, constructed so, that it will conform to optimal size based on criterion of choice.

9.5 Network Models and Critical Infrastructure

By **CI** we mean as per crisis law (240/2014 Sb.) [64]) infrastructure whose: *interruption or limiting functionality would have significant impact of security of the state, endanger basic needs of its citizens, public health or states economy.*



Example

Define your own network, generate it graph using software and compute shortest path between two nodes.



Self evaluation questions

1. Describe way the mark-up algorithms work (for example for finding shortest path)
2. How do we solve shortest path problem in R?
3. Describe function of Kruskal's algorithm.



Networks and critical infrastructure

In this section we will focus on adding security dimension on **Critical Infrastructure (CI)** to place whole problematic of network models in much broader scope.

Considering that the CI is extremely broad problematic, we have no chance to cover it all. Because of that we will focus only of some interesting features that are relevant to what we already learned about the network models and what will be useful during your further study to better understand CI security or the way problems spread through such networks.

Lot of these infrastructures is networked, like telecommunications, various networks in energetic, traffic networks, etc. Network analysis allows us to explore some important properties of such networks. From this perspective we may also define CI also like complex system with strong intra- and inter-sectoral links. To get overall view about behavior of such networks we can't analyze its elements, like nodes, one by one, because complex system is more than just sum of its parts.

For analytical purposes network models play tremendous role. We can define such network similarly to eq. (9.1) [50].

$$G(t) = N(t), L(t), f(t) : J(t) \quad (9.1)$$

Where t is time, N node in network, L edge, f topology specification and finally J is an algorithm describing node and edges behavior in network. Interpretation of building parts of eq. (9.1) is dependent on the network, which we try to model. For example for railway network the nodes may represent train stations or sliding rails, edges can represent railway itself.

For long time it was presumed, that CI networks are randomly distribute. Barabasi [30] based on study of node distribution on the Internet proved, that this is not the case. Nodes on the Internet and in many other networks use topology we call *scale-free networks*.

These networks are typical by its high tolerance to failure of the random nodes. Error (outage) caused by such failure usually does not have any measurable impact on the infrastructure as whole. But in such network there are also special nodes, called *hubs*, to failure of which the network is very sensitive. If the hub fails, impact on the network is usually significant.

We can clearly see differences between random and scale-free network on fig. 9.15.

The prevailing view today is that most, if not all, critical infrastructure networks are scale-free networks.

Mathematically we can specify by degree of distribution of the links between the nodes, which has exponential characteristics (9.2) [50]:

$$P(k) \sim k^{-\gamma} \quad (9.2)$$

where $P(k)$ is probability, that the node is in vicinity of other nodes in network, γ is distribution coefficient, usually $\gamma \geq 1$. This coefficient must be measured empirically on case per case basis.

Non-randomness in organization of such networks can make them vulnerable to targeted attacks. So, if the scale-free (CI) networks are not randomly distributed, there must be some kind of master

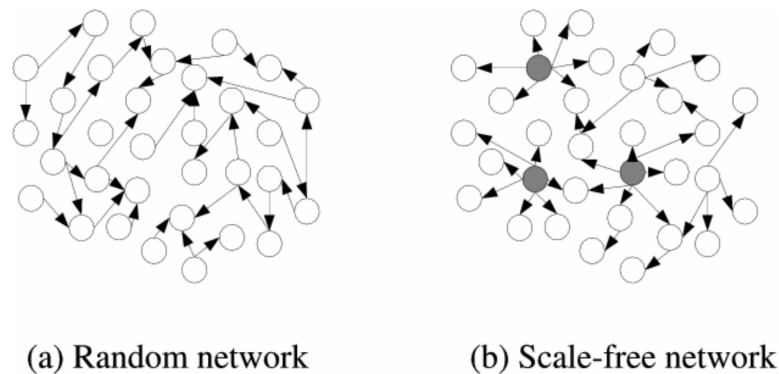


Figure 9.15: Random vs scale-free network (courtesy of [37])

plan behind them, which (or who) introduced the organization into the network, right? Well, not exactly, or maybe yes, but in different way than you may think. The organization, we are seeing in scale-free networks today is a side product of long-term repeated application of simple optimization principles. That in turn led to "organizing" of the network. Since the organization was not a goal and nobody is directly responsible for it, we may also say, that the organization emerged gradually. We call such process *self-organization* or *emergence effect*.

So, what exactly is that self-organization? Nice demonstration has been developed by [57], who developed simplified termites behavior model. We can visualize the model, see fig. 9.16. Left part of the figure shows randomly generated living space of the termites (red squares) and stones (black squares), the termites are going the "build" their termite mounds. In right part of the figure we see state of this space after many iterations of the algorithm.

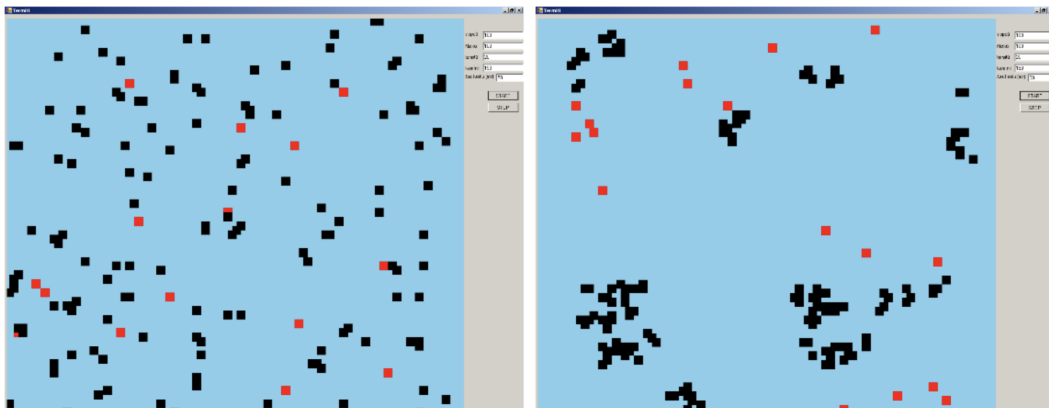


Figure 9.16: Simple model of termites behavior, left starting (random) state, right "end" (self-organized) state later (generated using [68])

The later (iteration wise) are we in computation the higher level of organization is visible in the model. Randomly distributed stones start to concentrate on "piles". Since we are using termites analogy, we can call them termites mounds. These mounds are nonrandom. We cannot say in advance, where they are going to form, only that they will form ... every time. Rules of the model are very simple. Resnick used only four simple rules [57]:

1. termites randomly move through its living space
2. if the termite finds the stone, it will pick it up
3. if the termite is holding the stone and finds another one, it will drop it
4. termite randomly move through its living space (repeat from step 1)

So whole complex organization is result of repeating only these four rules.

Termite model is obviously not network model, the self-organization principle remains same. For scale-free networks principle of repeated investments is typical. We could formulate it in following

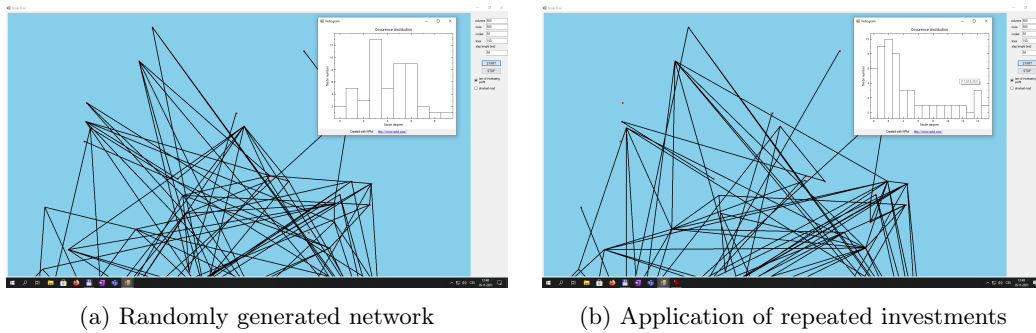


Figure 9.17: Applying law of repeated investments for generating scale-free network (generated using [67])

way [50]:

1. randomly select edge in the network
2. randomly select node from network, which is not in the edge
3. replace original edge by new one formed between two nodes with largest amount of connected edges
4. repeat from step 1)

Also in this case we can demonstrate principle in simplified form, see fig. 9.17. Application of the approach will gradually transform originally random network to organized one. Approach prefers creation and gradual increase in importance of hubs. Note also the distribution of nodes degrees at beginning and at the end of simulation.

By the node's degree we mean number of links (edges), the node is connected to other nodes in network. At beginning the distribution is approximately random. That is clearly visible in bar charts. See bell curve typical for normal distribution of probability. By application self-organizing rules, this distribution starts to change. Now it is more like poisson's distribution. We can interpret that as a preference of small number of highly connected nodes in network on one side and high number of the nodes that are connected to only one other node.

Summary

This section of text was intended more as inspiration than anything else. Pursuing knowledge in variety of fields may prove to be beneficial in other areas.

We learned some interesting information on critical infrastructures networks. We know now that they are mostly scale-free networks. These are typical by existence on small number of highly interconnected nodes. We call them hubs. Scale-free networks are highly resistant to failures of random nodes, but may be vulnerable to failures of hub nodes.

Into this high level of organization CI networks typologies evolved gradually on their own using repeated application of self-organizing principles.

Self-evaluation questions

1. Define critical infrastructure
2. What are difference between random and scale-free network?
3. What is self-organization?
4. How are distributed nodes in scale-free network?
5. Explain principle of repeated investments.

Chapter 10

Balance models



Study guide

When deciding during various emergencies we should come from deep, in past prepared knowledge/information about the organization we are trying to save. One of basic functions of the company (at least the manufacturing one) is to produce the products for sale.

Balance models, we will deal with in this chapter, are capable to bring us this insight by modeling whole chain of production – allowing us to describe how the raw materials are transformed into the form of half-finished products and these into final products.

Such information is usable for evaluation of emergencies impacts on production capability of the company, so we can develop contingency plans optimizing the post emergency recovery effort.

After reading through this chapter you will

Know

- What the balance models are
- How do we construct one
- And how can we solve one



Time required to study chapter

For getting through this chapter you will need at least 30 minutes.

10.1 Basics of balance modeling

Balance models are used for exact description of interconnection in production cycle in the company thus allowing us to describe the way the inputs (raw materials, half-finished products) transform on outputs (final products). Balance models are used by economists for long time for planning of production capacity and planning of resources needs of the production.

From technical point of view the models are based on family of models developed in 60-ties of last century by American economist Vasilij Leontief, called *input-output models*. Leontief developed these models to describe and quantify dependencies among various sectors of economy. Output of one sector serves as input of another sector. Leontiesf validated the models on USA economical data from World War II.

For his work, Leontief was awarded Nobel price for economy in 1973.

Today variety of such models exists, for their common properties we call them *leonties's models*. Balance models are representative of such approach and we can use them on level of one company to better understand various transformations of energies, raw materials, semi-products and products in the production process.

There are also other leontief's models to evaluate possible impacts of critical infrastructure elements disruption. For purposes of **CI** modeling huge amount of inter-sectoral data need to be collected on regional basis, so we will only briefly discuss these model in section *Security applications of the balance models* available latter in this chapter.

In emergency planning we can use this type of models for modelling of the emergencies impact on our production capability, especially how will take effect disabling or destruction of part of technology in the company on its ability to produce. Based on these results we may plan way of effective recovery from emergencies.

Lets discuss small example of the problem. On fig. 10.1 we have block schema of the simplified production process of the powder fire extinguishers. The structure of the production process is well known before we usually start with balance models – since the organization has to know how to produce the goods to survive on the market.

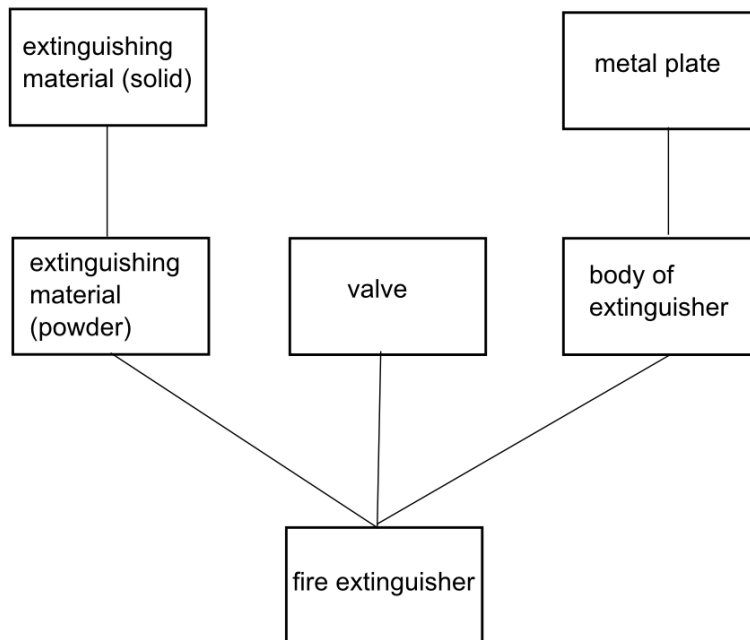


Figure 10.1: Production of the Powder fire extinguisher

Balance models are used to mathematically describe the production process by describing the way the production inputs transform into the goods or half-finished products (semi-product). The difference between the two are in its usage. We use semi-products to produce other semi-products and product, but the (final) products, are finished and in model always go outside the model.

It is possible to express interconnection between raw materials, half-finished products and products by following equation 10.1:

$$x = y + Ax \quad (10.1)$$

where x is overall production of the given product, y is the production expedited outside modeled system, Ax is amount of products used for production of different products and finally A is a matrix of specific consumption of semi-products

To apply the general model eq. (10.1), we need to know the way inputs are being transformed to outputs inside the production process. To do that we need to identify specific:

- raw materials - these are inputs

- semi-products - these are being produced during production process and are being utilized for production purposes (to produce another product). Semi product can be also sent outside of the model
- (finished) products - all of these go outside of the model (they are not used to produce something else in the model)

When we talk about consumption in the production process we mean consumption in the modeled part of the production process. From this point of view we may also see raw materials as a final product of some other production process, which we do not see in our model. Similarly final products may be utilized in other production processes, which we again do not see in our model.

One of basic questions for the modeling is the decision on the scope of the balance model. So we may decide to model single production line, whole organization or even entire supply chain depending on the type of information (purpose) we want to derive from the model.

Equation 10.1 is written in matrix form. If we express the products from figure 10.1, we get following interpretation of the model:

- x_1 extinguishing material (solid)
- x_2 extinguishing material (powder)
- x_3 valve
- x_4 metal plate
- x_5 body of extinguisher
- x_6 fire extinguisher

We can express equation (10.1) using system of equation (10.2). To specify the balance model, we have to state specific consumption of the half-finished products, see table 10.1.

Table 10.1: Specific consumption

Product	Half-finished product	Specific consumption
Fire extinguisher	Extinguishing material (powder)	10 kg/1 part
	valve	1 part/1 part
	body of extinguisher	1 part/1 part
Extinguishing material (powder)	Extinguishing material (solid)	1 kg/kg
Body of extinguisher	Metal plate	0,25 plate

$$\begin{aligned}
 x_6 &= y_6 + 10x_2 + x_3 + x_5 \\
 x_2 &= y_2 + x_1 \\
 x_5 &= y_5 + 0,25x_4 \\
 x_1 &= y_1 \\
 x_3 &= y_3 \\
 x_4 &= y_4
 \end{aligned} \tag{10.2}$$

This equation system is easily solvable, when we understand that the production values are defined by commissions and half finished products may be completely substituted by its balance equations. Simple production systems are even solvable by hand, for more complex scenarios we usually prefer using spreadsheet.

Equation (10.1) may be also expressed in this way (10.3):

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,25 & 0 & 0 \\ 0 & 10 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \tag{10.3}$$

The balance model higher is very simple. In practice we often formulate several models describing different aspects of the production process. We split it into several smaller, more manageable models. Event these small models may consist of hundreds variable. Out illustrative example is very simple

as it consist only from one final product, two semi-products and three raw materials - together six variables.

Apart from using basic balance model to quantify number of required products and semi-products, we usually use also additional model to compute raw materials needs, energetic requirements and technological limitations.

All these additional models require computed product balance. So the computation always starts with computing (10.1). From this we may derive raw materials needs which are required to produce them, see eq. 10.4.

$$Bx = s \quad (10.4)$$

where x is a vector of the overall production of the given product, s is vector of the overall need of the raw materials to produce all products x and B is a matrix of the technical coefficients describing raw materials usage to produce product x .

Similarly capacity model (10.5) for the production can be used.

$$Cx = f \quad (10.5)$$

where x is vector of the overall production of the given product, f is vector of the requirements on production machines, etc. and C is a matrix of coefficients describing the needed capacity of given machine to produce unit of product x

As you can see the model will not answer the question if the there is enough capacities available to fulfill the production needs of the company. The results have to be interpreted – we get the information on how large the required capacities are. It is up to us to know if we have large enough “real” capacities and if not to solve the problem. Perhaps we can rent capacities elsewhere, the model by itself would not know.

We interpret models described by equation (10.1) and (10.4) in same way.

When constructing and using the model our limitation is only our fantasy. We can work with alternative scenarios, incorporate results into other models. We can model dynamically to explore whole fan of possible scenarios. We can do many things.



Balance models

Example of balance model is uploaded to <http://lms.vsb.cz> in XLS format (MS Excel). The example is solved in two different ways (equal result wise).



Questions

1. What are balance models good for?
2. Explain balance equation of the product.
3. Can you specify parts of the balance model (what can be computed by it)?
4. How do we interpret the model (what information does it compute and how do we work with it)?

10.2 Application of balance modeling to security problems

From previous section we already know, that the purpose of the balance models is to allow us understand balances structure describing transformation of materials and energies in modeled system. That information can be relevant for planning purposes as it provides different perspectives to:

- **Bussines Continuity Planning (BCP)**
- in accident management planning - for quantification of the accident on the production process
- recovery planning after the emergencies

- and possibly other types of plans

We can add presumed impacts of the emergencies by introducing boundary conditions to the model. Then we can use model to quantify effect on production process. This type of information will allow us to identify parts of the process where the organization can benefit most by better planning and investing in protective measures.

As a student of security related study program, you already have (or should have) relatively in depth knowledge on risk. If we consider one of simplest models risk models $R = P \cdot C$ defining the risk as probability (P) of consequence (C) occurrence, we need a way to quantify the consequence. Possible damages to technology and buildings can be estimated as a costs to repair or replace the damaged, but what of a time the organization will be unable to produce?

Balance models are one way to shed some light to the problem. There is also another benefit. These models are often routinely used in **Enterprise Resource Planning (ERP)**, so the results of such models are usually easy to communicate with organization's management. In turn this can allow us to more easily find support for necessary investment into protective measures.

For applications in **CI** some input-output models have been developed. One of more popular is **Input-output Interoperability Model (IIM)**. The model has been developed by Setola et al. [61] in 2009. From mathematical point of view this model is almost same as classical I-O models, see eq. (10.6).

$$x = Ax + c \quad (10.6)$$

where x is vector of production in sectors of interest in economy, A is consumption matrix allowing us to model relations (dependence) between the sections and finally c is a vector of external demand, which represent the production going out of modeled system.

The benefit of IIM is that we can use it in simplified form, which considers only the intensity of the relations between sectors of **CI**. In such case we may limit $x \in \langle 0; 1 \rangle$. For modeling of cascade effects, the disruption propagation across the sectors of **CI**, $x = 0$ will mean complete interruption of the "communication" to dependent sectors, while $x = 1$ will mean that the disruption in sector will have no effect on operation of dependent sectors (disruption does not propagate to these sectors).

For IIM model we can compute *dependency index* (10.7).

$$\gamma_i = \sum_{j=1}^n a_{ij}^* \quad (10.7)$$

Technically we just sum the rows of matrix A .

Dependency index expresses robustness of infrastructure considering its interoperability with other section. It represents maximal interoperability in infrastructure i in case, that other sectors of **CI** are not functional.

The lower the declinr in γ_i (when $\gamma_i < 1$), the better the infrastructure can maintain its operation and provide services to other infrastructures.

Summing columns of matrix A we can get the value characterizing influence of the sector (10.8), also known as *influence gain*.

$$\rho_j = \sum_{i \neq j} a_{ij} \quad (10.8)$$

High values of ρ_j mean, that the interoperability of j sector introduces significant degradation of the system, when its disturbed.

Today, there are also available more sophisticated modeling tools, allowing us to chain partial models one after the another to get full picture on consequences of modeled event to the region.

Example of such system is HAZUS [42], which is being developed and extensively used by FEMA. Model chain is illustrated on fig. 10.2.

HAZUS is based on geographical data managed by **Geographic Information System (GIS)**. It allows estimation of possible polygons of expected consequences of the modeled event (i.e. hurricane) and integrates it with stored information on infrastructure present in the area.

Model of hurricane has been used in fig. 10.2. In left part of the figure, colored polygons are used for different wind speeds. Together with information on the infrastructure, the model can choose

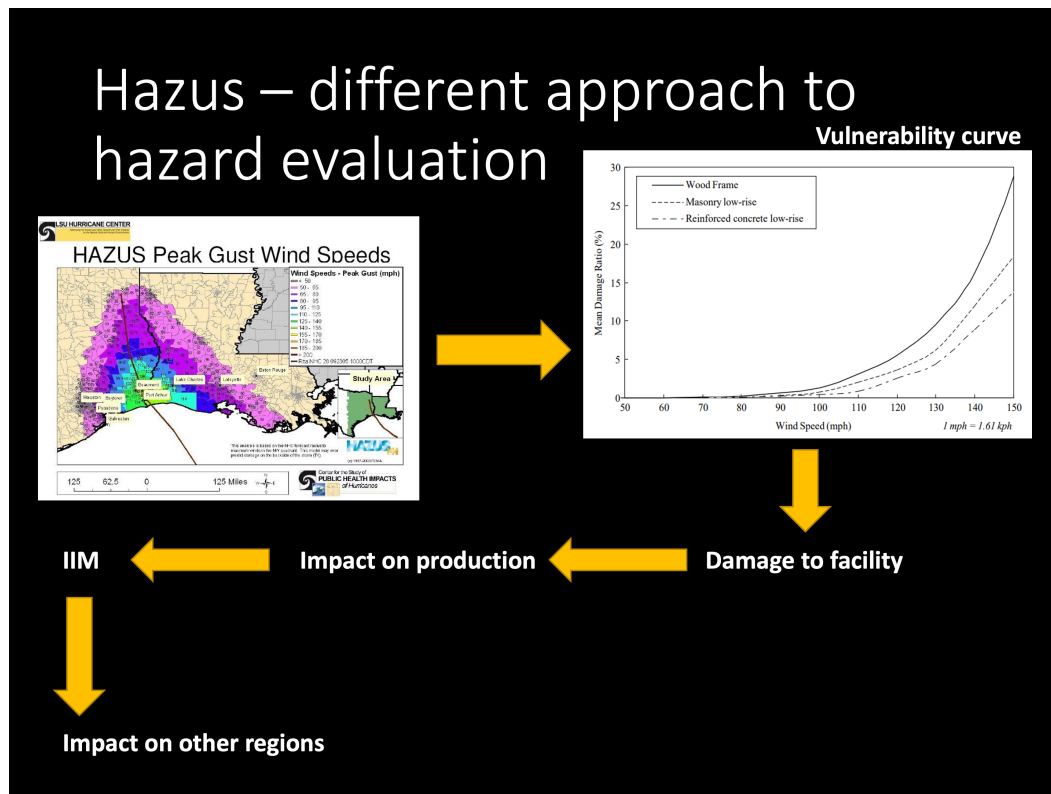


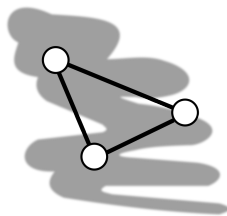
Figure 10.2: HAZUS - model chain (courtesy of [42])

appropriate vulnerability curve to evaluate what kind of damages will the wind cause to the infrastructure.

Chaining the models together - maps, polygons characterizing the event, consequence to infrastructure computation, IIM to see how the disruption will spread provides complex information to support strategic decision making for local governments and institutions like FEMA to form public policies, standards and plan to prepare for such events.

HAZUS itself is available free of charge. To get the copy the registration is needed with specification of the purpose. It is a logical choice, if we consider, that the models allowing us to plan our effort to minimize the impact of the event, can be also used to plan the event to be maximally effective.

To operate it, you will need ArcGIS and geographical information - maps, infrastructure, etc. for the region at appropriate resolution level. Most of the countries do have these dataset available in some form.



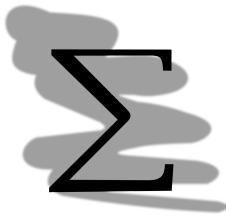
Vulnerability curves

Vulnerability curves are one of most important tools we have to estimate consequences of accident, disruptions of various kind. They map strength of the effect with potential to create damages (i. e. wind speed) to actual damages. Main disadvantage is, that we need to construct them separately for different types of events and different types of infrastructure.

To do that extensive information on damages from previous occurrences of event is required.

Even as vulnerability curve construction is not easy, there are many available in existing literature. During your study, you can gain basic outline of these in course *Consequence modeling for emergencies* or in textbook to this course [70]^a.

^aat present time in Czech only



Summary

Balance models are application of input-output models used for modeling of the relations between inputs and outputs inside of production process. Balance models consist of:

- product model,
- raw material model
- capacities model.

First we compute product model as all other models use it as its input.

For purposes of emergency planning we can use this type of models the better understand consequences of disruption to the production process as result of some kind of accident or other type of event.

Chapter 11

Linear Programming



Study guide

Another type of problems, we have to solve, are those described by system of constraints expressed by equations. For such type of problems we are searching for solution inside the constrained solution space minimizing or maximizing using effectiveness function.

After studying this chapter you will know

How

- to solve linear programming problem using simplex method



Time required to study chapter

For getting through this chapter you will need at least an hour.

We line linear programming into group of algorithms for mathematical solution of optimization problems called *mathematical programming*. Linear programming is called linear, because we solve the problem using system of linear equations.

In linear programming we work with one basic effectiveness function which we optimize. We usually do that by maximizing or minimizing system of the linearly expressed constraints of our problem. Using these constraints we limit the space of possible solutions – so we may actually come to some conclusion.

We may express effectiveness function using following equation (11.1).

$$optimz = \sum_{j=1}^n c_j x_j \quad (11.1)$$

Where z effectiveness function

Optim required optimization max or min

c_j evaluation of variables in effectiveness function (for example price, variable costs etc.)

x_j optimized variable

To limit the solution space of the effectiveness function z we may use three types of the constraints. You can see its mathematical expression in equation (11.2).

$$\begin{aligned} \sum_{j=1}^n a_{ij} x_j &\leq b_i & i = 1, 2, \dots, k \\ \sum_{j=1}^n a_{ij} x_j &= b_i & i = k + 1, k + 2, \dots, k + p \\ \sum_{j=1}^n a_{ij} x_j &\geq b_i & i = k + p + 1, k + p + 2, \dots, k + p + s \\ x_j &\geq 0 & j = 1, 2, \dots, n \end{aligned} \quad (11.2)$$

Where

a_{ij} technical coefficient of the model, these are set independently for each model and they do not change then

x_j optimized variable

b_i right sides of the constrains, these are set in dependence on model (capacity constrains etc.)

The optimized variables must have zero or positive value. This constrain is based on fact, that we usually can't produce negative amount of whatever. Allowing for negative values would lead to non-standard problem, which would not be solvable by linear programming (at least not by method we will use).

First lower then or equal constrain (\leq) notes the situation, that the solution of the presented problem is constrained by availability of some kind of goods or material and the amount of it is limited. The equal sign ($=$) notes the situation, when there are known production processes (or similar), where we know exactly how they do come out. For example we must know exactly how much of raw materials is needed to produce unit of the desired product.

Finally greater then or equal (\geq) describes the situation, when we try to introduce into model some kind of obligation – for example we have already agreed to supply certain amount of the goods – that amount would be the minimal value and we can go only higher from there.

So what are the problems which are solvable by this method? Primary use of this method is by economy for optimization of the manufacturing programme (for sake of profit), but it is general method, which is usable generally – for example for optimization of venting gas from one source using several venting stations, for optimization of transportation of the good using different trucks with different capacity etc.

So how exactly do we formulate the problem and how do we solve it. As first step it is necessary to formulate the model using equations (11.1) and (11.2) – as system of effectiveness function and constrains equations, as in the next example:

$$\max z = 2x_1 - 3x_2 + 4x_3 \quad (11.3)$$

$$\begin{aligned} 4x_1 - 3x_2 + x_3 &\leq 3 \\ x_1 + x_2 + x_3 &\leq 10 \\ 2x_1 + x_2 - x_3 &\leq 10 \end{aligned} \quad (11.4)$$

Such system of equations is solvable using variety of available algorithms as:

1. primary algorithm (simplex method)
2. dual algorithm
3. ...

We will use only the simplex method as most famous representative of linear programming algorithms for solution of this type of problems.

Following text is rather harsh, especially if you heard of linear programming just few moments ago, that's why, you should try nice tutorial by Stephen Waner [63], which is in English and will interactively lead you step by step to learning the method.

So let's solve the problem using simplex method. First step is to transform the constrains to equations. We do that by adding additional variable d_i if the operator was \leq . If the operator was \geq , we have to formally transform it to \leq by multiplying whole equation by -1. Our example would look as follows.

$$\begin{aligned} 4x_1 - 3x_2 + x_3 + d_1 &= 3 \\ x_1 + x_2 + x_3 + d_2 &= 10 \\ 2x_1 + x_2 - x_3 + d_3 &= 10 \\ -2x_1 + 3x_2 - 4x_3 + z &= 0 \end{aligned} \quad (11.5)$$

Additional variables, if computed in optimal solution as not zero represent amount of resource b_i (see (11.1), which will remain, which will not be used. System of equations higher can be transformed to matrix form (see table 11.1).

With each table there is associated just one *basic solution*. Basic solution is one of virtually infinite number of possible solution. To find such solution we search for clean columns, in which there is only

Table 11.1: Working table

x_1	x_2	x_3	d_1	d_2	d_3	z	
4	-3	1	1	0	0	0	3
1	1	1	0	1	0	0	10
2	1	-1	0	0	1	0	10
-2	3	-4	0	0	0	1	0

Table 11.2: Iteration 0 – working table – basic solution

x_1	x_2	x_3	d_1	d_2	d_3	z		Basic solution
4	-3	1	1	0	0	0	3	$d_1 = 3/1 = 3$
1	1	1	0	1	0	0	10	$d_2 = 10/1 = 10$
2	1	-1	0	0	1	0	10	$d_3 = 10/1 = 10$
-2	3	-4	0	0	0	1	0	$z = 0/1 = 0$

one nonzero value in the row. Variables in these columns are active, while the remaining variables are not active. Basic solution of our table would be as follows (see table 11.2).

As you can see in table 11.2, only variables d_1 , d_2 , d_3 and z are active all other variable are inactive (x_1 , x_2 and x_3) and we set them to zero. Active variables form up basic solution for this iteration. For example we compute the value of d_1 dividing right side of equation with value for d_1 in its column so $d_1 = 3/1 = 3$.

Basic solution for this iteration would be:

$$\begin{aligned}
 d_1 &= 3 \\
 d_2, d_3 &= 10 \\
 x_1, x_2, x_3, z &= 0
 \end{aligned}
 \tag{11.6}$$

Well such solution is indeed possible, but it is clearly way too far from optimal solution – which tells us, that our computation will have to continue. We have to transform table after chosen pivot. We search for pivot using two simple criterions:

1. Choosing of the column (for pivot) – we choose the column with most negative number in effectiveness function (last row of the table). The lowest number is -4 in x_3 column.
2. Pivot has to be always positive number with highest testing ratio. Testing ratio is the ratio of right side of the equation and value of the column. We compute testing ratio in table 11.3.

Table 11.3: Testing ratio

x_1	x_2	x_3	d_1	d_2	d_3	z		Testing ratio
4	-3	1	1	0	0	0	3	$3/1 = 3$
1	1	1	0	1	0	0	10	$10/1 = 10$
2	1	-1	0	0	1	0	10	
-2	3	-4	0	0	0	1	0	

Pivot is in the intersect of the chosen column and row. In table 11.3, it is highlighted in bold italic (in the crossection of the highlighted row and column).

In next step, we have to adjust the table so, that the pivot column is zeroed out, excluding pivot row (see table 21).

Table 11.4: Iteration 2

x_1	x_2	x_3	d_1	d_2	d_3	z		adjustment
3	-4	0	1	-1	0	0	-7	- row 2
1	1	1	0	1	0	0	10	
3	2	0	0	1	1	0	20	+ row 2
2	7	0	0	4	0	1	40	+ 4 * row 2

Because there are no negative numbers in effectiveness function row, the basic solution of this iteration is also the optimal solution of the task. If there was a negative value in last row, we would repeat the whole process over and over again.

Our solution is:

$$\begin{aligned} x_1, x_2 &= 0 \\ x_3 &= 10 \\ d_1 &= -7 \\ d_3 &= 20 \\ d_4 &= 40 \end{aligned} \quad (11.7)$$

Of course, it would be highly ineffective if we had to compute the solution by hand, that's why we usually use linear programming calculator, such as [24]. Usually the programs for mathematical computations such as MathLab, SciLab or R support linear programming too. There are also smaller single purpose programs specialized on linear programming such as Finite mathematics utility: simplex method tool [6].

The equation system for the problem is written directly into window of the WWW browser of your choice, see fig. 11.1. See definition and results ... yes they are different based on type of algorithm and the way it is implemented.

Type your linear programming problem below. (Press "Example" to see how to set it up.)

Maximize $p = 2x - 3y + 4z$ subject to
 $4x - 3y + z \leq 3$
 $x + y + z \leq 10$
 $2x + y + z \leq 10$

Solution:
 Optimal Solution: $p = 27.75$; $x = 0$, $y = 1.75$, $z = 8.25$

Solve Example Erase Everything Rounding: 6 significant digits
 Mode: Decimal Fraction Integer

The tableaux will appear here.

Tableau #1

x	y	z	s1	s2	s3	p	
4	-3	1	1	0	0	0	3
1	1	1	0	1	0	0	10
2	0	0	0	0	1	0	10
-2	3	-4	0	0	0	1	0

Tableau #2

x	y	z	s1	s2	s3	p	
4	-3	1	1	0	0	0	3
-3	4	0	-1	1	0	0	7
2	0	0	0	0	1	0	10
14	-9	0	4	0	0	1	12

Tableau #3

Figure 11.1: Finite mathematics utility: simplex method tool

We can also demonstrate computation of the linear programming problem in the R. We will use same example, but this time around we will compute it using *lpSolve* package [33]. Remember to install it first using `install.packages()` function, or via GUI in RStudio.

Výpis 11.1: Linear programming example solution using *lpSolve* package

```

1  library(lpSolve)
2  f.obj <- c(2, -3, 4)
3  f.con <- matrix(
4    c(4, -3, 1,
5      1, 1, 1,
6      2, 1, -1),
7    nrow = 3, byrow = TRUE)
8  f.dir <- c("<=", "<=", "<=")
9  f.rhs <- c(3, 10, 10)
10
11 result <- lp("max", f.obj, f.con, f.dir, f.rhs, compute.sens=TRUE)
12 result$solution # Variables final values
13 # sensitivities
14 result$sens.coef.from
15 result$sens.coef.to
16
17 result$duals # Duals of the constraints and variables are mixed
18 # Duals lower and upper limits
19 result$duals.from
20 result$duals.to

```

Both web app and lpSolve came with exactly same solution: $x_1 = 0, x_2 = 1.75, x_3 = 8.25$. If both approaches yield same results, why bother with "programming" in R. Main reason is that it provides much higher flexibility. It allows us to compute much larger tasks and also explore the results in much greater detail.

For example it allows us to analyze sensitivity of the solution to the changes of the coefficients. For that we can use `sens.coef.from` and `sens.coef.to` to find lower and upper bound of the solution. It allows us to specify range of the objective's function coefficients, for which the computed solution is still optimal.

In our case $x_1 \in < -1.0e + 30; 9.25e + 00 >$, $x_2 \in < -1.2e + 01; 4.00e + 00 >$ and $x_3 \in < 1.0e + 00; 1.00e + 30 >$. The computation using the package is purely technical. That means that the `lp()` function responsible for solving the program and computing the sensitivities, has no knowledge of true meaning of what has been computed. If the coefficients represented some production process, i.e. number of products 1, 2 and 3 we should produce to maximize the profit, then values < 0 have no meaning. Sensitivities may be computed way outside of the ranges, that are possible in practice (in interpretation of the model).

That is also one of reasons we had to explicitly set `lp()` to compute it using `compute.sens = TRUE` parameter, as the function does not compute sensitivities by default.

So what does it mean? How do we interpret it. To answer this question, let's consider much simpler task to compute composition of the vegetables the farmer should plant in the spring. Let's limit selection to only two vegetables, so we can interpret the results in 2D, using graph. This example is adapted from Ong [55], which discusses the problem in much more detail.

It also provides introduction to the R and examples in statistics and solving of optimization problems. While at the time writing of this textbook, the resource was in draft state only, what was available was very good, so the resource¹ can be recommended for further study.

Back to our example

$$\begin{aligned}
 \max p &= 0.15x_1 + 0.40x_2 \\
 x_1 + x_2 &\leq 200 \\
 0.20x_1 + 0.70x_2 &\leq 100
 \end{aligned} \tag{11.8}$$

Optimal solution for the problem would be $x_1 = 80$ and $x_2 = 120$, with sensitivities for x_1 coefficient $\in < 0.1142857; 0.400 >$ and x_2 coefficient $\in < 0.15; 0.525 >$. Graphically we can see the solution on fig. 11.2.

Since we are working in 2D we can interpret the constraints easily as the lines which limit possible solution space. Green highlighted area represent all possible solutions for which the constraints hold. Blue point then represents optimal solution.

If one of the coefficient in objective function change so drastically, that it would go out of range identified by sensitivity analysis, the solution would shift. If the coefficients in our example represent unit selling price of the vegetable, that selling price would be only an estimate due to demand and availability. If coefficient for x_1 changed from 0.15 to 0.1, which is outside of range, the price for

¹<https://desmond-ong.github.io/stats-notes/>

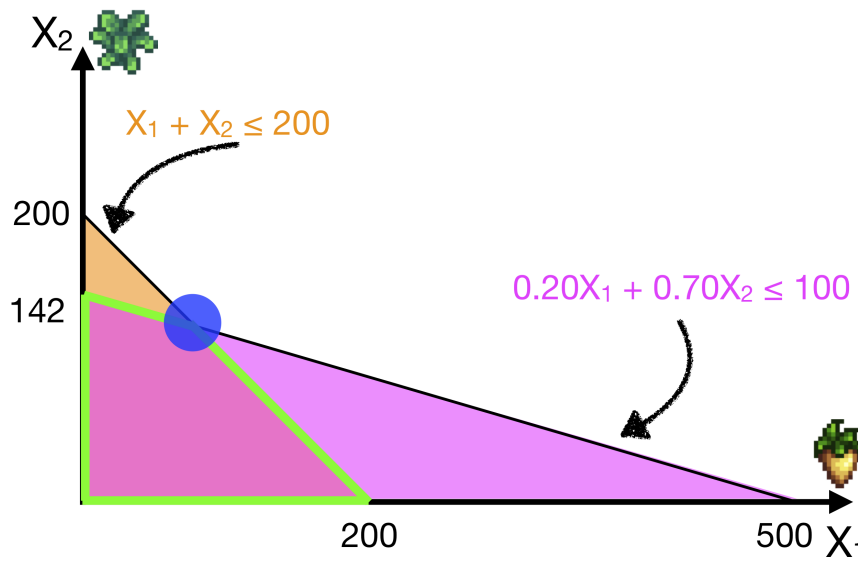


Figure 11.2: Farming problem - constraints and optimal solution visualization (courtesy of Ong [55])

vegetable x_1 would be so low, that it would no longer be profitable to sell it. So all production would shift to x_2 .

Another form of sensitivity analysis is analysis using *shadow prices*. Ong [55] defines shadow price of a constraint as the change in the objective function value per unit-increase in the right-hand-side value of that constraint with all other things being equal (*ceteris paribus*). Let's use farming example once again and change one constrain from $0.2x_1 + 0.7x_2 \leq 100$ to $0.2x_1 + 0.7x_2 \leq 101$.

We can see the difference on graph, fig. 11.3. The line representing the constrain moved slightly right, but that also slightly moved the optimum.

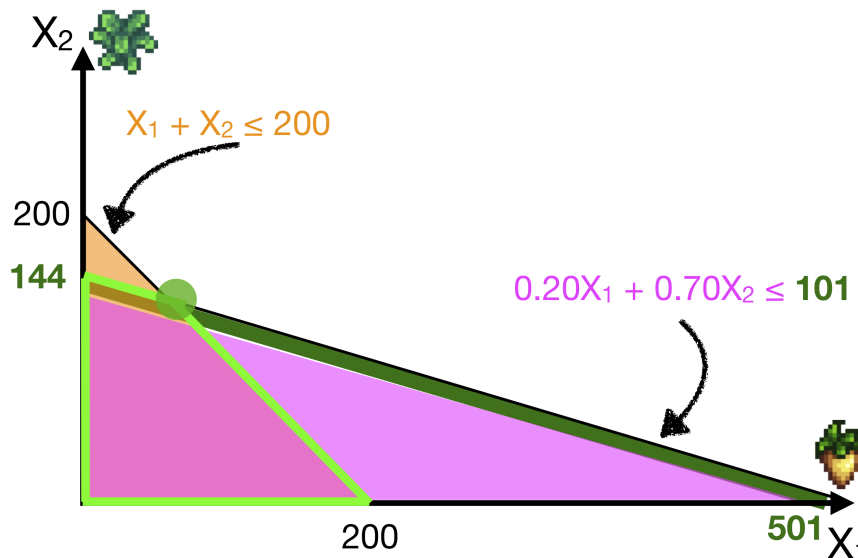


Figure 11.3: Farming problem - shadow price demonstration (courtesy of Ong [55])

New optimum would be $x_1 = 78$ and $x_2 = 122$, leading to increase of profit of 0.5 to 60.5. Computed 0.5 is the shadow price.

Now let's go back to R and see what kind of answers provides lpSolve's `lp()` function. We call shadow prices by calling property `duals` of computed results. `$duals` provide vector with number of elements equal to $2x$ number of constrains used in linear programming problem.

First three represent shadow price for the three constrains, in our case 1.75, 2.25 and 0.00. The

last three numbers $(-7.25, 0, 0)$ are shadow prices of implicit non negativity constrains $x_1 \geq 0$, $x_2 \geq 0$ and $x_3 \geq 0$.



Linear programming task

Two students Ann and Charles work X and Y hours weekly. Together they may work only 40 hours a week. Considering rules for the brigade-workers, Ann may work maximally 8 hours longer then Charles, but Charles may only work 6 hour longer then Ann. Additional constrain is $18 \leq 2y + x$. Try to find optimal solution for situations:

1. If hour pay of Ann is 15 EUR and Charles's 17 EUR - find maximal combined pay
2. If hour pay of Ann is 17 EUR and Charles's 15 EUR - find maximal combined pay
3. If both of them get 16 EUR - find maximal combined pay



Linear programming task - solution

Ann 15 EUR (y) and Charles 17 EUR (x)

Variable represents hours each of them will work.

Effectiveness function: $\max z = 15x + 17y$

Both may not exceed 40 hours a week: $x + y \leq 40$

Ann may work 8 hours longer then Charles: $x - y \leq 8$

Charles may work 6 hours longer then Ann: $-x + y \leq 6$

Additional constrain: $x + 2y \geq 18$

Optimal solution: Ann 17 hours, Charles 23 hours, they will earn 646 EUR

Solution second sub task:

Try to construct the constrains yourself.

Result: Ann 24 hours, Charles 16 hours, they will earn 648 EUR

Solution third sub task:

Result: Ann 17 hours, Charles 23 hours, they will earn 640 EUR



Linear programming task 2

The company produces two products (X and Y) and for this, it is using two machines (A and B). For production of each unit of product X, it is necessary to use 50 minutes of machine A and 30 minutes of machine B. For producing unit of product Y we need 24 minutes of machine A and 33 minutes of machine 2

At the beginning of the week, there are 30 units of product X and 90 units of product 90 B in the store. Available time of machine A is 40 hours and B 35 hours.

Demand for product X this week will be 75 units and Y 95 units. Company's politic states that we maximize sum of products X and Y in store at the end of the week.

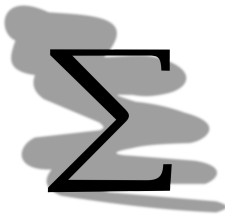


Linear programming task 2 - solution

Produce 45 units of product X and 6 units of product Y (in store at the end of the week 51 units)

**Not sure if you can solve the linear programming problems?**

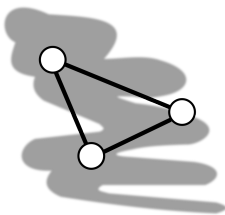
Good news, the Internet is virtually inexhaustible source of the examples, you can practice on. Solve them by both R and Simplex method tool.

**Summary**

Linear programming is used for solving of optimization problems expressed by objective function (which needs to be optimized) and its constraints expressed as a system of linear equations or inequalities.

To solve it, we use often so called *primary algorithm*, also known under name *simplex method*. The method works with iterations. Each iteration has connected one solution called *basic solution*. With each iteration we improve on the solution.

Most of the times we compute linear programming problems using software. It is both faster and also allows us to get additional information, for example for sensitivity analysis.

**Connection to other models and approaches**

One of reasons we choose to discuss the linear programming in this text book is its versatility. Since the model is entirely agnostic to meaning of the coefficients and variables of the model, it is possible to transform lot of problems to the problems of linear programming. We will see that in last two chapters focusing on Game theory and Localization models.

Chapter 12

Basics of the Game Theory



Study guide

So far we focused on the decision making based on “hard”, measurable data, but most decisions take place in dynamic environment, where other people are trying to achieve their goals. Game theory provides us with understanding of how these interactions between the “players” actually work.

After studying this chapter you will know

- what is game theory a what is it good for
- how to solve simple problem
- various model situations and its specifics



Time required to study chapter

For getting through this chapter you will need at least 30 minutes.

Up to now we described the decision or preparation to decision as process using objective “hard” characteristic, when there was no opponent to the decision with different goals than we. The decision simply hasn’t been made against human being. Such situation is not entirely common. So we need a tool to deal with this type of situation too.

Theory describing these decision situations is called game theory. Decision situation is described here as a game, where all decision makers are players. For demonstration of the theory, we usually use theoretical problem called prisoners dilemma.

The prisoner dilemma is following: *Two prisoners are awaiting their trial and they can’t speak with each other. So they think about what to say, when the trial comes. If the prisoner A frames prisoner B and prisoner B will say nothing, prisoner A will get free sentence while prisoner B gets hard sentence. If both prisoners will frame each other, both of them will get hard sentence. If both prisoners will not speak at all, they will both get low sentence. Finally if prisoner B frames prisoner A and prisoner A will not speak, prisoner B will get free while prisoner A will get hard sentence. What should both prisoners do?*

We can describe this situation in tabular form.

We can formulate inverse table for prisoner B, but for this type of the decision it is not necessary.

The goal of both prisoners is to maximize their profit in situation, when they do not have information on which strategy the opponent chooses. So we choose one based on what we think the opponent will do. Such decision is not entirely rational.

The goal of the game theory is to cultivate the game so the players do not choose for the strategy maximizing their profits but the one which is optimizing their profits. We are searching for optimal strategy.

Table 12.1: Decision strategies – prisoner A

		Prisoner A	
		speak	Speak
Prisoner B	Not speak	normal sentence	low sentence
	Speak	hard sentence	hard sentence

If we expressed the evaluation in numerical form, the numbers would form up matrix of the profits A about witch we could write the following (12.1):

$$\max_i \min_j a_{ij} \leq \min_i \max_j a_{ij} \quad (12.1)$$

Gros writes about the situation following [45]: ... *highest from minimal profits, which can first player secure can't be larger then highest profit, which the second player tries to minimize.*

In situation when there is a point representing equality of both sides of (12.1) we say that the matrix of the profits has a saddle point. This point represents optimal strategy, the player should choose if he wants to optimize his profit.

Let's demonstrate this on our prisoners dilemma. First we have to evaluate the profit of the strategies in numerical form. In our case it would be low sentence 50 points, freedom 100 points, hard sentence 0 points.

Table 12.2: Searching for saddle point

		Prisoner A		\min_j
		Not speak	Speak	
Prisoner B	Not speak	50	100	50
	Speak	0	0	0
\max_i		50	100	

Saddle point (in table bold) we got this way recommends prisoner A not to speak. If the prisoner B knows game theory, he will choose this strategy and both will optimize their profits.

Such simple games are called games of *two players in normal state*. Unfortunately not all games have such simple solution (the saddle point) – in such case we talk about *mixed extension of two players game*. Under these difficult circumstances we do not search for single optimal strategy, but for mix of strategies and probabilities by which we should change them. We search for mixed strategies. All games expressible by matrix have solution in **mixed strategies**.

Such game is transformable into form solvable using methods of linear programming.

If all decision situations have had saddle point we would see the benefits of its usage more clearly. In situation, when we have problems to quantify possible reactions on “multiple fronts” as we are rarely in situation when we have single, clearly defined, “enemy”. In reality there may dozens of them all pursuing their agendas. In such situations using game theory is much harder and its benefits much harder to see. So why are we even discussing this problematics as we clearly have no room for in depth analysis of all aspects of the game theory.

The short answer would be – to change a mind set of you as a student. And to do that we will introduce model types of the games with known outcomes. There are dozens of games, but we will focus just on two of them.

1. game of chicken
2. stag hunt

Game of chicken in games theory is based on real-world, dangerous game, favored by less responsible youth, which involves two strong cars and racing against each other. The one who swerve “chickens out”, letting the other to win. Not swerving means frontal crash against each other in full speed, which is basically death sentence for both of them.

In the model game both player have same car and are have also same capabilities as drivers and of course, they both want to win the race. Lets quantify the outcomes of the decision situation:

- win race = 10
- loose race = -10

- no win situation = 1
- crash = -100

With limited strategies as of swerve or race we can construct the matrix of the outcomes, see tab. 12.3. Since there are two players I will put in matrix outcomes from point of view of both players, so we can better understand the problem.

Table 12.3: Game of chicken

		Racer A	
		swerve	race
Racer B	swerve	1, 1	-10, 10
	race	10, -10	-100, -100

It is obvious, that both players can not win. They can both die, but that's usually something we try to avoid. Optimal strategies lie in forcing other player to chicken out. Unfortunately the players can't guarantee that they will be able to force other one to swerve there is no optimal strategy.

Logical choice would be to swerve under such circumstances for both players. Also there is a solution in symmetric Nash equilibrium – provided that both players change their strategies randomly.

In this case the negative results are really negative (we can hardly imagine more adverse negative effect than death), but we may come to different scenario using game of chicken type of the game.

Lets think about the scenario of collective bargaining in some kind of company. Unions bargain against the management for higher salaries. If they both push – the agreement will not be possible and the worker will go on strike, the company will not produce (and will not pay for worker on strike). If it goes on for long, the company will go bankrupt and the workers will be without the work – everybody loses.

When both sides give something away from their demands the workers will gain something while management has to pay something. Other options lead to the victory of one side and defeat of the other side (either management or union).

Another type of the game is **stag hunt**. This game is derived from hypothetical situation of stag hunt, where the hunters wait in hiding for the game. But it doesn't show up for long time. If the stag hunt is successful, then all of the hunters will eat, but the game is not guaranteed to come. Then a hare comes instead. If the hunter guns it down, he can eat it, but it is not large enough to fill stomachs of all the hunters. So what should the hunter choose – be selfish or cooperate with others?

Lets construct the matrix of the outcomes. We will simplify the situation for two hunters and again we will bring the point of the view of both players in the matrix, see table 12.4.

Table 12.4: Stag hunt

		Hunter A	
		cooperate	defect
Hunter B	cooperate	2, 2	0, 3
	defect	3, 0	1, 1

If the hunters cooperate, they will both eat, if one of them defects, then the one defecting gets all benefits of the betrayal but his fellow comrade in hunting will get nothing. If both of them decide not to cooperate (like continuing the hunt on their own), both of get some benefit from the decision, but not as large as in case they cooperate.

Basically the stag hunt situation in real world is the situation when two (or more) parties have to cooperate to achieve common goal. For example when protecting n endangered species two neighborhood countries arrange project to save the species from dying out. If both of them fulfill their obligations, the species will be saved. If one of them withdraws its support it will get some benefits as the money, it saved by withdrawing, can be used elsewhere to further other goals of the country. The other country will sink its money in now futile attempt to save the species and fails. If both countries withdraw their support, they may spend money in different areas, but the species dies out.

For last (3rd) edition of the textbook we add bonus game, *public goods game*. This type of game has been described in experimental economy for better understanding of the problem of *stowaway*.

The game explores relation and motivations between the individuals contributing to overall benefit of society. Stowaway does not contribute to this benefit, but utilizes investment of others in it.

Let's consider following situation, the game participants commit to invest 20 USD for improvement of well-being of society. Multiplication effect ensures, that any investment into the society will increase by 20 % and will be returned to game participants in the form of better services, etc. One of participants decides not to invest (despite his prior commitment). Since all society members use public goods, the participant who did not invest will use both improved public goods and the his not invested money.

Since the benefits of the investment are distributed equally, those who contributed will get lower benefits then was their original investment.

You can see the situation graphically on fig. 12.1.

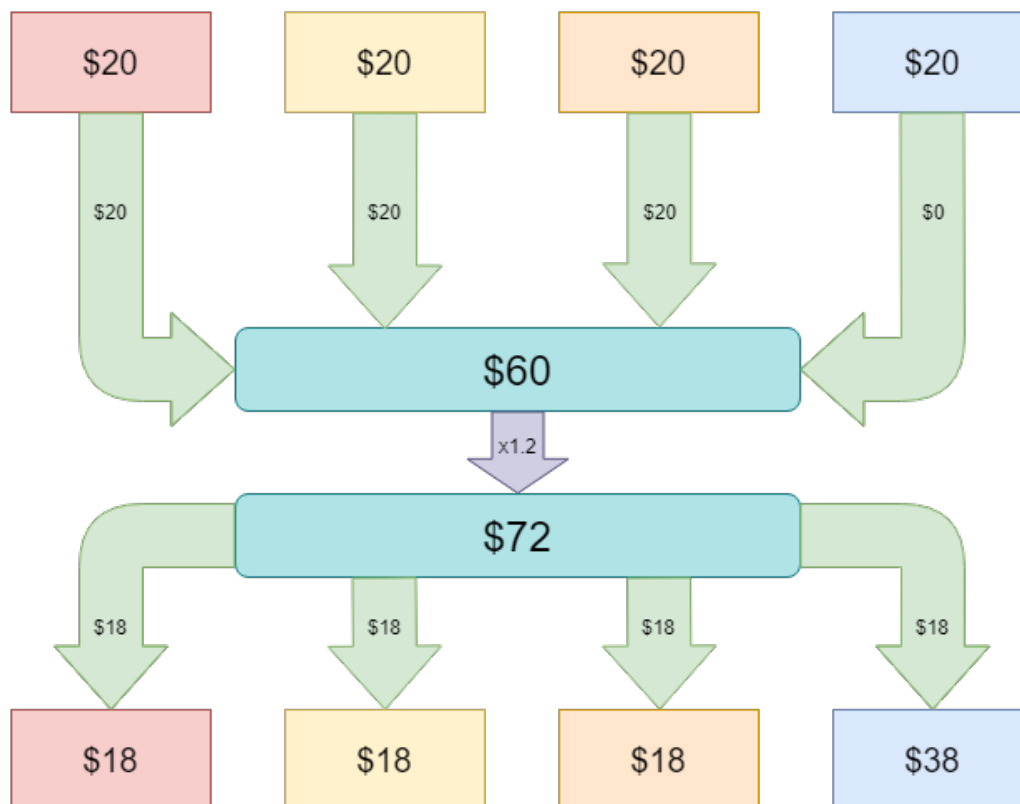


Figure 12.1: Public Goods Game (courtesy of [47])

We can think about COVID-19 vaccination in terms of public goods game. People who decide not to get vaccinated are protected to certain degree by other people who decided to vaccinate themselves, at same time they do not contribute to building immunity of the community and do not share the discomfort connected with the process (side effects of vaccination, arm pain ...).

Also there are many more types of the games. As a good start in studying of its basics you can use *Wikipedia's List of games in Game Theory* [16] page.



So what do you think?

What type of the game would be the situation of financial crisis of many countries in the eurozone and European Stability Mechanism? Is it stag hunt or game of chicken? What would be the most important moments of such decision? Please note that this question has no "right" answer. It is here for you take perhaps slightly different approach to it.



Questions

1. What is difference between stag hunt game and the game of chicken?
2. What is saddle point and how do we find it?
3. Do all games have the solution in saddle point and if not what type of the solution do we look for in them?
4. Describe prisoner's dilemma problem.

Chapter 13

Localization models



Study guide

Another optimization problem is the problem of physical localization of the object in the space.

After studying this chapter you will know

- what are localization models
- how to solve simple localization problems
- how to approach the solution of more difficult problems



Time required to study chapter

You will need at least 30 minutes to study basic theory of localization models.

Localization models exist to help us solve problem of placement of some kind of object in space. We do not place the object randomly, we try to follow some kind optimization criterion usually derived from effectivity of the investment. Typical problem could be to cover some specified area with 5G signal. We can do that placing **Base Transceiver Station (BTS)** to send the signal. Our goal would be to find optimal placement for BTS's to cover the area with minimal investment. In other words we would try to place minimal required BTS to get maximal coverage.

Solution of the problem depends on the way we define it. Do we place single object, which is going to be communicating with other objects. In network analogy we would be speaking about adding new node in existing network. Problem could be different, we may need to place multiple object. The solution will also differ if we work in 2D or 3D space. We can be limited in selection of places we may use - in this problem we only select best placement from some kind of selection.

Finally do we know how many objects need to be placed? The number itself may be a variable we need to find.

What types of the problem are we talking here about:

1. In the area we have already three manufacturing facilities, company's **CEO** wants us to find suitable place for storage where the company will temporarily store its production from above mentioned production facilities.
2. After the latest floods the area government found out that there are parts of the area, which are not covered by the wireless warning system. The task is to find suitable placement for new units of the warning system. Also the proposed investment should be cost effective (largest coverage with minimum of the warning system units).
3. ...

To find a solution, we have to specify optimization criterion. Such criterion is usually connected

to the cost of connection between the already localized objects and the new (proposed) objects. We will compute such cost N_{ij} for each connection between objects i and j .

The costs themselves are usually function transportation distance d_{ij} . Based on presumption that to transport the goods on longer distances costs more then to do the same on short distances. The amount of the goods transported x_{ij} is also important for costs estimation. Third and last value, which affects costs is the type of the vehicle used for transportation c_{ij} . Different types of the transport have different costs of usage.

Mathematically we can express the costs between the existing and new object as in equation (13.1).

$$N_{ij} = x_{ij}d_{ij}c_{ij} = d_{ij}w_{ij} \quad (13.1)$$

Since x and c values are from computational point of view constants, we can substitute them by value w , representing weight of the connection. To solve the problem we will have to optimize d value only.

Connection between new objects, if we have to place more then one object can be expressed as on equation (13.2).

$$N_{ik} = x_{ik}d_{ik}c_{ik} = d_{ik}w_{ik} \quad (13.2)$$

Cost function then may be expressed as a sum of the costs, see (13.3).

$$\min z = \sum_{i=1}^m \sum_{j=1}^n N_{ij} + \sum_{i=1}^n \sum_{k=1}^n N_{ik} \quad (13.3)$$

If we are localizing one object only, whole sum of N_{ik} equals to zero simplifying greatly the optimization problem.

As we stated higher, the crucial point of the optimization is distance value. There are three basic types of distances. We choose between them based on problem characteristic, or more specifically characteristic of transportation. Lets visualize the various distances, see fig. 13.1.

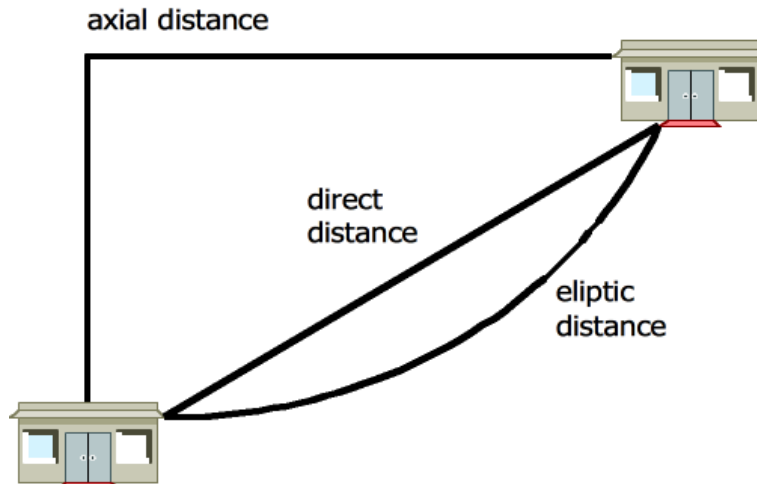


Figure 13.1: Three basic types of the distances

Mathematically we can express them using equations (13.4 – 13.7).

The first distance we usually imagine it direct distance. In localization models this distance is not so common, at least if the problem is to localize the object which should be reachable on the ground. Only rarely can we use direct route under such conditions. Perhaps if we are working on “macro” scale, for example we could be choosing city to place new logistic centre, then the road network could be perhaps approximated by direct lines.

Usage of direct distance is common for wireless communications, where we are not limited by terrain segmentations.

The equation to compute the direct distance is notoriously well known, see equation (13.4). In localization models we also use its slight modification adding k constant to compensate the fact that

in reality the direct route is not direct entirely (small curves, it goes over the hill), which lengthens it, see equation (13.5).

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (13.4)$$

$$d_{ij} = k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (13.5)$$

Axis distance can be used in situation where we have orderly road system. That is typical for newly build cities or its parts. Look for example on the road systems of the New York, see fig. 13.2. Such distance can be computed using equation (13.6).

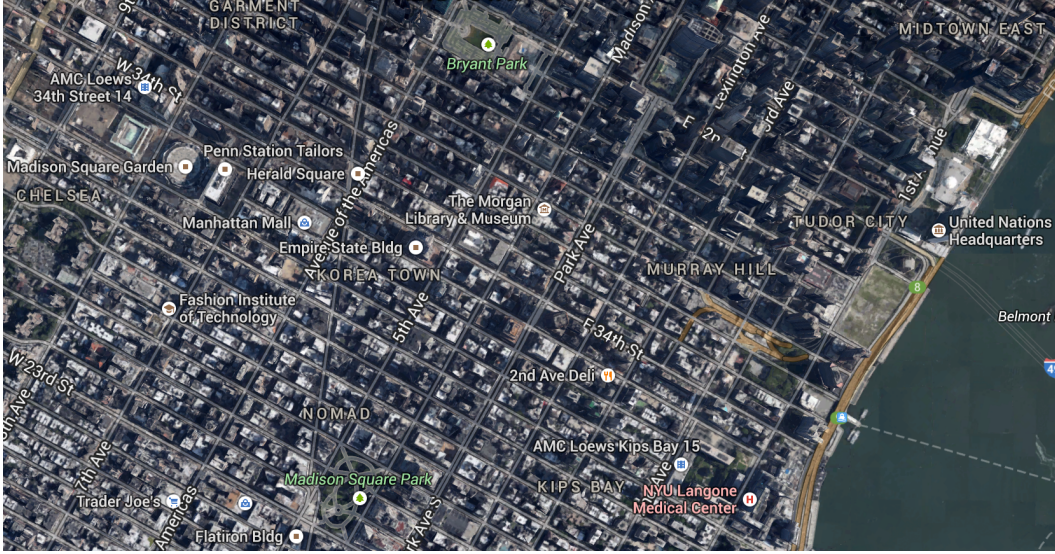


Figure 13.2: Road network of New York (source Google Maps)

$$d_{ij} = |x_i - x_j| + |y_i - y_j| \quad (13.6)$$

Lastly we can use quadratic distance as computed in equation (13.7).

$$d_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 \quad (13.7)$$

The most simple example we can demonstrate the problem solution would be localizing one object in 2D using axial distances. We would be optimizing following expression (13.8).

$$\min z = \sum_{j=1}^n |x - x_j| + |y - y_j| \quad (13.8)$$

Sum of additions can be easily expressed as an addition of the sums allowing us to separate x and y coordinate, see (13.9).

$$\min z = \sum_{j=1}^n w_j |x - x_j| + \sum_{j=1}^n w_j |y - y_j| \quad (13.9)$$

The solution, geometrically, would be in the middle. As we are working with existing road system we are trying to find coordinate for which the sum of the weights w on the right and left (for x coordinate) and higher and lower (for the y coordinate) approximately equals.

We can do that easily in tabular form. Lets find optimal location for new object, so that it may communicate effectively with four other existing objects A-D, see table 13.1 for their coordinates and weights connected to them.

Now if we want to perform the computation, we have to order the object by X and Y coordinate and choose the coordinate which exceeds half of the sum of the weights, see tab. 13.2.

Table 13.1: Localization of existing objects and estimation of the weights (courtesy of [45])

Object	x	y	w
A	1	8	50
B	4	2	100
C	9	2	80
D	8	5	70

Table 13.2: Localization of existing objects and estimation of the weights (courtesy of [45])

Order by X column				
Object	x	y	w	$\sum w$
A	1	8	50	50
B	4	2	100	150
D	8	5	70	220
C	9	2	80	300
Order by Y column				
Object	x	y	w	$\sum w$
B	4	2	100	100
C	9	2	80	180
D	8	5	70	250
A	1	8	50	300

Under given circumstances the new object shall be localized on coordinated X=8 and Y=2.

When using quadratic distance then the objective function we are optimizing looks as in equation (13.10).

$$\min z = \sum_{j=1}^n w_j ((x - x_j)^2 + (y - y_j)^2) \quad (13.10)$$

We can search for local minima of the function by deriving the equation (13.10) by coordinate variables, see (13.11) and putting them equal to zero. The equation (13.11) has analytical solution (13.12), which is then easily computable using any spreadsheet program.

$$\begin{aligned} \frac{\partial z}{\partial x} &= -2 \sum_{j=1}^n w_j (x - x_j) = 0 \\ \frac{\partial z}{\partial y} &= -2 \sum_{j=1}^n w_j (y - y_j) = 0 \end{aligned} \quad (13.11)$$

$$\begin{aligned} x &= \frac{\sum_{j=1}^n w_j x_j}{\sum_{j=1}^n w_j} \\ y &= \frac{\sum_{j=1}^n w_j y_j}{\sum_{j=1}^n w_j} \end{aligned} \quad (13.12)$$

Finally for direct distance we are optimizing the equation (13.13). Unfortunately partial derivations (13.14) do not have analytical solution, so we have to compute them using numerical methods.

$$\min z = \sum_{j=1}^n w_j \sqrt{(x - x_j)^2 + (y - y_j)^2} \quad (13.13)$$

$$\begin{aligned} \frac{\partial z}{\partial x} &= \sum_{j=1}^n \frac{x - x_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \\ \frac{\partial z}{\partial y} &= \sum_{j=1}^n \frac{y - y_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \end{aligned} \quad (13.14)$$

To effectively compute the coordinates we will use the substitution (13.15).

$$f_j(x, y) = \frac{w_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2 + \varepsilon}} \quad (13.15)$$

Epsilon in equation (13.15) should allow us to go around the possible equality of the coordinates, which would lead to situation, when we would divide by zero.

In iterations we go from the point of optimal solution for quadratic distance and compute the next set the coordinates using equation (13.16).

$$x^{(k+1)} = \frac{\sum_{j=1}^n x_j f_j(x^{(k)}, y^{(k)})}{\sum_{j=1}^n f_j(x^{(k)}, y^{(k)}), y^{(k+1)} = \frac{\sum_{j=1}^n y_j f_j(x^{(k)}, y^{(k)})}{\sum_{j=1}^n f_j(x^{(k)}, y^{(k)})} \quad (13.16)$$

Using the old and new coordinated we compute how much the iteration contributed to the improvement of the solution (25).

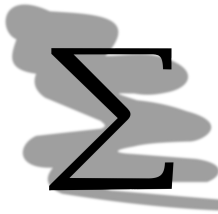
$$\Delta z = z^{(k)} - z^{k+1} \quad (13.17)$$

With each iteration, the improvement is smaller and smaller. So we are basically optimizing in kilometres, then hundreds of meters, meters, decimetres . . . It is usually not effective to reach higher precision, as the benefits of such improvement would be marginal.

Last set of coordinates is as solution.

When computing the localization of multiple new objects, doing se in hand is practically impossible, well maybe it is more impractical then impossible, so we try to use software tools to help us.

Summary



Localization model are optimization model allowing us to find optimal placement for new objects. Optimization criterion is usually derived from travel distance between the objects as distance is often directly related to the costs.

Computation differs based on number of objects we place and relief we take in account - 2D or 3D.

Questions



1. What are localization models?
2. What type of the problems do we solve by it?
3. Are there differences in look on the distances in these models?
4. How do we in general approach the solution of the localization model when localizing single object?
5. Can you specify how iterative method for computation of optimal placement of the object works?

Chapter 14

Regression models



Study guide

Another possibility to get additional information is to analyze numerical data describing some kind of problem, time series and use the information to predict at least important trends in data. Such information can improve significantly decision making.

This chapter is intended as a bridge to other courses and science - in this case statistics.

You can study *Statistics* in detail in course with same name, available for you in 2nd or 3rd study year, depending on what study program you study.



Time required for study

To go through the chapter you will need approximately 20 minutes.

This chapter is more motivational than anything else. There are plenty of techniques and approaches we can use in statistics and the space available in this textbook is very limited. So consider it as a very short introduction to Statistics and then go and continue your study either in specialized course or plethora of study materials available. There is for example Hands on ... series of books explaining in condensed form with lot of examples in R problematic of time series [49] or machine learning [35] and there are many more similar publications.

In this chapter we will focus on basics of *regression analysis*. As part of it we will analyze k potential explanatory variables to find out whether they contribute to explaining depended variable. Translated to common English - we have a dataset with numeric data and we will interleave it with curve in such way that it represents data in dataset as closely as possible.

Finding the curve (regression function) will allow us to compute the values which were not part of the original dataset. Regression model in general (linear regression) can be expressed as eq. (14.1).

$$y = \eta + \epsilon \quad (14.1)$$

where y dependent (explained variable), η nonrandom part of estimate, ϵ random part of estimate.

Mathematically we are capable to estimate only nonrandom part η . About random part of estimate ϵ we usually presume that it follows normal probability distribution and thus we can presume that the errors introduced by this randomness will not significantly skew the estimate for sufficiently sized datasets.

For η we choose appropriate regression model, for example linear regression $\eta = \beta_0 + \beta_1 x_1$ or $\eta = \beta_0 + \beta_1 x_1 + \beta_2 x_2$, etc. There are also other types of regression models, since this text should serve only as rudimentary introduction and there are whole courses and books dealing with this topic.

In general form the linear model can be expressed in following way (14.2).

$$\eta = \sum \beta_i f_i \quad (14.2)$$

where β is regression parameter, f regressor (function of explanatory variables)

Main goal of regression analysis is to estimate the regression parameters. One of most often used methods used to estimate regression parameters is least squares method. With it we replace regression parameters β with their estimates b . We are searching for such regression function, for which sum of squared error is lowest.

If we are searching for parameters of line (simplest example), we can express the model as (14.3):

$$Y = b_0 + b_1 x_1 \quad (14.3)$$

Sum of error squares can be computed by evaluating differences between data in dataset and the regression model.

We can compute the regression parameters by partially deriving sum of (14.4) by regression parameters, which leads to system of equation we can compute easily.

$$Q = \sum_{j=1}^n (y_j - Y_j)^2 = \sum_{j=1}^n (y_j - b_0 - b_1 x_j)^2 \quad (14.4)$$

After we estimate regression parameters, we need to decide, whether the model sufficiently corresponds to reality. For this purpose we can use so called *determinant index* (14.5) or *correlation index* (14.6).

$$I^2 = \frac{s_T}{s_Y} = \frac{\sum (Y_j - \bar{y})^2}{\sum (y_j - \bar{y})^2} \quad (14.5)$$

$$I^2 = 1 - \frac{s_R}{s_Y} = 1 - \frac{\sum (y_j - Y_j)^2}{\sum (y_j - \bar{y})^2} \quad (14.6)$$

Determinant index $I^2 \in < 0; 1 >$ and can be interpreted as percentage of how well estimated regression model explains variance of measured values of y .

Adequacy of the model can be evaluated using *F-test* and *partial t-tests*. F-test is used for evaluation of null hypothesis stating that apart of parameter b_0 all other parameters are zero, in other words insignificant for explanation of the y . To evaluate the hypothesis we will use testing criterion (14.7).

$$F = \frac{\frac{s_T}{p-1}}{\frac{s_R}{n-p}} \quad (14.7)$$

Null hypothesis H_0 on *significance level* α can be rejected, if $F > F_{1-\alpha}$.

Partial t-tests can be used to evaluate significance of regression parameters. We can use test criterion (14.8) for that.

$$t_i = \frac{b_i}{s(b_i)} \quad (14.8)$$



Self test questions

1. Explain hypothesis testing.
2. What is linear regression?

Chapter 15

Artificial intelligence



Study guide

Lately artificial intelligence based modes, especially neural networks, gained traction for analytic purposes as well as automation of some processes. In this chapter we will only focus on *expert systems* and *neural networks*..

Again, similarly to previous chapter, this chapter is only intended as context building for your knowledge. So do not expect to become expert after reading just about two pages of text on this topic.

And again we recommend our students to continue in their studies of this topic by selecting appropriate courses during their study or from wealth of resources available on the topic.



Time required for study

Well how quickly can you read two pages of text :-)? So lets say 20 minutes?

Similarly to statistical methods, methods based on artificial intelligence can be very effective support tool for our decision making. While in this chapter we will discuss only expert systems and neural networks, other models are readily available. Especially in area of neural networks we see in last few years extreme increase in usage, up to the point that modern processors have hardware for accelerations of neural networks utilization.

But before that we will start more deep in the history with *expert systems*. These systems were introduced in 60-ties of last century and peaked in 80-ties. End goal of these systems was to create such systems, which enables its user to access knowledge on expert in problem's domain. It tries to solve problem of expert availability at a moment notice.

Expert system presents baseline for decision making. At later time, when the human expert arrives, course can be amended accordingly, if needed.

Each expert system to be usable needs to implement its *knowledge base* in problem domain and logic to process this knowledge. Since computers still are not capable of working with loosely formatted data, the knowledge must be expressed in strictly formalized form, so that the expert system is able to work with it and provide answers.

Encoding of knowledge base is a task for *knowledge engineer*. This task is critical for success of the system and is also extremely hard, even for middle-sized system. knowledge engineer needs to deal especially with following problems:

1. problem domain expert needs to provide the knowledge engineer insight on the way (s-)he works. That is problematic as most experts like to keep their knowledge to themselves and also, even if they would like to help

2. experts might also be unable to provide information in such way, the the engineer is able to formalize it for the system as lot of this knowledge is intuitive and "automated" for the expert (veni, vidi, vici style).

That is also reason why expert system in its original form are rarely used today. Thou, expert systems gave birth to **Bussines Rules Management System (BRMS)**, which are extensively used in large organizations, banks, etc. to automate certain workflows based on defined rule sets which are similar to knowledge base of expert system.

Knowledge in expert system can be divided to declarative and procedural. We also call declarative knowledge as facts. If our expert system was classified the animals, the fact could be: dog has four legs.

Procedural knowledge define the way information will be derived in the expert system. Procedural knowledge is often expressed by rules. The rules use defined facts. For example if the classified creature is dog, then it has four legs.

Expert system during consultation with its user present set of questions to gather missing facts and compares it with the facts stored in its knowledge base to derive answers (using rules). Base on these rules it formulates its answers

Neural networks use different principle. Instead of trying to formally express all necessary knowledge of the expert, neural networks try to emulate the way biological organisms learn and solve problems. Critical for success are the ability to learn and ability to recall it when needed.

Neural network works as a black box, witch after the adaptation process provides answers.

Neural networks are almost as old as expert systems, but opposed to them their golden age is right now. In the past usage of such networks was limited by slow computers. Today hardware based accelerators exist to let us compute extremely large neural networks allowing us to solve lot of very different problems.

When talking about modern neural network we often use term *deep learning*. The depth is achieved by using many layers of neural networks with many different functions.

These are some more known types of deep learning neural networks:

- *convolutional* - for classification problems, image detection, etc.
- *recursive* - for problems like sentiment analysis, language translation, etc.
- *autoencoders* - for data compression, image transformation, etc.

Neural network transforms the inputs to outputs through pushing the signal through many layers of the the network. Links between the neurons in layers are weighted, so the signal changes as it goes through them.

Out of three types of networks mentioned above only autoencoders do not adapt by comparing output of the last layer of the network with true value and using the difference (error) to modify the weights on links of the network.

Adaptation process is iterative. After each iteration error is computed and weights adjusted in hopes to provide better outcome in next iteration. Adaptation process continues either until the global error of the networks is lower then set threshold, or preset number of iterations (epochs) is reached.

Ability of network to adapt depends directly on its type and configuration. Generally we can say, that the larger the network is the better chance it has to adapt successfully. But note that increases in size of then network grow computational requirements to adapt it exponentially. So for complex problem it may be required to pre-process the data and to experiment with network configuration to find combination of parameters which work fine for the problem.

Also note that complex problems are usually not considered solvable on CPU, because the adaptation process would simply take too much time. Instead graphical cards (especially those from NVidia) or accelerators are used for adaptation process. Specialized hardware allows us to adapts in reasonable time order of magnitude larger networks, then those we could adapt on CPU only.

For truly extremely complicated problems, usage of supercomputers may be necessary.



Self-evaluation questions

1. Try to explain adaptation process of neural network.
2. Explain problems with creating knowledge base of the expert system.

Annex

Annex 1 - Sensitivity Analysis for decision tree 2

Here we present basic R script to print graph for visualization of the costs sensitivity on changes in expected treatment cost during "small" epidemic of influenza.

Výpis 15.1: Sensitivity graph for overall costs

```

1  x <- 250:310
2  N <- 0.75 * (0.1 * x + 0.3 * 400 + 0.6 * 600)
3  plot(x, N, type="b", pch=21,
4  main="Influence of treatment costs of small epidemic on overall costs",
5  xlab="treatment costs [mil. EUR]",
6  ylab="overall costs [mil. EUR]")
7  abline(lm(N~x))

```

Annex 2 - Graphviz

One of possibilities to visualize the networks is to use open source tool GraphViz [11]. Structure of the network and its properties, important to appropriately draw the network, must be known in advance. Using general graphical programs to draw the network might be possible, but is inconvenient as the network does not have all true properties of the original network. See for example in hand made fig. ?? and automatically generated ??). Both are similar but not same.

If the purpose was only to provide illustrative example, the difference would not be significant, but if we used the network as a base for some other analyzes, automatically generated one provides much better value. This is also reason, why such specialized programs even exist. GraphViz is one of many such programs. It is open source and is available for many operating systems, though not for all of them in latest stable version.

We can work with the program either through the command line (terminal) or using GUI. Terminal interface is useful in situation when we need to generate lot of network visualizations. Batch processing is using some kind of script is usually faster than trying to do the same by clicking through the GUI.

We can define the structure of the network by specifying sequences of nodes linked by edges, for example 1 -> 2 -> 4 -> 7 -> 8 -> 11. We frame the path by identification of the graph, see listing below.

If the graph should not be directed we will use "-" instead of "->".

Whole definition for network from fig. ?? looks like this:

Výpis 15.2: Network structure for analytic purposes

```

1  digraph G{
2    1 -> 2 -> 4 -> 7 -> 8 -> 11;
3    1 -> 4 -> 6 -> 7;
4    1 -> 3 -> 5 -> 6 -> 10 -> 11;
5    4 -> 5;
6    5 -> 9 -> 10;
7    9 -> 11;
8  }

```

Easy (relatively speaking), right? GraphViz uses DOT library to generate graph of the network. Our results will be slightly different than network on fig. ??, as GraphViz places the nodes on its

own using the information on graph structure we provided it. That should not border us, the other, relevant, properties of the network remain unchanged.

We can see the result on fig. 15.1.

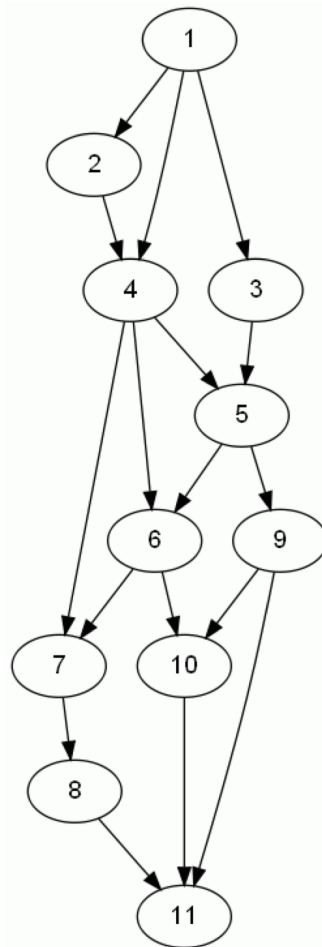


Figure 15.1: Oriented network using DOT language

To solve some problems, such as finding shortest path, we need undirected graph. By replacing oriented lines "->" with unoriented ones "-", we can easily achieve the goal, see listing below. Visualization of this network is available on fig. 15.2. We will use Neato library to generate the.

Výpis 15.3: Neato generated network structure

```

1 graph G{
2   1 -- 2 -- 4 -- 7 -- 8 -- 11;
3   1 -- 4 -- 6 -- 7;
4   1 -- 3 -- 5 -- 6 -- 10 -- 11;
5   4 -- 5;
6   5 -- 9 -- 10;
7   9 -- 11;
8 }
  
```

Even in this form our network has not enough information to allow us to compute shortest path. We also need to provide information on length of the edges. Amended notation is provided in next listing:

Výpis 15.4: Network specification using edges capacity

```

1 graph G{
2   1 -- 2 [len=10];
3   2 -- 4 [len=8];
4   4 -- 7 [len=6];
  
```

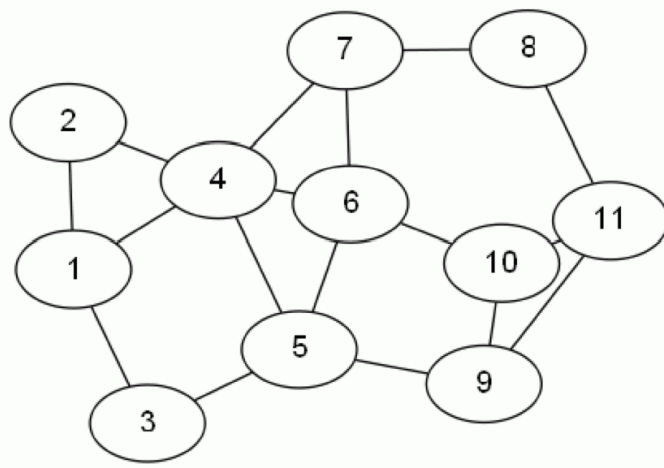


Figure 15.2: Undirected network generated by Neato library

```

5     7 -- 8 [len=11];
6     8 -- 11 [len=12];
7     1 -- 4 [len=12];
8     4 -- 6 [len=9];
9     6 -- 7 [len=6];
10    1 -- 3 [len=8];
11    3 -- 5 [len=10];
12    5 -- 6 [len=12];
13    6 -- 10 [len=10];
14    10 -- 11 [len=10];
15    4 -- 5 [len=6];
16    5 -- 9 [len=11];
17    9 -- 10 [len=6];
18    9 -- 11 [len=9];
19 }

```

Annex 3 - AHP - YAML file cars.ahp

In AHP chapter we only provided fraction of the file. Now we provide it all. You can also download the file from <https://lms.vsb.cz>.

Výpis 15.5: Full listing of YAML file cars.ahp

```

1  Version: 2.0
2
3  Alternatives: &alternatives
4    Accord Sedan:
5      Purchase Price: 20360
6      MPG: 31
7      Residual Value: 0,52
8      Safety class: midsize car
9      Cargo Capacity: 14
10     Passenger Capacity: 5
11     Surb Weight: 3289
12     crash rating: 4* in side impact front
13     60K tire cost: 700
14     Brakes Cost: 1x
15     Consumer Report: +++
16   Accord Hybrid:
17     Purchase Price: 31090
18     MPG: 35
19     Residual Value: 0,46
20     Safety class: midsize car
21     Cargo Capacity: 14
22     Passenger Capacity: 5
23     Surb Weight: 3501

```

```

24     crash rating: 4* in side impact front
25     60K tire cost: 700
26     Brakes Cost: 1x
27     Consumer Report: +++
28   Pilot SUV:
29     Purchase Price: 27595
30     MPG: 22
31     Residual Value: 0,44
32     Safety class: midsize car
33     Cargo Capacity: 87,6
34     Passenger Capacity: 8
35     Surb Weight: 4264
36     crash rating: 4* in rollover
37     60K tire cost: 1400
38     Brakes Cost: 2x
39     Consumer Report: +++
40   CR-V SUV:
41     Purchase Price: 20700
42     MPG: 27
43     Residual Value: 0,55
44     Safety class: small SUV
45     Cargo Capacity: 72,9
46     Passenger Capacity: 5
47     Surb Weight: 3389
48     crash rating: 4* in rollover
49     60K tire cost: 1600
50     Brakes Cost: 2x
51     Consumer Report: +++
52   Element SUV:
53     Purchase Price: 18980
54     MPG: 25
55     Residual Value: 0,48
56     Safety class: small SUV
57     Cargo Capacity: 74,6
58     Passenger Capacity: 4
59     Surb Weight: 3433
60     crash rating: 3* in rollover
61     60K tire cost: 1300
62     Brakes Cost: 2x
63     Consumer Report: ++
64   Odyssey Minivan:
65     Purchase Price: 25645
66     MPG: 26
67     Residual Value: 0,48
68     Safety class: minivan
69     Cargo Capacity: 147,4
70     Passenger Capacity: 8
71     Surb Weight: 4385
72     crash rating: 5*
73     60K tire cost: 2400
74     Brakes Cost: 2x
75     Consumer Report: +
76
77   Goal:
78     preferences:
79       pairwise:
80         - [Cost, Safety, 3]
81         - [Cost, Style, 7]
82         - [Cost, Capacity, 3]
83         - [Safety, Style, 9]
84         - [Safety, Capacity, 1]
85         - [Style, Capacity, 1/7]
86     children:
87       Cost:
88         preferences:
89           pairwise:
90             - [Purchase Price, Fuel Costs, 2]
91             - [Purchase Price, Maintenance Costs, 5]
92             - [Purchase Price, Resale Value, 3]
93             - [Fuel Costs, Maintenance Costs, 2]
94             - [Fuel Costs, Resale Value, 2]
95             - [Maintenance Costs, Resale Value, 1/2]

```



```

96 children:
97   Purchase Price:
98   preferences:
99     pairwise:
100     - [Accord Sedan, Accord Hybrid, 9]
101     - [Accord Sedan, Pilot SUV, 9]
102     - [Accord Sedan, CR-V SUV, 1]
103     - [Accord Sedan, Element SUV, 1/2]
104     - [Accord Sedan, Odyssey Minivan, 5]
105     - [Accord Hybrid, Pilot SUV, 1]
106     - [Accord Hybrid, CR-V SUV, 1/9]
107     - [Accord Hybrid, Element SUV, 1/9]
108     - [Accord Hybrid, Odyssey Minivan, 1/7]
109     - [Pilot SUV, CR-V SUV, 1/9]
110     - [Pilot SUV, Element SUV, 1/9]
111     - [Pilot SUV, Odyssey Minivan, 1/9]
112     - [CR-V SUV, Odyssey Minivan, 5]
113     - [CR-V SUV, Element SUV, 1/2]
114     - [Element SUV, Odyssey Minivan, 6]
115   children: *alternatives
116 Fuel Costs:
117   preferences:
118     pairwise:
119     - [Accord Sedan, Accord Hybrid, 1/3]
120     - [Accord Sedan, Pilot SUV, 5]
121     - [Accord Sedan, CR-V SUV, 3]
122     - [Accord Sedan, Element SUV, 4]
123     - [Accord Sedan, Odyssey Minivan, 3]
124     - [Accord Hybrid, Pilot SUV, 9]
125     - [Accord Hybrid, CR-V SUV, 5]
126     - [Accord Hybrid, Element SUV, 7]
127     - [Accord Hybrid, Odyssey Minivan, 6]
128     - [Pilot SUV, CR-V SUV, 1/4]
129     - [Pilot SUV, Element SUV, 1/3]
130     - [Pilot SUV, Odyssey Minivan, 1/4]
131     - [CR-V SUV, Element SUV, 2]
132     - [CR-V SUV, Odyssey Minivan, 1]
133     - [Element SUV, Odyssey Minivan, 1]
134   children: *alternatives
135 Maintenance Costs:
136   preferences:
137     pairwise:
138     - [Accord Sedan, Accord Hybrid, 2]
139     - [Accord Sedan, Pilot SUV, 4]
140     - [Accord Sedan, CR-V SUV, 4]
141     - [Accord Sedan, Element SUV, 4]
142     - [Accord Sedan, Odyssey Minivan, 5]
143     - [Accord Hybrid, Pilot SUV, 4]
144     - [Accord Hybrid, CR-V SUV, 4]
145     - [Accord Hybrid, Element SUV, 4]
146     - [Accord Hybrid, Odyssey Minivan, 5]
147     - [Pilot SUV, CR-V SUV, 1]
148     - [Pilot SUV, Element SUV, 2]
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Glossary

- AHP** Analytic Hierarchy Process.
- AI** Artificial Intelligence.
- ANP** Analytic Network Process.
- BCP** Bussines Continuity Planning.
- BRMS** Bussines Rules Management System.
- BTS** Base Transceiver Station.
- CEO** Chief Executive Officer.
- CI** Critical Infrastructure.
- CPM** Critical Path Method.
- CSU** Český statistický úřad (Czech Statistical Office).
- CVM** Contingent Value Method.
- ERP** Enterprise Resource Planning.
- GIS** Geographic Information System.
- GPS** Global Positioning System.
- GUI** Graphical User Interface.
- IIM** Input-output Interoperability Model.
- JDK** Java Development Kit.
- JRE** Java Runtime Environment.
- LMS** Learning Management System.
- MCA** Multicriterial Analysis.
- TCM** Travel Cost Method.
- UML** Universal Modeling Language.
- WPM** Weighted Product Method.
- WSM** Weighted Sum Method.
- WTA** Willigness to Accept.
- WTP** Willigness to Pay.
- XML** Extensive Markup Language.
- YAML** YAML Ain't Markup Language.

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