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Modelling of Decision Processes

textbook
2nd edition



Modelling of Decision Processes

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this text has not gone through edition process

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Introduction

Dear student, you are holding in your hands a textbook on the subject of modelling of the decision processes. Basic goal of this textbook is to supply you with various methods for decision support.

This textbook has been derived from Czech version of it, which has been published first in 2006 in Ostrava and has been prepared for e-learning. As English version of this textbook has been written rather in rush, so it is more syllabus then a classical textbook.

Also there are subtle differences between the two language versions. There are also other changes in the text as the english version of the text is a recent one.

In the text there are some symbols, which are intended to support you while studying. These symbols are:



Study guide

It is used to help you familiarize yourself with basic goals of the chapter, so you know what you should be looking out for :-).



Time required to study chapter

Is estimated time necessary to understand (learn) all of the concepts in the chapter. The time estimation should be looked upon only as rough estimation to help you better organize time. So do not be disturbed if the learning requires more or less time.



Definition, note, important details

Accompanying this icon there will be text which the author finds so important, that he puts exclamation mark to it. Please pay close attention to it.



Test your knowledge

On the end of each chapter there are several questions or tasks you can use to self-evaluate your knowledge. These questions are not exhaustive to the theme of the chapter, so if you know all answers to the presented questions . . . it is a good start :-), if not go back few pages and search for answer – chances are you did not understand some important concept.

Also this book has been typeset in LaTeX to introduce some features to improve your learning experience. The text has list of figures and tables, also it has the index with most important concepts with their location.

**Take a break**

Well done, you worked hard, so it is time to take break.

**Example or task**

In this textbook, there are many examples but also many tasks available for you to compute by hand or using the computer. Take your time to compute them – it is a good indicator of whether you understand the problematic or not.

I hope that you will find information in this textbook useful.

Pavel Šenovský

Whats new in second edition of this textbook

- added introductory chapter one
- all subchapters now have questions to help you self-evaluate your knowledge on given problematic or tasks for chapters which are more practically oriented
- added practical software related chapter to project management
- added chapter for software support for Ford-Fulkerson and Dijkstra algorithms
- added chapter for software support of networks renderings based on the definition of the network structure
- added discussion of software tools for computation of linear programming problems
- many smaller corrections and changes.

Chapter 1

Introduction to decision making



Study guide

In this chapter we will learn something about decision making. While decision making is part of our every day life – so we could think that we humans are rather good at it. Making a decision is hard, especially in situations which are complicated – we require help and decision support methods are fully capable to supply such help.

After finishing of study of this chapter you will

Know

- What is a decision

Be capable

- to distinguish between different types of decision situations



Time required to study chapter

For getting through this chapter you will need at least one hour ... you know only something small for starter.

Every day, every hour or perhaps even nearly every moment after we awake and before we go sleep we perform consciously and unconsciously countless decisions. Most of them are small and we make them automatically without even thinking of them, for example when we walk we automatically change the direction of our steps to go around obstacles, we greet people we know and such.

Some decisions are harder, for example we are hungry, we have to decide when and more importantly what to eat. While such decision can be very hard at the times it is not the decision we would usually require assistance of sophisticated decision support tools. Such tools we dedicate to situations with even higher level of complexity – we choose between merchandise of high value or different technologies, choosing between different protective measures ... this is the kind of situations we are going to examine.

All decision situations regardless of what are we choosing between have common points – we choose between choices characterized by different properties of the choices resulting into different outcome of the choice.

These outcomes and its predictability is first moment we can use to distinguish between the decision situations. We speak about so called *deterministic* choices when know exactly what will be the result of our choice. In ideal world, we would know what results will our choices have, unfortunately we do not live in such world, so most of the choices are *stochastic* in nature. With stochastic choices we can only predict with certain probability outcome of our choices. Basically we are not sure of the outcome and this uncertainty has to be addressed by the decision support methods we are using.

So what is the benefit of using decision support methods and tools. Well, using established method is like using highway instead of making your own way through the thick forest. Decision support methods are documented approaches some else used before us to solve the problem we are solving now. So by using method we may reach goal more easily, with fewer resources and the outcome will come in expected format, that is if we use it as we should – for the problems the method has been verified and are considering possible constrains of the method, if such are known.

Another benefit is that the methods in general also enable us to make emotional distance between us and our results. Sometimes we look at the problem and some kind of solution we like more then the others at the first sight. Such “hunch” may be correct especially if we are experienced at solving that kind of situations, but it may be also misleading. Using the method help us maintain our objectivity.

Lets go back to variants and differences between them, because even such type of the difference may be used are criterions characterizing the variant. Basic division could be between choices characterized by single criterion and those which require usage of multiple criteria. From point of view of decision support methods we distinguish between monocriterial and multicriterial methods.

When using single criterion we usually use money. Financial value is logical choice as we are capable to transfer many things to it: cost, benefits, value of property, health or even life. Good example of the monocriterial method would be decision trees we will discus in next chapter, while **multicriterial analysis (MCA)** is a method on it own.

Even the way we express the values of the criterions may lead to usage of different methods supporting the decision. Basically we may express the criterion in three different ways:

1. numeric value – as in continuous numerical variable, for example age, charge time, price,
2. ordinal value – may be number or string, for example 0 – no, 1 – yes or
3. freeform answer – meaning that the criterion valuation for the variant will lead to paragraphs of text

It is only logical that different types of the criterion valuation will be more or less processable by the methods. For example numeric values are processable by statistical methods such as regression analysis, while ordinal values are better processed by data mining methods such as decision trees or decision rules.



Please note

In previous sentence we were discussing decision trees, please note that there are two different methods of such name on, which will be discussed in next chapter, is monocriterial decision support tool while the second can be used in data mining and works best with ordinal values. This text is not focused on data mining, so we will not discuss this method more in detail.

Also note that there are many more methods such as statistical methods or methods based on **artificial intelligence (AI)**, which will also not be discussed in this text, even knowing that these methods are very useful in supporting our decision. The choice not to discuss them here is organizational. Universities usually organize dedicated study modules for statistics, data mining or **AI** (neural networks, expert systems etc.).



Study guide

In your professional life you will have to deal with many different decision situations requiring the usage of different approaches to reach successful solution – so build up portfolio of methods you are familiar with and capable of using. The more different the methods will be the better.

In this textbook, I try to supply the tools for many typologically different decision situations. So we can say, that the problem typology itself may be used as criterion to distinguish methods supporting our decision.

On the beginning of the chapter we were discussing classical choice – when deciding between different variants as in buying different product. What if the nature of decision (and the difference

of the variants) is not in buying of new product, but rather in change of its operational parameters – setting up in different way. In such cases we talk about *optimization problems*.

Again, there can be many types of optimization based on what are we trying to optimize. For example when we have technology and are searching for its optimal setup, we may use one of methods of *mathematical programming* such as linear programming (solvable for example by primary algorithm also known as simplex method).

But when we are deciding of how to best place for example broadcast towers of local wireless telephone operator we use different set of methods known as localization models. So different problems may have different set of tools available to solve them.

Lets try to finally use one of such tools.



Questions

1. What characterizes decision situations?
2. What is difference between stochastic and deterministic decisions?
3. Specify types of criterions (there are 3 of them).
4. Can nature of the problem, we are trying to decide, have impact on the methods effectively usable to solve it – can you give example?
5. Look back at your study plan and try to find methods usable as support for decision making, you already know.

Chapter 2

Decision trees



Study guide

In this chapter we will learn of how to use one of simplest tools for decision support – *decision trees*.

After finishing of study of this chapter you will Know

- What decision trees are
- When and how to use them

You will also learn about decision situations.



Time required to study chapter

For getting through this chapter you will need at least one hour and you will need about two hours for calculation of examples.

One of most simple tools available for decision support are *decision trees*. Decision trees are oriented graphs, which are by its look similar to the tree. As normal tree, decision tree has one root and branches itself to leafs.

The root of the tree represents a basic choice between variants. The branching then represents partial decisions we may do during the decision making. What do we mean by partial decisions? In our decision we usually do not perform single decision – but a series of small decisions. For example we want to by a car, the basic choice may be between the car producers – Ford, Škoda, Volkswagen but there would be also other decisions leading to our dream car – what type will it be – SUV, hatchback, limousine and what about the engine? To finish the choice we must perform all these small decisions (branching on the decision tree). Then finally we reach the outcome of our decision – leaf node.

Writing decision situation into form of the decision tree helps us by visualizing the situation in the easily to understand way so we may focus on partial decisions while not losing the bigger picture of the decision situation.

Decision trees are monocriterial method, so each variant of the choice is characterized by only one criterion. The criterion itself must be common over the whole tree regardless of the nature of partial decisions. It is logical requirement, otherwise we would not be able to compute the value of the decision as whole.

Such universal criterion is usually only one – money. We are capable to translate many different variables to financial value. For some of them there are even available the tables to make the process easier (damages on fauna and flora). The optimal solution to our problem then would be the variant with minimal or maximal value of the criterion.

Even with money as common denominator we may face both maximalization and minimalization

problems. For example the we usually like to minimalize the costs while maximalizing the benefits or profits.

When making decision or partial decision we may or may not know the outcome of that decision. Decision in situation of certainty means, that we fully know what will happen, if we make the choice. Unfortunately that is usually not the case, so we have to deal with some kind of uncertainty in most of the case.

Decision in situation of uncertainty means, that we cannot state with 100% certainty, what exactly will happen, if we make a that choice. But usually we are at least capable to establish probabilities of the outcome.

Both situations are solvable by decision trees, but of different type. In situation of certainty we use *deterministic decision trees* and in situation of uncertainty we use *stochastic decision trees*. General example of both types of the trees is on the fig. 2.1.

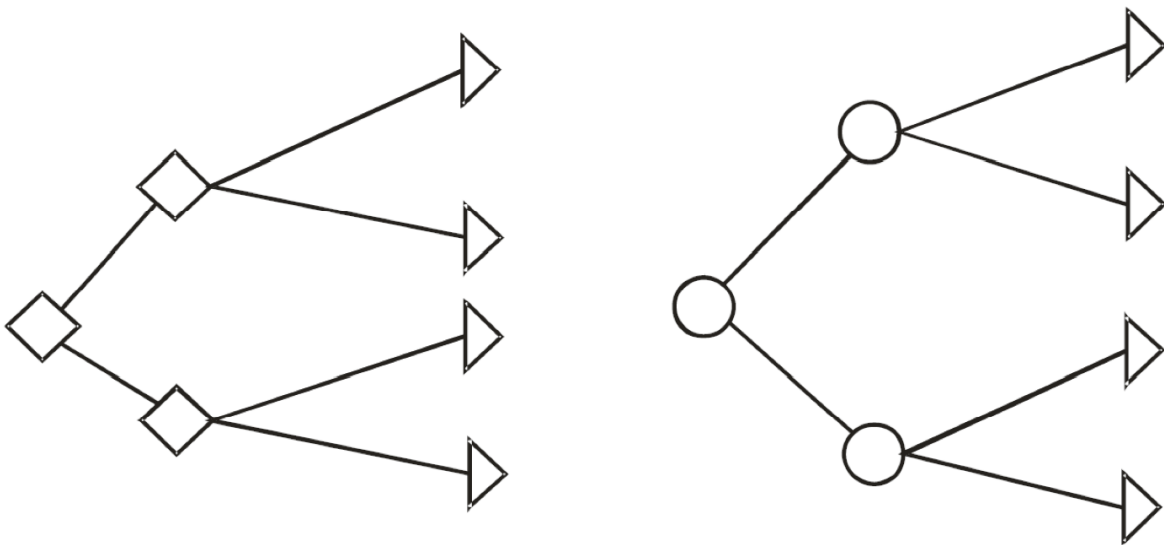


Figure 2.1: Deterministic (left) and stochastic (right) decision trees

Decision trees consist of the nodes and edges as we stated higher. The nodes differ for partial decisions based on whether the outcome of decision is known in advance. So as we have stochastic and deterministic decisions, we also have stochastic and deterministic nodes. The shape of the nodes differs for both types of the decisions.

For deterministic nodes we are using diamond or square shape, for stochastic nodes we are using circle shape and for leaf nodes, we are using triangle shape. The decision tree is stochastic if it has at least one stochastic node. For the decision tree to be deterministic, it has to consist only from deterministic nodes.

The root node is usually deterministic as it represents the decision between basic variants, which are then more developed further down the tree. Also it makes little sense to create the decision tree using stochastic nodes only – in such situation, our decision would have absolutely no value, as we would have been fully in the hands of the chance.

Position of the node in the X axis of the graph should also correspond to the time of the decision, the node represents. The nodes representing partial decisions made farther in future should be farther right then those made in near future. Position of the nodes along X axis is for sake of better readability of the graph, the computation of the tree is affected by the structure of tree only, not by its graphical representation.

Also to improve readability of the decision tree, we are often numbering the nodes. Numbering enables us to more easily document the tree or perform more elaborate computations outside of the tree, while making it easy to identify part of the tree we are working with.

Lets try to finally use it!

Let's solve the example together. Considering the situation described higher, we may construct the decision tree similarly to fig. 2.2.



Example 1: Deterministic decision tree

Company XYZ must decide whether to or not to reconstruct its business premises. At present time the company is capable to sell its products for 2,5 mil. EUR, during reconstruction the company would have to temporarily move into alternative buildings, from which only 1 mil. EUR worth of its products could be sold.

The reconstruction is expected to take one year and will cost 10 mil. EUR. In new business premises the company hopes to sell its products worth 3 mil. EUR per year.

Recommend whether to or not to reconstruct, considering 10 years time horizon.

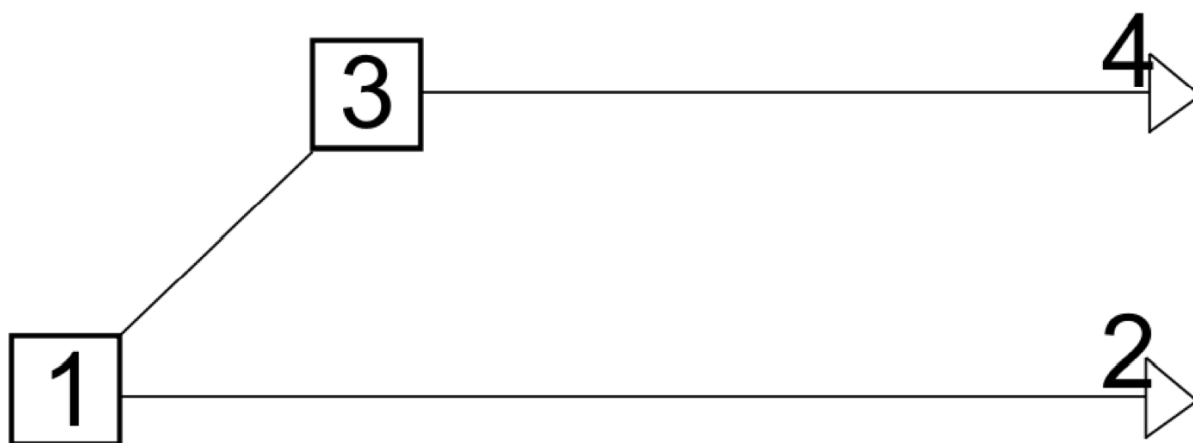


Figure 2.2: Deterministic decision tree construction

The decision tree on fig. 2.2 is rather simple – that is not so surprising as we already know, that the deterministic decision trees do not describe real decision situations well. So the example on fig. 2.2 is typical school problem just to demonstrate the way of computation.

Lets look at the tree. Its node 1 represents our decision to or not to reconstruct business premises. Node 3 only marks one year reconstruction period and does not represent the decision by itself.

We can calculate the net profit in the leaf nodes for 10 years period: **Node 2: we did not reconstruct**

Profit: 2,5 mil EUR * 10 = **25 mil. EUR**

Node 4: we reconstructed

Profit:

reconstruction period: 1 mil. EUR

Post reconstruction: 3 * 9 mil. EUR = 27 mil. EUR

All in all: **28 mil. EUR**

Costs:

Reconstruction: 10 mil. EUR

Gain: 27 – 10 = 17 mil. EUR

So if we reconstruct, the profit will be 17 mil. EUR, while if we will not reconstruct, the profit will be 25 mil. EUR. Since its usually better to have 25 mil. Then 17 mil. - we mus recommend for company not to reconstruct its premises while specified financial constrains remain.

Lets try another example to demonstrate simple stochastic decision tree problem and its solution.

Well, the example 2 is also “shoolary” example – so there are some simplifications to make the computation easy. The largest one is that the vaccination will be 100% effective, could be realized “overnight” for whole population and will will function immediately. Real vaccination needs long (usually over october and november) vaccination campaign, only approximately 25% of the europe’s population is vaccinated (approximately 5% in Czech republic) and the body needs few weeks to create enough antibodies to protect itself effectively. Also the effectiveness of the vaccination is not 100%, but slightly above 60% - so do not take the example to seriously.



Example 2: Stochastic decision tree

Government committee considers economical usefulness of preventive influenza vaccination program. For such program the committee considers “early warning” system, which will cost 120 mil EUR and will permit to detect early start of influenza epidemic. After the early warning, the vaccination program will start. If the vaccination program will not be realized, we estimate the cost of medication 280 mil. EUR with probability of 0,1, 400 mil EUR with probability of 0,3 and 600 mil. EUR with probability of 0,6. The vaccination program will cost 400 mil EUR and probability of influenza coming is 0,75.

We consider the vaccine to function immediately and be 100% effective.

As example 2 is stochastic tree, we are working with probabilities and there are some rules we have to follow. The probabilities themselves may be expressed by one of three ways as probabilities in interval $<0; 1>$ (this is our case), as the percentual chance of occurrence in given time interval or as frequency of occurrences.

For example if the event repeats itself once per ten years, then the probability for time interval of ten years would be 0,1 (1/10) or 10%.

The decision trees work only with probabilities in interval $<0;1>$, so if we had probabilities in different format, we would have to convert them.

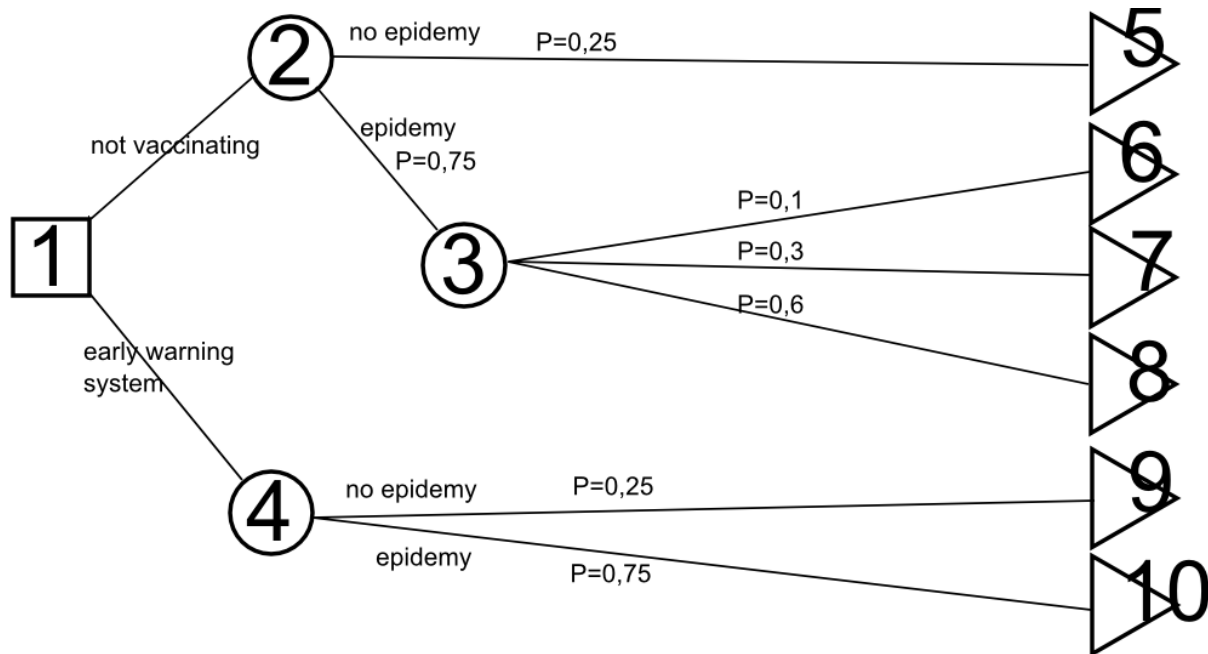


Figure 2.3: Stochastic tree construction

The construction process should be pretty obvious from the fig. 2.3, so let's start computing the gains of variants. The computation process, at least in its first phase will be exactly same as in deterministic tree – so we will ignore the probabilities for now.

Costs:

- Not vaccinating and no epidemic: 0EUR (1-2-5)
- Not vaccination and epidemic with low costs: -280EUR (1-2-3-6)
- Not vaccination and epidemic with middle costs: -400EUR (1-2-3-7)
- Not vaccination and epidemic with high costs: -600EUR (1-2-3-8)
- Early warning system and no epidemic: -120EUR (1-4-9)
- Early warning system and epidemic: -520EUR (1-4-9)

If the decision tree would have been deterministic, we would end at this point evaluating the costs. Unfortunately we have to consider the probabilities too, as we do not have the whole process under

our control.

We have to compute the costs of stochastic nodes all the way back to node 1. We will compute the cost of such nodes in such way that we will multiply the costs of branches leaving the node by its probabilities and summarize them.

Cost node 3: $(-280 * 0,1) + (-400 * 0,3) + (-600 * 0,6) = -508$ mil. EUR

Cost node 2: $0 * 0,25 + (-508 * 0,75) = -381$ mil. EUR

Cost node 4: $(-120 * 0,25) + (-520 * 0,75) = -420$ mil. EUR

Considering the nodes 2 a 4 we decide that we should not apply the vaccination program, as the costs connected to it are way larger (-420 mil EUR), then the costs of medication (-381 mil EUR).

You may try to compute some tasks yourself.



Task 1

MDS is company oriented onto geologic evaluation of the land. MDG has the possibility to buy a land for 3 mil EUR. If MDS buys the land, it will perform geologic evaluation of it. Such evaluation costs about 1 mil. EUR and will result:

- Manganese 1% chance
- Gold 0,05% chance
- Silver 0,2% chance

If the one metal is found, no other metal may be found. If the manganese is found, the land may be sold for 30 mil EUR, if gold for 250 mil EUR and silver for 150 mil EUR.

MDG may also pay 750 000EUR for right to realize 3 day preliminary evaluation. Past experience show, that 3 day ecaluation will cost 250 000EUR and increases possibility, that there is metal deposit to 50%. Chances to find the metals is:

- Manganese 3%
- Gold 2%
- Silver 1%

If 3 day test does not show deposit existence, chances that there actually is metal deposit is very low.

- Manganese 0,75%
- Gold 0,04%
- Silver 0,175%

What will you recommend?



Solution: task 1

3 day test and if it is successful buy



Task 2

Company decides, if it is going to take part in selection procedure. They estimate, that only the preparation for selection will cost 10 000EUR. If it is going to participate, they estimate that there is 50% chance that they will advance to second list. In second list it will be necessary to supply more information (estimated cost 5000EUR).

Company estimates, that the cost of the work and material for realisation of the contract will be 127 000EUR. Company thinks about three variants of offer: 155 000EUR, 170 000EUR, 190 000EUR. Probability of success will be: 0.9, 0.75, 0.35.

Should company take part in selection procedure and if yes at what price.

**Solution: task 2**

take part in selection procedure, if selected into short list go for middle price (170 000EUR)

**Other tasks you may want to practice on**

On Moodle supporting module “Modelling of Decision Processes” there are also available other tasks for computations. See if you can solve them all.

The module is available at <http://prometheus.vsb.cz> [3].

The Examples and task has been taken from [21].

**Questions**

1. What is the difference between stochastic and deterministic decision tree?
2. Is it possible to use deterministic nodes in stochastic decision tree?
3. What is the use of decision trees?
4. Is it possible to classify decision trees into multi-criterion analytic methods?
5. What is the sign used for stochastic node?

Chapter 3

Multi – Criterion analysis



Study guide

In this chapter you will learn something about various methods of analysis of the decision situation, when we are optimizing decision based on multiple criteria. We will focus on decision analysis, which we will examine both theoretically and practically.

After reading this chapter you will understand how to

- Create decision analysis

know

- multi – criterion analysis
- Method of pair comparison



Time required to study chapter

For getting through this chapter you will need at least two hours.

Decision tree we were discussing in previous chapter is without any doubt very useful tool, but it is capable to use single criterion only. What if we actually need to use other decision criteria too, perhaps even many more criteria? And what if these criteria are measured using different units. The various criteria themselves may also differ in usability for decision making – some of them may be more important than the others.

Under such circumstances usage of decision trees is out of the question. We have to use specialized type of analysis called **MCA**.

So what benefits do we get in general, when using **MCA**?

- we may compare criteria measured in different units,
- we may assign different importance to the criterions,
- we may evaluate both benefits and risks connected to variants both independently and together.
- MCA allows us to easier achieve objectivity in decision making – to diminish effects of our bias.

So how exactly MCA achieves such different goals?

3.1 Decision analysis

Main goal of decision analysis is to process available information into tabular form and if possible to objectively analyse the problem. Our goal is usually to identify or two optimal variants, representing solution to our problem.

MCA in its written form is a supporting document for manager's decision. Manager himself is usually not part of the analytic team. Analytic team, especially for complex problem is usually assembled ad hoc, so the team members have large portfolio of specializations to cover whole problem domain. Manager presents the problem to the team and then awaits the results. Manager does not need to be specialist in problem domain and the analytical team has to take this fact into account.

The result – the analysis, has to be general enough for manager to understand it and agree with its conclusions (and possibly also constrains of the decision itself).

The analysis itself is made in several steps:

1. Definition of the problem
2. Information analysis
3. Evaluating benefits of variants
4. Evaluating risks of variants
5. Final evaluation – recommendation of variants.

Our first task is to *define the problem*. Correct definition of the problem allows us to specify the criteria, which we can use to evaluate the variants. Definition of the problem may be problematic itself as the original definition is performed by manager outside or analytic team. The analytical team has to make sure that its members understand the problem same way as the manager and each other. Also they have to make sure that there is an agreement on the goals of the analysis.

If such agreement does not exist or is questionable – the problem must be redefined to resolve all controversies and questions regarding the problem. To do that analytical team has to communicate problems with task definition with the manager. The redefinition itself is performed by the manager alone again. Failing to resolve such problem may result in the rejection of the results of the analysis as whole or the need to remake large parts of the analysis at later time – which requires time, human resources and presents additional costs of the decision, we would like to evade whenever possible.

By *analysing of information* we mean gathering all available information significant to problem solution and its evaluation. There are different types of information we can use – *analytical information* describing various interesting properties of the variants and decision situation and *thematic information*.

Analytical informations are the informations adding to our understanding of the problem. They may consist of the values, catalogues, need specification, description of present state, business offers. All these informations support us on our way to reach the goal the manager defined for the analytic team.

Thematic informations are typologically same as analytical with single, but very important difference – they do not help us in reaching goal. So what are these informations good for? They map alternative routes to similar goals.

For example the manager sets the goal to choose the new car to buy for ... something. Analytical information would be on prices, car producers, type of the engine, size of the car, fuel consumption, safety ... Thematic informations be these on alternative way of financing – perhaps it would be financially interesting not to buy the car “cash”, but lease it or even rent it. What if getting new car was not necessary at all. We might think about overhauling old car.

So the thematic information basically describes the road not taken during decision process. Such information may be useful in case the original goal is unachievable. Even if the analysis is good and its result accepted by the manager, the optimal variant may not be realizable (purchasable, rentable, ...) due to many different circumstances.

The company may change its orientation due to different situation on the market, leaving original goal behind. The company may not have enough money to “make it happen” or the negotiations with bank, producer, contractor, ... may fail on either side. At that moment the company stands before the choice not to pursue the goal at all or choose different approach.

Thematic informations may become analytic information at that moment, forming base of the new decision situation, with slightly different goals.

Evaluation of variant benefits

To evaluate variant benefits we have to define evaluation criteria, we will use and quantify them in some form. Different criteria may be quantified using different units. Such is the nature of the world around us and the way we use to describe it. We know, that the battery in mobile phone will hold X minutes, that the screen resolution may be measured in pixels, the price in Czech Crowns (CZK) or Euros (EUR).

We call such description natural. Unfortunately it is not possible to directly compare the criterions in different units. So that would be our first task after quantifying the criteria.

First we have to create table (matrix) with criteria in its natural units (see table 1). To increase the objectivity of the evaluation process, we use C1 – Cn instead of criterion names and V1 – Vn instead of variant names. Actually it is not possible to completely elude the connection to “real” names at this point, as we need to gather all necessary data needed for evaluation. But following this principle in other tables can help help us significantly.

Table 3.1: Quantification of the criteria of the variants

Criteria	Unit	V1	V2	...	Vn
C1	[min]	10	12	...	9
C2	-	Good	Average	...	Excellent
...
Cn	[kg]	0,5	0,4	...	0,55

To be capable to work with these criteria, we have to remove these natural units. Such removal is possible if we translate criteria from natural units to percentage of our satisfaction with the value. This translation solves two problems to us – firstly we will have single unit for all criteria (percentage) and secondly we elegantly solve different direction of the optimization. So while we maximize some criteria, we may also want to minimize other criteria. Translating the values to percentage solves this problem too.

As the units and criteria will have different meaning we also have to describe value system used in establishing criteria values. We have to say what properties must variant have for criterion C2 have to be valued as excellent.

Example of criterion transfer to percents could be: C1 may be benchmark for some computer task. Nine minutes as best time will represent 100%, while 12 minutes (variant 2) will be only 10%.

Establishing of the percentage of satisfaction will enable us to compare the criterions as the unit will be same.

With introducing of percent values, we may transform table 3.1 into form of simple benefits (see table 3.2).

Table 3.2: Table of simple benefits

Criteria	V1	V2	...	Vn
C1	80	10	...	100
C2	75	50	...	100
...
Cn	60	100	...	50

Now we have criterions with exactly same unit (percentage), so theoretically comparing these values is possible. That is under presumption that the importance of all these criterions is exactly the same. In other words, if we believe that all criteria contribute to finding solution to the problem in exact same way we may just start to comparing them. Needless to say, this is not exactly real world scenario, as we usually regard some criteria more important than the others. We have to find a way to evaluate criteria importance.

It is entirely possible, that the criteria preferences (weights) will be part of problem documentation – the team got from manager, but if not, we will have to find another way to explore significance of the criterions. Before we do that let me point out one very important thing regarding evaluating criterions.

What is exactly the dependence in criteria? Lets consider following scenario: we are considering buying of the new monitor to our desktop computer. Among other criteria we would like to evaluate also ecological aspect of such device. When searching for possible criteria we found following:



Criteria

In this chapter we focus on “simple” form of **MCA**. That means that we presume that the **criteria are independent to each other**. If there was some kind of dependence between the criteria, we could still use **MCA**, but we would need to form up and work with criteria hierarchies, which is of course more complicated.

- energy star certification,
- TCO certification
- easiness of disassembly
- used materials
- environmentally friendly

Now when we know, that we are trying to demonstrate dependence between the criteria, it is rather easy to identify the problem – the monitor is environmentally friendly if it has low power consumption (energy star), low radiation emissions (TCO), is easy to disassemble and its produces is using materials which encumber the environment less then other materials he could legally use.

We may express the situation graphically by forming the hierarchy of the criteria used in analysis, see figure 3.1.

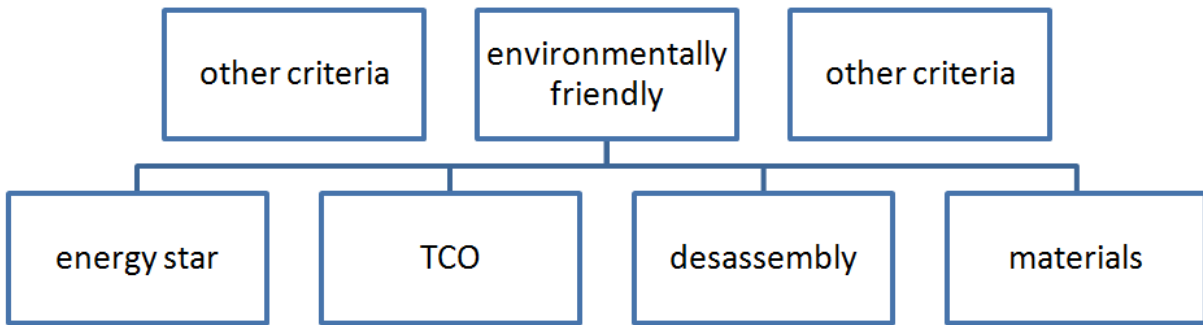


Figure 3.1: Forming up of the criteria hierarchies

To evade the problem, we have to resolve interdependencies, so we use either environmentally friendly or use subordinate criteria only.

Lets go back to the problem of establishing criteria importance. There are many various ways of how to approach the problem. Perhaps similar problem has been solved by somebody in the company or the report is available in some kind of journal or over the Internet.

Importance, expressed by weights assigned to the criteria, may be developed information gathered form problem domain’s experts (see Delphi oracle method in this textbook, for clues of how to approach the problem). We may also try to establish importance directly by comparing pairs of criteria and deciding on which is more important then the other. We call this method *pairwise comparison*.

Let’s compare criteria C1 – C5. To do that we need to construct the comparison triangle.

C1				
	C2			
C2		C1		
	C2		C4	
C3		C2		C1
	C4		C4	
C4		C3		
	C5			
C5				

We construct the triangle column after column. Firs column has all criteria in it and we write it down without any evaluation. In the second we compare criteria next to each other, the C1 and C2

– we prefer C2, C2 and C3 – we prefer C2, C3 and C4 – we prefer C4, C4 and C5 – we prefer C5. In third column we compare first criterion with third, second with fourth, third with fifth ... In the fourth column we compare first with fourth, second with fifth and lastly in fifth column we compare first with last.

If we had more criteria we would employ same strategy to express our preferences. Obviously, there would be more columns needed to finish the task.

Having constructed the comparison triangle, we have to interpret the results comparison. Let's make a table for it. We will count number of occurrences in the triangle. We may count occurrences in the first column, but nothing happens if we leave it out, as it has exactly one occurrence of each criterion and that's why it will not influence the final order of criteria by importance, see table 3.3.

Table 3.3: Computation of the weights

Criteria	No. of occurrences	Order	Weights
C1	2	2	2
C2	3	1	3
C3	1	3	1
C4	3	1	3
C5	1	3	1

In table 3.3, there are weights computed as inverse order to criteria order by importance. Most preferred criterion will get highest weight. In some cases, this way weights estimation may be too rigid, so we can modify it to get larger (or smaller) gap between the weights. What we can't, or better said – we shouldn't, modify is the order itself – so that more important criterion can't have lower weights than those behind it.

Applying the weights on matrix of simple benefits we get the matrix of weighted benefits (see table 3.4).

Table 3.4: Table of weighted benefits

Criteria	Weights	V1	V2	M
C1	2	80 x 2 = 160	20	200
C2	3	75 x 3 = 225	150	300
C3	1
C4	3
C5	1
\sum		385	170	500
U	%	77	34	100

We compute the weighted gains as product of weight and simple benefit for criterion of given variant. We also construct extra maximum variant M, which should represent an ideal variant. Ideal variant fulfils the criterion on 100%, weighted gains then are simply weights x 100 for this ideal variant.

Ideal variant will help us find, how close are actual variants to this ideal. Computing this, we will get first results of the analysis.

Alternatively we may use maximal value of the criterion for M variant instead of presuming, that the ideal variant will fulfil our expectations to 100%. The reasoning behind this is to try to compare the existing variant to the M variant having best available properties instead of ideal properties.

Choosing best available instead of ideal will lead to higher scores for evaluated variants in comparison to situation, when we perform the comparison to the ideal variant.

We may order the variants based on sum of weighted gains – and recommend the one with highest score. That doesn't mean that our analysis is complete. Next we will compare risks connected to the variants (see table 3.5).

Risk in this context is probability of occurrence of negative event or effect in given period of time. This period of time must be same for every risk, so we can actually compare them.

Similarly to gains, also risk may be weighted (see table 3.6). The weights for risks may be established in similar way as the weights for the benefits.

Similarly to matrix of weighted gains we introduce new variant Vmax, with probability of occurrence 100%. We use this variant as etalon to compare our variant. Of course we should be minimizing

Table 3.5: Matrix of simple risks

Risk	V1	V2	Vn
R1	10%	8%	...
R2	15%	25%	...
Rn
...

Table 3.6: Matrix of weighted risks

Risk	Weights	V1		V2		Vmax	
		P	GT	P	GT	P	GT
R1	1	0,1	0,1	0,08	0,08	1	1
R2	2	0,15	0,3	0,25	0,5	1	2
\sum			0,4		0,58		3
GT	%		13,3		19,3		100

risks when interpreting this table.

Again Vmax may be also constructed based on worst available scenario, using highest GT value for given risk.

When choosing of what to recommend we usually follow one of strategies:

1. Maximal
2. Minimal
3. Optimal

Maximal strategy prefers maximisation of the gains regardless of possible risks connected to the variant. Our recommendation in this strategy is derived from results computed in table of weighted benefits. We are searching for the variant with highest score.

Minimal strategy represents the situation, when we have complete aversion to the risk – so we are minimizing risks regardless of possible benefits we could get from realizing the variants. Our recommendation is based on results of the table of weighted risks.

Lastly we can compare risks and benefits optimizing our decision, see table 3.8.

Table 3.7: Gains vs. Risks

Order	1st	2nd
Benefits	V1	V2
Risks	V1	V2

Table 3.8: Final Effect

	V1	V2
Gain (U)	77	34
Risk (R)	13,3	19,3
Final Effect (U-R)	63,7	14,7
Final Effect (U/R)	5,79	1,76

As we can see both minimal and maximal strategy did recommend same variant (V1), optimal strategy may only result into recommendation of this variant too, otherwise we would have to compute final effect of the variant (see tab. 3.8).

As final effect is computed in two different way. In our case both approaches lead to recommending of V1, but in cases when maximal and minimal strategies differ in recommendation it may be different. In such cases we recommend one variant as optimal and the other one as its alternative.

Also it is good practice to write down executive summary for management, which consist of recapitulation of whole analysis and its recommendations.



Mistakes?

Well if you look back at the chapter, the process of the **MCA** is pretty strain forward. The analysis structure is firm and we use only basic mathematical operations, so we can not make a mistake, can we?

Unfortunately my experiences say that you can and it is not hard at all. It is rather easy to make mistakes both logical and numerical. Typical logical mistake would be to presume that we are maximizing all criteria, another one would be to maximize the risks.

These kind of mistakes come from lack of focus and automatic writing without thinking. Best defense against it is to thing logically proofread parts of the analysis to make sure to constrains hold.

Numeric problems come from bad translation of the criterion's natural units into matrix of simple benefits or simply making a mistake during the tables computation. Best help is to use spreadsheet such as MS Excel, LibreOffice or any other to support you with computations. Using such programs will automatically recompute all tables as required, leaving to you only the task to copy them to the report.

Both types of mistakes come from heavy underestimating of the complexity of the analysis – so do not underestimate complexity of **MCA**.

Quiz:



1. Define problem with the hierarchies of criteria and how can we resolve it.
2. How can we estimate weights of the benefits?
3. How is working pairwise comparison?

3.2 Case study of decision analysis



Case study of decision analysis

Let's make a short decision analysis on some simple problem. Such problem may be choosing of WWW photo album. We can base our analysis on article [24].

Please note that this decision analysis is for demonstration purposes only and does not represent actual state of the art in the area of web photo albums.

Analytical part

Goals: *Realize photo album on Internet*

Analytical information

- Safety
- Number of photo albums the service allows us to create
- Size of the album
- Cooperation with photo shops for printing of photos

Thematic information

- Program your own photo album using services of ISP (Internet service provider)
- use open source photo album on your own hosting.
- Do not realize photo album at all

Stating gains

Variants

V1 – Album.cz

V2 – Volný album

V3 – Tiscali foto

V4 – Foto-album

V5 – Rajče.NET

Table 3.9: Criteria

Criterion	Unit	V1	V2	V3	V4	V5
Capacity	[MB]	30	40	30	200	Not limited
No. of albums	-	5	5	Not limited	Not limited	Not limited
Formats	-	3	5	2	1	1
Services	-	Good	Excellent	Excellent	-	-
Photo shop	0/1	1	1	1	0	0

Legend

Capacity – in MB, influences number and quality of photographs we can store in the albums, so higher is better.

Maximal number of photo albums: photo albums allow its user to organize the photographs into contextual categories called albums. Number of these albums says how much freedom we have to organize our photo collection, more is clearly better here.

Formats – means graphical formats supported by system. More is better here, but this criterion is not critical as we usually use JPEG format and JPEG is supported by all album providers.

Services – additional services above the standard requirement of photo album providers. For evaluation as good, there must at least some services, and for excellent there must be plenty of them. This criterion is supporting only.

Photo shop – is there an agreement between photo album company and some photo shop?

Note:

Please note the commentary to the criteria. It describes the nature of the criterion as well as the way of evaluating it. Remember, that manager or anybody else who has to read the analysis may have problem decrypting your thoughts. Anticipate that and provide commentaries to the tables documenting the what you do and why.

Table 3.10: Matrix of simple benefits

Criterion	V1	V2	V3	V4	V5
Capacity	50	55	50	90	100
No. of albums	50	50	100	100	100
Formats	70	100	60	50	50
Services	50	100	100	0	0
Photo shop	100	100	100	0	0

For weight estimation we will perform pair comparison.

K1
 K1
 K2 K1
 K2 K1
 K3 K2 K1
 K3 K2
 K4 K5
 K5

Capacity and maximal number of photo albums are most important criteria, that's why I increased the weights for them. Criterion photo shop is important too, so I increased its weight too, but not so drastically. Other criteria weights remained untouched.

Risk estimation

R1 – not enough disc space

R2 – camera does not support required data format

R3 – availability of service (lack of it)

Table 3.11: Weight estimation

Criterion	Occurrences	Order	Weight
Capacity	5	1	10
No. of albums	3	2	8
Formats	1	4	2
Services	0	5	1
Photo shop	2	3	5

Table 3.12: Matrix of weighted gains

Criterion	Weight	V1	V2	V3	V4	V5	M
Capacity	10	500	550	500	900	1000	1000
No. of albums	8	400	400	800	800	800	800
Formats	2	140	200	120	100	100	200
Services	1	50	100	100	0	0	100
Photo shop	5	500	500	500	0	0	500
\sum		1590	1750	2020	1800	1900	2600
U	[%]	61,15	67,31	77,69	69,23	73,07	100

Table 3.13: Matrix of simple risks

Risk	V1	V2	V3	V4	V5
Capacity	90	80	90	20	0
Format	1	1	1	1	1
Outages	1	1	1	1	1

As format and outage risks are same for all variants, they do not help us make a choice, so we can remove them from analysis.

Note:

In table 3.13 I made it easier for me to perform the risk analysis. As format and outages are same for all variants I may just ignore them. That is not exactly real-world scenario, I can do it, as the use case if for scholarly purposes. But when you develop your semestral project or perform MCA in your professional carrier, do not take shortcuts – it will devalue your effort in remainder of the analysis.

Results

Table 3.14: Gains vs. Risks

	1	2	3	4	5
Gain	V3	V5	V4	V2	V1
Risks	V5	V4	V2	V1, V3	

Table 3.15: Final effect

	V1	V2	V3	V4	V5
Gain (U)	61	67	77	69	73
Risk (R)	90	80	90	20	0
U – R	-29	-13	-13	49	73
U / R	0,67	0,7	0,85	3,45	73

Preference considering strategies:

1. Maximal – V3 with alternative V5
2. Minimal – V5 with alternative V4
3. Optimal – V5 with alternative V4

Verbal evaluation of variants:

In evaluation, there were best following variants

V3 – Tiscali foto
V4 – Foto-album
V5 – Rajče.NET

Tiscali foto scored with its offer of additional services, which outweighed lower capacity. Problems with capacity were fully revealed by risk analysis, where this variant was worst.

Photo album Rajče.NET got its points in basic characteristics, but it does not offer any additional services as photo shop cooperation etc. If we decide to realize this variant we have to remember this and find alternative photo printing services provider.

I recommend realizing variant Rajče.NET.



Semestral project

Perform MCA on problem of your choice as your semestral project. There must be minimally 5 variants characterised by 5 criterions. Consider also minimally 3 risks.

Upload result of your work on <http://prometheus.vsb.cz> module Modelling of Decision Processes for evaluation.

Chapter 4

Other methods usable in general decision situation



Study guide

In this chapter you will learn about other supporting methods, which can help in almost any decision situation.

After reading this chapter you will understand how to

- use brainmaps
- use Delphi oracle method

know

- what problems are connected to data gathered by questionnaires and how to minimize them



Time required to study chapter

For getting through this chapter you will need at least one hours, perhaps more if you try to solve the tasks practically with software support.

4.1 Method of Delphi oracle

In some cases, when we decide about specific, complex problems, with high amount of uncertainty it is advised to use method of so called Delphi oracle. This method is based on averaging of expert opinions in the required field.

The method requires that we prepare set of questions for expert to answer and then we statistically process the results. The main advantage of this method is that it eliminates extreme opinions in the set. Unfortunately resulting majority opinion may not be correct.

Problem may be the experts themselves. They may know each other; they may be influenced by a common knowledge, articles, perhaps even the education on same school. As it is a normal for human beings they tend to support their opinions based on opinions of others. Under normal circumstances that may not be a problem or may be even encouraged, but in situation of extreme uncertainty it is possible to put single source not verified information into circulation – which can be believed. The experts could reassure each other, that they also red/heard of it – so it can be believed, which is not true as we need independent source of information to believe it.

Book [25] even names the phenomenon – it calls it *data incest* and demonstrates it on simple example:

We have several independently operating agents, who sharing information. Agent 1 calls agent 2, that he saw something which with probability of 50% could be target. Agent 2 sends information to agent 3, that there may be target in the area, agent 3 believes agent two, that there is target in the area and tells agent 1, that there is a target in the area. Agent 1 reinforces his probability of seeing target to 100% and reacts accordingly; as the information from second “independent” source confirmed his believe.

Similar examples may be seen in different areas from intelligence communities to lobby organizations etc.

Unfortunately verification of the information is still more a matter of ethic of each specialist than anything else and in certain cases is even hard to do, especially in situation when we simply do not have enough information available. That’s why we have to carefully choose the specialist and questions we will ask.

Evaluation of the questions unwinds itself from the tape of the question, well perhaps rather type of answer. Typologically we can have three types of answers:

1. Numeric
2. Choice from predetermined variants
3. Free text answer.

It is obvious, that all types of the answer must be processed in different way.

For numeric answers we may compute averages, medians, minimum, maximum etc. When, there are some extreme values, we may want to remove them, as they will have large impact on results. If you remove some answers, than you have to write reason for it in the analysis, otherwise there would be risk of losing objectivity and possibility to lose credibility of the analysis. There is also risk of removing extreme answer, which is right (however improbable).

When evaluation choices from predetermined variants, we can’t compute averages etc., or we could, but it would not make any sense. When evaluating such type of answers we should focus on number of occurrences of the opinion or the percentage of opinions. Percentage may be used for example to estimate results of the election etc.

Free text answers are hardest to evaluate. Problem with them is that all answers will be original. When evaluating we have to analyse the answers and try to find some common points or interesting opinions. Free form answers may inspire us to perform more in-depth analysis (or study) of question’s subject. If we do not intent to spent lot of time on answers evaluation, we should evade this type of questions.

Even if we manage to choose the experts in such way that they are independent, the answers to our questions can still be misleading. The problem can be with personal connection of the expert to the problematics – the problem of **bias**.

Hypothetical bias happens when the person answering the questions sees the situation as purely hypothetical and strongly believes that it cannot happen. In such cases person has a habit to downplay the answer, which may lead to underestimating the effect.

We can minimize risk of the hypothetical bias by formulating the questions in way that will clearly connect it to the real-world scenarios. We may achieve this by introducing the scenario prior to starting the questions. The more real-world the questions are, the less likely is hypothetical bias to happen, unfortunately also the so called strategic bias is more likely to happen.

Strategic bias happens in event the person answering the questions not only believes that the question is about real situation, but also he believes that certain type of the answer will bring him some kind of benefit. For example he may intentionally overestimate the risk to motivate someone to realize protective measures regardless whether they are needed or not.

The preparation of the questionnaires is hard. We have to think out of how much real-world the questions will be. We have to also decide the form the questionnaire will have. Will it be sent by e-mail, will we ask random people on streets, or will it have for of interview (face to face, over the phone, chat, ...)?

In other words – making it all happen is hard, so why should we even try? There are many decision situations where there is virtually no information available in “hard data” to support our decision – we have simply no other option then “fish” for the opinions.

There can be also opposing extreme – there is too much of information and we are unable to sort it out. In this case the method may help us find what is important or what should we focus on. We can use such information to narrow down the problem and find analytical method to solve it. In standard

situations it is usually better to focus on usage of other methods – as they have better chance to achieve accurate solution.



Questions

1. How works Delphi oracle method?
2. What kind of answers can Delphi oracle method bring and how do we process it (are there differences in processing between the types of the answers)?
3. What is data incest and how can we avoid it?
4. What is difference between hypothetical and strategic bias?

4.2 Brainstorming and brain mapping

Brainstorming of one of most used methods for analysing (not only) decision situation. This method works best for smaller teams trying to find all aspects of the given situation. As such it is especially useful in beginning stages of decision making.

The method itself is not usable to identifying optimal solution of the problem, but rather to describe it or identify all important aspects of it. The solution itself must be reached using different analytical method.

When brainstorming, there is always one person in charge, who set the stage for brainstorming session, defines the problem and writes down opinions and ideas of other team members.

Brainstorming session works in that way, that everybody in the team speaks up their ideas and opinions on given theme. Multiple opinions of different people should allow describing the problem from many possible points of view. If speciality portfolio of the team members is wide enough, there is a very good chance to identify all important aspects of the problem. Though there is never a guarantee, that we covered whole problem. That's why we have to approach the problem open minded and be prepared to reevaluate our work so far when some new kind of information shows up.

Opinions are written down and as next step we can connect them and work them out in similar way. Resulting interconnected graph is called **mind map**.

Mind maps are often developed using mind mapping software, such as FreeMind [7] (open source) or MindManager [14] (proprietary software). Mind maps in computer are usually developed in different way. Only single person works on the mind map at the time, and when ready he sends it to his colleagues to work on. Until it is fully developed.

While mind maps can be used instead of black board or paper during brain storming sessions, it is also very usable to make a private mind map – just to help organize mind on the problem for one team member only. So the mind maps do not need to be shared. Capabilities of the software to link to other documents, notes, pictures and other resources only support such notion.



Mind map example

Let's try to construct short mind map around the theme – modelling of decision processes, see fig. 4.1.

As you can see on fig. 4.1, we are starting with single noun describing our problem. From it we derive various aspects of the problem and describe it again by noun. The number of the nodes in the map is virtually unlimited. Even large mind map is easy to work with, because the nodes can be folded do hide large parts of the map, when you do not need it.

We can link different mind maps together to form huge, self describing structures. Because we can also link leaf nodes to the documents on local computer or on network (LAN or Internet), the mind map is not limited only to interconnected brainstorming nodes. Better orientation is also possible by using icons visually describing the node.

FreeMind as a tool written in programming language JAVA is capable to function as JAVA applet and allows for diagrams to be viewed inside web browser over the Internet. Of course, that on the

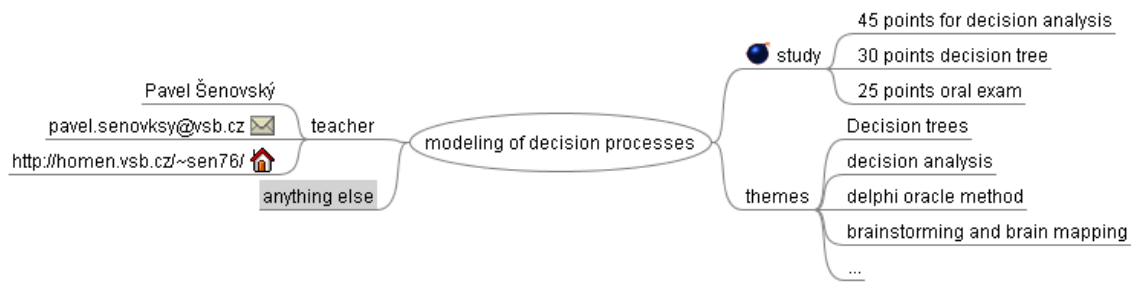


Figure 4.1: Mind map example

other hand also means, that you will need the **Java Runtime Environment (JRE)** installed. Fortunately Oracle provides JRE free of charge for many platforms at its dedicated web page [10].

There are available many more free or proprietary softwares for this tasks.



Create mind map

Download FreeMind (or any other freely available tool for the task) and create mind map of emergency: accident of tank transporting methane under pressure.



Questions

1. What is brain map good for?
2. When in decision process should brainstorming session take place?
3. What is the role of team leader during brainstorming session?

Chapter 5

Network models



Study guide

In this chapter we will acquaint ourselves with application of the network models for description of the activities as part of the project realization as a basic tool of manager for identification of problematic activities inside of project – allows him to redevelop them.

After reading this chapter you will

Know

- what the network graph is
- what the progress chart is
- how is used CPM method



Time required to study chapter

For getting through the theory in this chapter you will need at least an hour, but if you try network models practically, reserve few hours for your experiments.

5.1 CPM method in project management

Network models are commonly used models for analysis of so called *critical path method (CPM)*. Network models are also used for visualization of interconnected project activities, for analysis of distribution networks etc.

When talking about distribution network, the configuration of network is derived from physical configuration of modeled network. When modeling project, the configuration of model is derived from activities necessary to finish the project and time connection between them.

Before we dwell into network model construction, let's define several terms first, we will use on next few pages. First of them is management. The term management is derived from French word *manège* - as circular arena for horses training or French word *managere* – in English management. There are many definitions of management; one of definitions used more often is that management is completing the goals using others. Another definition of management is that management is mobilization of all company's resources to accomplish planed goals.

Management as science branch is considered to be arts, that's why it is not completely possible to for "hard" mathematic methods to hold all the answers we need. Such methods are usable as important support tools. Its usage will not enable the manager to magically succeed whatever he is doing, but if used properly it can increase his chances for success.

Another term we will discuss in this chapter is *project*. Project means chain of activities, which lead to the goal of the project. For each project we have clearly stated start and finish dates, we also have only limited resources to achieve the project goals.

The activities of the project have to take place in certain order. It is possible to express such order in tabular form. Let's demonstrate the approach on example of simplified building construction (see table 5.1).

Table 5.1: Activities of the project

	activities	followers	Predecessors	Length [days]
A	Foundation excavation	B	-	2
B	Concreting of excavations	C	A	10
C	Fabric of the building	D	B	17
D	Roof	E,F,G,H	C	5
E	Electro installation	I	D	5
F	Piping (water, gas)	I	D	5
G	heating	I	D	5
H	windows	I	D	14
I	Facade rendering, flooring	J	E,F,G,H	21
J	Building acceptance	-	I	1

We can describe the sequence by specification of followers (activities which have to follow after finishing present activity) and predecessors (activities which have to be finished before present activity can start).

The both approaches are completely equivalent by its meaning; they differ by the direction of construction. When considering followers we are going from start to finish, while when considering predecessors we do opposite – going from finish back to start.

Some software tools for project management such as Microsoft Project [13], open source Libre Project [18] or Open Workbench [17] allow the user to choose follower or predecessors approach as needed or as preferred by user and even to change then between each other – followers and predecessors will convert accordingly.



Software support

Creating the diagrams by hand is tedious task, which takes time and you can easily make a mistake. That's why there are many software tools to support us in project management.

The leader in the branch of project management is Microsoft with its MS Project Suite [13]. It is proprietary software and is usually considered as standard for such type of the software. Proprietary nature allows MS Project to cooperate with other softwares in MS Office family and with Exchange to take project management one step closer to the project members by planning their work task and propagating them into their own calendars in MS Outlook.

There are also available various other softwares to support us which are open source. Example of such could ProjectLibre (formerly known as OpenProj, [18]). ProjectLibre is available on practically any platform (MS Windows, Linux or Mac OS). Alternative could be Open WorkBench [17], but only MS Windows operating system is supported and it has far less features than ProjectLibre.

You can find practical, hands on example in ProjectLibre in next chapter.

List of the activities in tabular form is not entirely usable for operative control of activities progress, resources planning etc. That's why the graphical interpretation of this information is actually chosen, in the form of *progress charts*, sometimes also called *Gantt chart* and *network graphs*.

As you can see on figure 5.1, the red road here is so called *critical path*. By critical part we mean sequence of the activities, for which any increase in the time over the planned time would automatically lead to increase the time needed for the project as whole. Because of this property it is necessary for manager to especially thoughtfully manage these activities.

On network graph the activities are represented by the edges. Edges E, F, G, H may occur at the same time, so they have to be finished before activity I takes place. To demonstrate it we add so called virtual activities (look for dotted arrows on figure 5.2).

These virtual actions do not represent real activities; we add them to graph only to improve its readability.

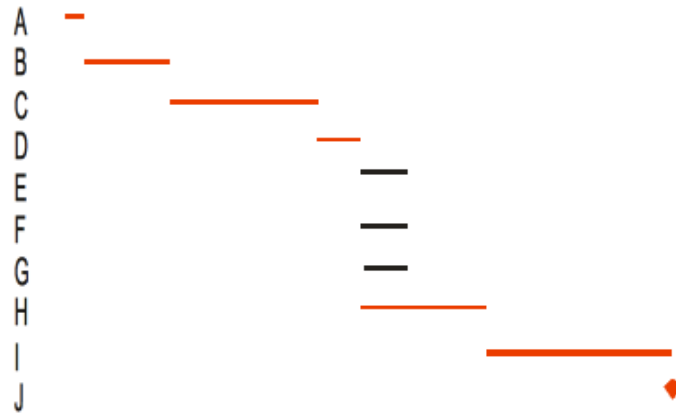


Figure 5.1: Progress chart of the project

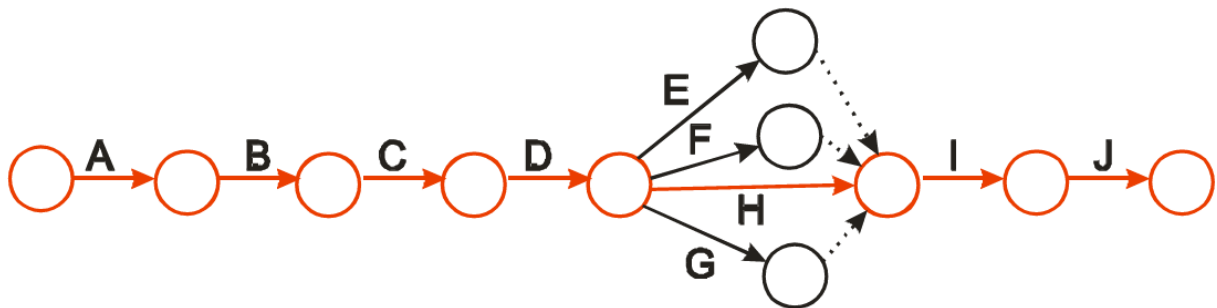


Figure 5.2: Network graph of the project

We can also note critical path in the graph. On figure 5.1, the critical path is noted in red color.

Maybe you ask why we use two types of graphs for single task (identification of critical path). The answer is that we use them both because identification of critical path is only one task the graphs are meant to solve. Primary goal of the network graph is to visualize interconnection between the activities of the project. Primary goal of the progress chart is to visualize activities from point of view of the time to allow its optimization. For this task all non-critical actions may be used, since we can increase the time needed for such non-critical task by sparing some resources (as people hours) and use them somewhere else, where they are perhaps needed more.

It's obvious that the time increase for noncritical activities can't be infinite, because by increasing length of the activity it may become critical under these changed circumstances.

Advantages of progress chart usage:

- identification of the critical path
- optimization of resources usage for non-critical activities
- overview of realized activities as a good source of information for effective management



Critical Path Method

Compute length of the critical part for example we worked with in this chapter.

5.2 ProjectLibre – software support to project management

First we to download the ProjectLibre from its homepage on SourceForge [18]. SourceForge will offer you download the latest version of the software available in binary form for your operating system.



Critical Path Method - solution

Possible paths

$$1 - A, B, C, D, E, I, J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61$$

$$2 - A, B, C, D, F, I, J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61$$

$$3 - A, B, C, D, H, I, J = 2 + 10 + 17 + 5 + 14 + 21 + 1 = 70$$

$$4 - A, B, C, D, G, I, J = 2 + 10 + 17 + 5 + 5 + 21 + 1 = 61$$

Critical path is the path 3 (A, B, C, D, H, I, J), project will last for 70 days.



Questions

1. What is CPM method?
2. Explain interconnection between the project activities from point of view of the follower and predecessors.
3. Why do we use both progress chart and network graph, wouldn't it suffice to use one of these?
4. Why do we try to find critical path?



Study guide

In this chapter we will be using ProjectLibre for project management, but most of what is written for it should apply also to any other project management software. Though **graphical user interface (GUI)** will be slightly different. So if you have installed different tool, you do not have to install ProjectLibre, if you do not want to.

Double clicking on the downloaded file will start the installation process. In case your operating system is not detected at all or is not detected correctly or you are simply downloading from different computer, you may list all available binaries in files.

At the time of writing this text (November 2012), latest available version was 1.5.1. After downloading and installing of the software, we may start to use. After opening ProjectLibre we have to create new project as we do not have existing project yet.

When creating new project, we have usually specify the project name, project manager and possibly some notes for us, see fig. 5.3. In fact writing these informations is not required, but it is considered as a good practice. Also different software suites support different project properties. For example MS Project allows you to specify what kind of calendar will the project use (common office format – 8 hours a day, 5 days in week, or will it be 24/7).

In this chapter we will try to recreate the example from previous chapter, so lets coll the project: Family house – construction. We will specify ourselves as project managers. You also may or may not want to add your own note. Clicking on OK will create the project.

The new project is automatically opened in Gantt chart mode, which allows us to specify the project tasks and the connection between them. Out of this information it automatically created the progress chart of the project.

We have already specified project tasks in table 5.1, with its name, connection to other tasks and its length. Lets with task definition – in Gantt chart we create the task by writing down its name into column name. When you create the task note that ProjectLibre automatically adds additional information to it – it guesses when the task will start (on the day of the beginning of the project), that its length will be approximately one day.

Also note that this information is automatically used for process chart on the right side of the screen. There are also other columns, which are empty so far – predecessors and resources, allowing us to specify the connection between the tasks and to assign human or material resource to the task.

Lets change length of the tasks, because right now it is one day with question mark. Question mark means uncertainty – so we are basically saying, that the task will require approximately one day. From table 16 we know that this is not the case, so lets make the corrections.

The new length can be assigned in days, weeks or months. Gantt chart presumes, that the value is in days, so writing $5 = 5d = 5 \text{ days} = 1w = 1 \text{ week}$ (this holds for office calendar). The time unit used to describe length of the task can be specified in days (d), weeks (w) or moths (m). Be aware that

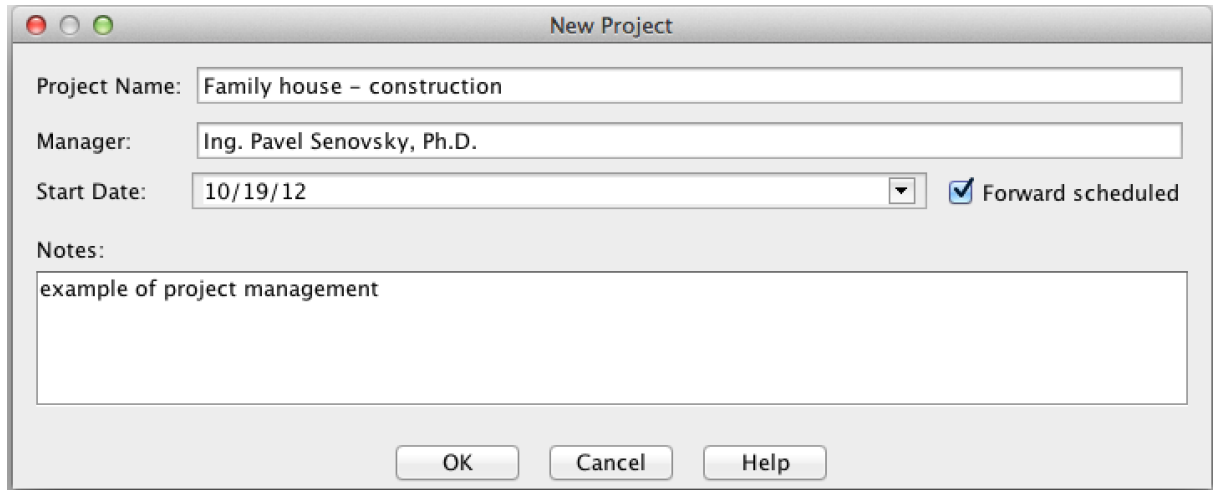


Figure 5.3: Create new project

the week may mean something different than you think. When normally speaking about the week, we usually mean monday – sunday (or sunday – saturday depending, where are you from), the meaning of the week in project management is different.

The difference is in what calendar is used – is it normal office calendar with working hours 7 - 15:30 monday to friday or is it different? Meaning that the week can have 5 days or 7 days for different tasks using different resources. In project management we also do not know exactly when we will start with the task as its predecessors may have finished earlier or later then expected, so the week in project management is described by number of working days only – with floating beginning and the end.

The information on tasks in table 5.1 and the way we have defined our tasks in Gantt diagram also differ by tasks identification. In table we identify the task alphabetically using capital letters A – J, in Gantt chart the tasks are identified by the number of the row they are written on (1-10). Defining the connection between the task would be slightly harder for us to do, so we will use capabilities of the software to insert new column for us to write the letter identification in it.

Adding new column is easy. We just click on the column where we want to have new column. Then we right click on it to display context menu allowing us to insert and remove the columns. We will insert one. As you can see (well if you are trying it on the computer :-), there is plenty of possibilities to choose from as the template of the new column. We will choose Text1 – which is general template allowing us to insert any text into it. After inserting the column, we will change its name by right clicking on the column and choosing Rename. The new name will be ID Letter. Now we can write down letter IDs from table 5.1 into our new column.

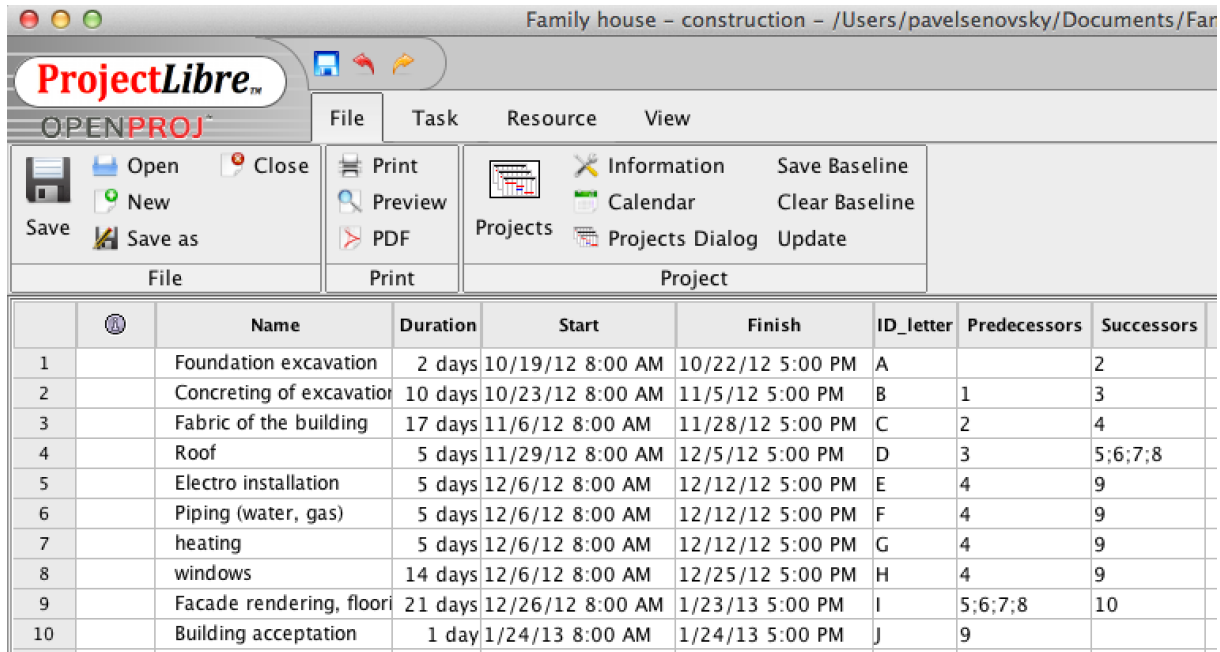
Now the only information missing from table 5.1 are the connections between the tasks. In Gantt chart we already have predecessors, so we can use these, but if you rather preferred usage of the followers – you can, by defining new column with successor template. The software will provide automatic translation to the other one.

In this text, I will use predecessors. Do not forget that we will have to identify the task by number, not by the letter in the new column we created. Also we will need to assign more then one predecessor for task I – we write all IDs in predecessor column and separate them semicolons. Well, the separation by using of comma works too – ProjectLibre just automatically replaces the commas with semicolons. In the end your Gantt chart should look similar to the figure 5.4.

By definition of the predecessors or successors we specified the connection between the tasks and the progress chart now makes some sense, see fig. 5.5.

On the progress chart you can see that the tasks have different sizes, are of different colour. The size of the task is related to the task length. So as we defined it, we have it, but once defined it doesn't mean that we stop working with it – on the contrary, our work as project managers has just started. Now we have to closely follow happenings of our project and how the tasks are finished. The software supports us by allowing us to follow percentage of completeness of the task.

To change the percentage we use the mouse. We move the mouse over the beginning of the task.



The screenshot shows the ProjectLibre interface with a table of task definitions. The table has columns for ID, Name, Duration, Start, Finish, ID_letter, Predecessors, and Successors. The tasks are listed as follows:

ID	Name	Duration	Start	Finish	ID_letter	Predecessors	Successors
1	Foundation excavation	2 days	10/19/12 8:00 AM	10/22/12 5:00 PM	A		2
2	Concreting of excavation	10 days	10/23/12 8:00 AM	11/5/12 5:00 PM	B	1	3
3	Fabric of the building	17 days	11/6/12 8:00 AM	11/28/12 5:00 PM	C	2	4
4	Roof	5 days	11/29/12 8:00 AM	12/5/12 5:00 PM	D	3	5;6;7;8
5	Electro installation	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	E	4	9
6	Piping (water, gas)	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	F	4	9
7	heating	5 days	12/6/12 8:00 AM	12/12/12 5:00 PM	G	4	9
8	windows	14 days	12/6/12 8:00 AM	12/25/12 5:00 PM	H	4	9
9	Facade rendering, floor	21 days	12/26/12 8:00 AM	1/23/13 5:00 PM	I	5;6;7;8	10
10	Building acceptance	1 day	1/24/13 8:00 AM	1/24/13 5:00 PM	J	9	

Figure 5.4: Gantt chart – task definitions

The mouse pointer will change into percent symbol and the arrow pointing right. Now click and pull to right to increase percentage.

Similarly we can change the beginning or the end of the task as needed. The remainder of the tasks will adjust itself as needed without the need to micromanage it.

And lastly we have the colour of the task. On our chart we have red and blue tasks. Task in blue are these which do have a time reserve. By time reserve we mean, that there is time available for us to lengthen the task without endangering whole project completion.

So the time reserve and manipulation with non-critical tasks allows us to free resources necessary to finish the critical (red) tasks on time.

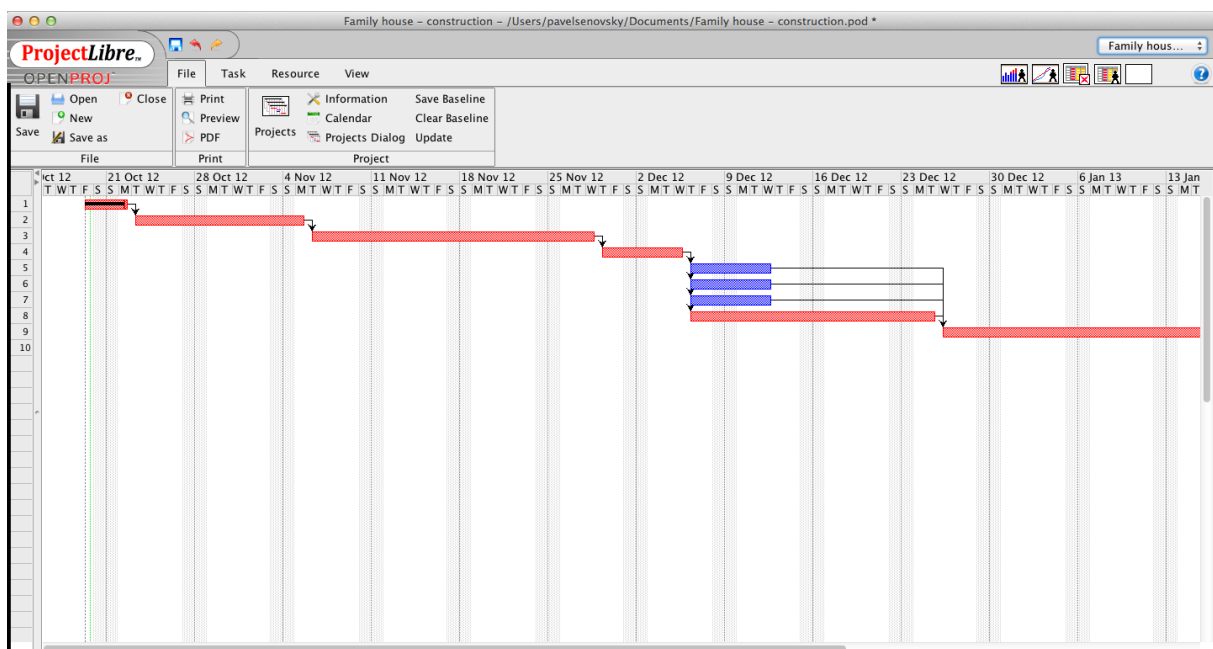


Figure 5.5: Gantt chart – progress chart

Alternative way of looking on the progress chart is by using network diagram. In network diagram we see the tasks as the nodes of the network. Connection between the tasks allow us identify which tasks must be completed before others may begin. Network diagram will be generated automatically based on the tasks definitions in Gantt chart.

Start network diagram on Task tab -> tool Network. You may see screenshot of our network on figure 5.6.

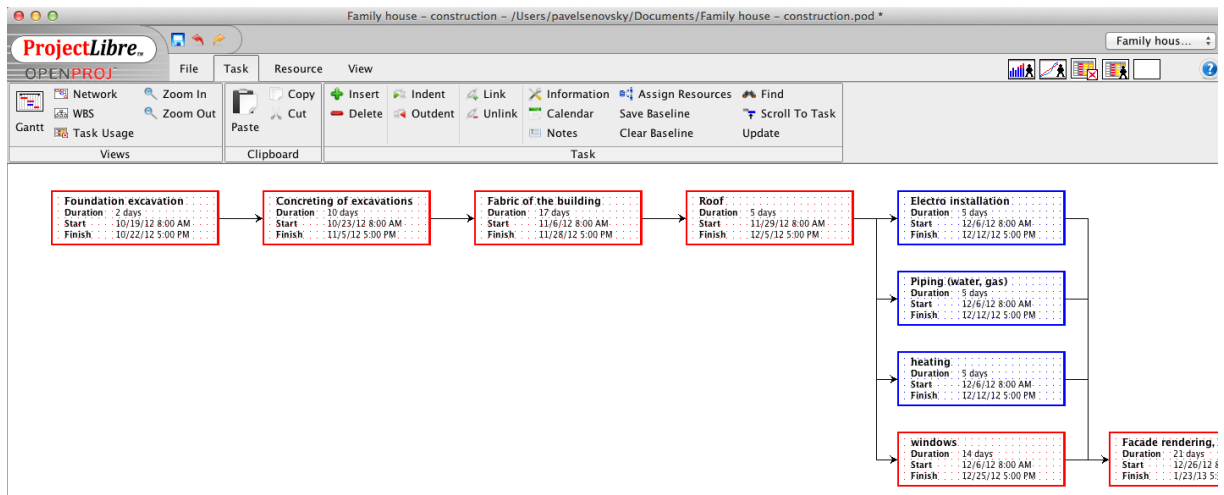


Figure 5.6: Project's network diagram

So why should we use both network diagram and progress chart when both basically show the same things? Well in fact they, but both in slightly different ways. Network diagram allows us to better see the network of the tasks structure, while progress chart allow us to better see and manipulate with reserves and manage the project when it starts.

We are talking here about the resources, can the software tools help us with these too? Yes it can. We can manage the resources available for the project on the resources tab (first tool from left on the tab).

Lets make three resources – worker 1, 2 and sand. For the resources we specify the type of the resources. The resources can be human beings available for the work, it can be also the material. Obviously resources of the different types have also different properties.

For human resources we choose type work. To such resources we can assign contact email a group them for example by the teams they are working in. For human resources we state max. units. Automatically software assigns 100% max. unit for the resource. Usually we interpret it so, that we have available 8 hours of resource's work. Depending on the resource usage we may have more or less available. Max. units should not exceed 150%, as we would be getting closer to the slavery work which is forbidden by law in many countries.

Each human resource has its standard pay on per hour basis and overtime pay again on per hour basis for the cases when the usage of the resource exceeds max. units. Overtime pay is usually higher then standard pay.

Cost per usage is the cost connected to the usage or the human resource, but can't be considered income of the resource. For example I might need to transport workers on the site, which costs me money – that would be cost per usage.

Accrue at column lets us set when we will pay the resource for his work. We may do it as prorated, at the beginning or the end. Prorated would mean, that we will pay the resource every day. In praxis we may use the scenario of usage of diggers to prepare sewer network. In such case we pay them on the end of each work day (if paid on the beginning they may not come to work).

At the beginning we pay in cases when there is some kind of resources procurement necessary on side of the resource. On the end means that we will pay after we check and approbate the work.

Every human resource may have its own calendar – standard, 24 hours and night shift.

In case of the material resources many columns do not have any sense, so we do not use them. What is the meaning of the remaining columns? Standard rate should be interpreted as the rate at which we procure unit of the resource. The units may be tons, kilograms, price for the piece, square

meters, ... Cost per use is similar to its human resource's counterpart – usually we need to move the material on site before we can use it and it costs money.

Example of the resources definition is on figure 5.7.

	Name	RBS	Type	E-mail Address	Material Label	Initials	Group	Max. Units	Standard Rate	Overtime Rate	Cost Per Use	Accrue At	Base Calendar
1	Worker 1		Work			W		100%	≈200.00/hour	≈300.00/hour	≈0.00	Prorated	Standard
2	Worker 2		Work			W		100%	≈230.00/hour	≈350.00/hour	≈0.00	Prorated	Standard
3	sand		Material			s			≈5000.00		≈0.00	Prorated	

Figure 5.7: Resources definition

Now lets go back to the Gantt chart. We will assign the resources to the tasks. Click on the task you wish to assign the resource to and click on the tool Assign resources to open new dialog with all resources, we have defined in resources table. Lets try to assign the worker 1 to all the tasks. You may speed the definition process by copying resource name in Resources column.

Lets see how exactly are we using the resource. First we will look at the resource's histogram. You may start it by clicking on histogram icon on upper right side of your program's window, or look at the figure 5.8.

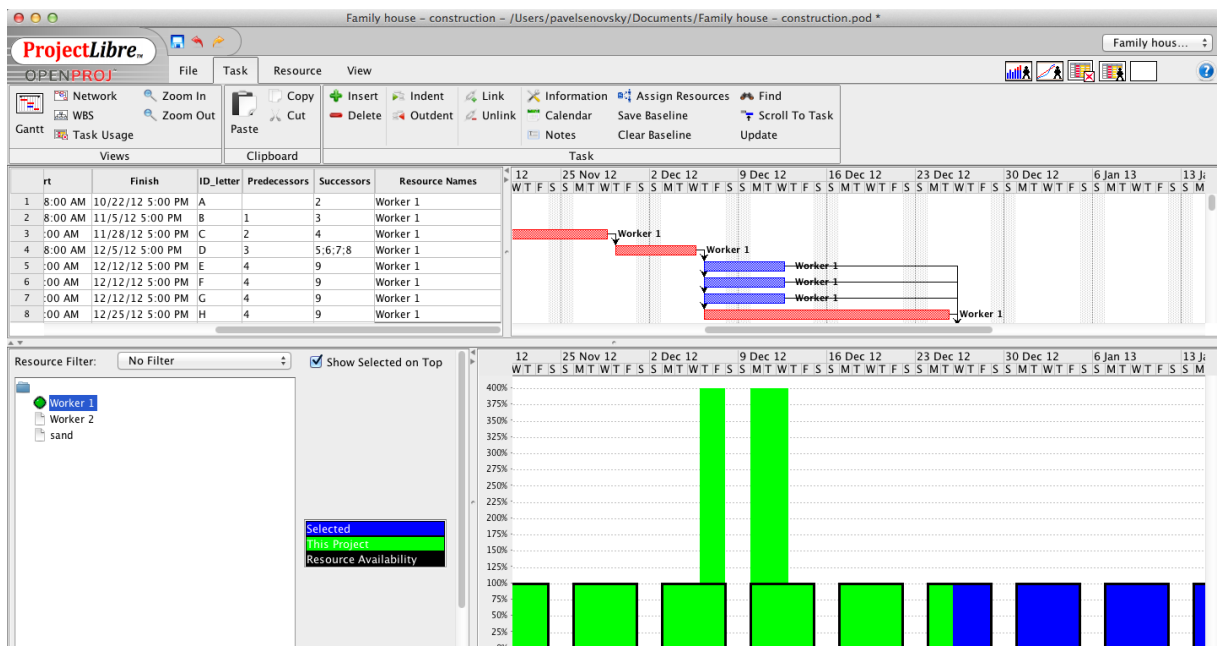


Figure 5.8: Resource's histogram

The histogram is connected to the Gantt chart, to navigate through it you have to mo the Gantt chart, histogram will move automatically. As you can see the problem is in tasks starting 6th December, we have four of them and we assigned worker one to all of them, which leads to our brutal overuse of the resource – we plan to use it on 400%, which is of course not possible.

Histogram thus helps us identify the problematic tasks in our project and allows us to solve the problem before the problem manifests itself in praxis.

We may look at the resources also by assigned working hours choosing Resource usage, see fig. 5.9.

And thats all, you will se in this textbook on project management. The software itself has more tools allowing you to better track monetary resources and many more aspects of the project manage-

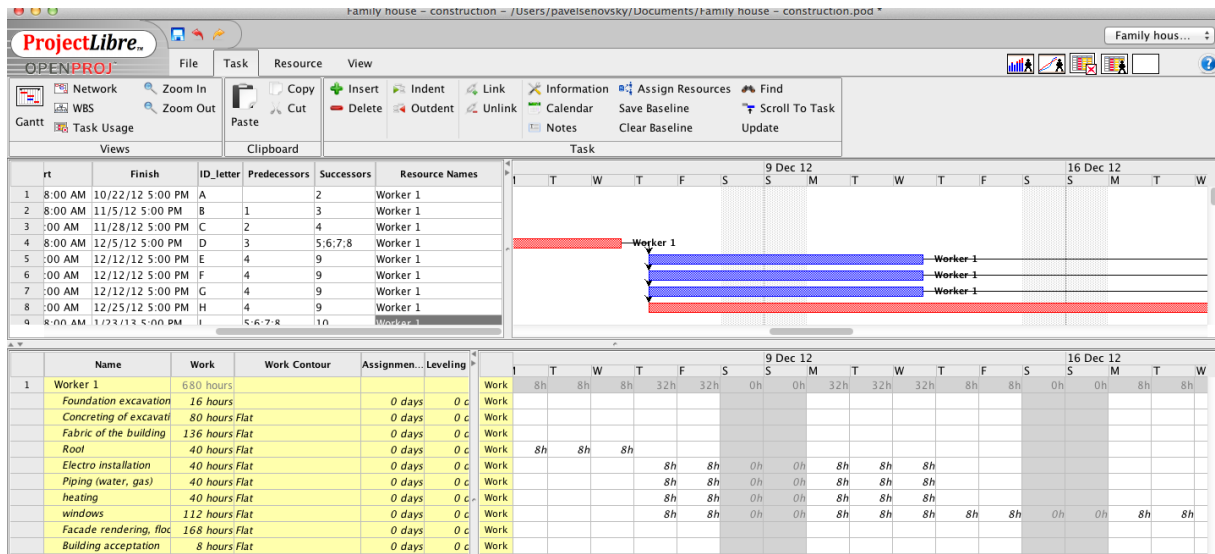


Figure 5.9: Resource usage

ment.



Project management

Create new project of your choice, if you do not want to think – create project for your study program this year, look at all of the terms, semestral projects, assignments...

5.3 Optimization in other network models

Project management has special needs, but the knowledge of CPM method may be used in different types of networks as well. For example we may want to analyse road (rail) network or some kind of pipe network. These networks need different types of answer – we may search for shortest road between two nodes or critical path from capacity point of view etc.

Such types of task are solvable if the modelled network closed – there are no outside connections to or from it. Another limitation of the model is that what comes in the first node must also go out from last node.

Let’s see how we can solve some simple problems algorithmically. First type of problem is searching for **maximal flow in network between two given nodes**. Problem with this kind of problem is that the edges between the nodes usually have different capacity. And to solve this problem combinatory would be for large networks virtually impossible as the difficulty of the problem will grow exponentially with number of nodes.

We can apply different approaches to solve it, for example **Ford-Fulkerson algorithm**, which works in following way [5]:

Inputs: Graph G with flow capacity c , source node s and the sink node t .

Output: flow f from s to t , which is maximal

1. $f(u,v) < 0$ for all edges of the network (u, v)
2. while there is path p from s to t in G_f , such that $c_f(u, v) > 0$ for all edges $(u, v) \in p$:
 - (a) find $c_f(p) = \min[c_f(u, v) | (u, v) \in p]$
 - (b) for each edge $(u, v) \in p$
 - i. $f(u, v) < -f(u, v) + c_f(p)$, send flow along the path
 - ii. $f(u, v) < -f(u, v) - c_f(p)$, the flow might me “returned later”

Imaging how Ford-Fulkerson's algorithm actually works is rather hard. You can see an example of Ford-Fulkerson's algorithm in Ford-Fulkerson Demo [6].



Software support for physical network computations

Computing the physical network using Ford-Fulkerson's algorithm, or any other by hand is rather hard, even for small networks. That's why there are many software tools which employ the algorithms and virtually separate us from the algorithm itself. You can find them in modern **Global Positioning System (GPS)** and many more applications.

We will use computer program for numerical computations called SciLab. We will leave the demonstration for the next chapter.

Another type of the problem could be *searching for shortest path between nodes*. For this type of problem algorithm of Dutch scientist Dijkstra is often used (*Dijkstra algorithm*). The algorithm is as follows [2]:

1. Create a distance list, a previous vertex list, a visited list, and a current vertex.
2. All the values in the distance list are set to infinity except the starting vertex which is set to zero.
3. All values in visited list are set to false.
4. All values in the previous list are set to a special value signifying that they are undefined, such as null.
5. Current vertex is set as the starting vertex.
6. Mark the current vertex as visited.
7. Update distance and previous lists based on those vertices which can be immediately reached from the current vertex.
8. Update the current vertex to the unvisited vertex that can be reached by the shortest path from the starting vertex.
9. Repeat (from step 6) until all nodes are visited.

Both algorithms work similarly – they reduce the network into the tree graph, for which the number of computations needed to find the answer is manageable. Dijkstra algorithm is also usually applied by specialized software.

There are about dozen of various algorithms used for different types of tasks. You can review them more in depth for example on Wikipedia in article List of Algorithms [?], which has dedicated section to it.

Questions



1. What are limitations of the network models?
2. What kind of problems are we able to solve by the models?
3. What is the difference between Ford-Fulkerson's and Dijkstra's algorithms?
4. What is common for these two algorithms?

5.4 SciLab – numerical computations

Lets prepare software tools so we can begin. We will be using SciLab [20] which is open source numerical computation tool similar to MathLab by its focus and even the integrated programming language is similar. You can download SciLab for your operating system from <http://scilab.org>.

SciLab package is available for Windows, Linux and Mac OS X operating systems in binary form easy to install – just follow the installation wizard. After installation you can run the SciLab start working with it.

You can work either in interactive regime by writing the commands directly into the program's console and seeing immediately the results, or you can prepare SCE file with batch of the commands, which will be run together – forming small program. See fig. 5.10 for SciLab screen shot.

Please note, that the program is localized into many languages and the language is assigned automatically at the start of the program in connection to operating system language preferences, so

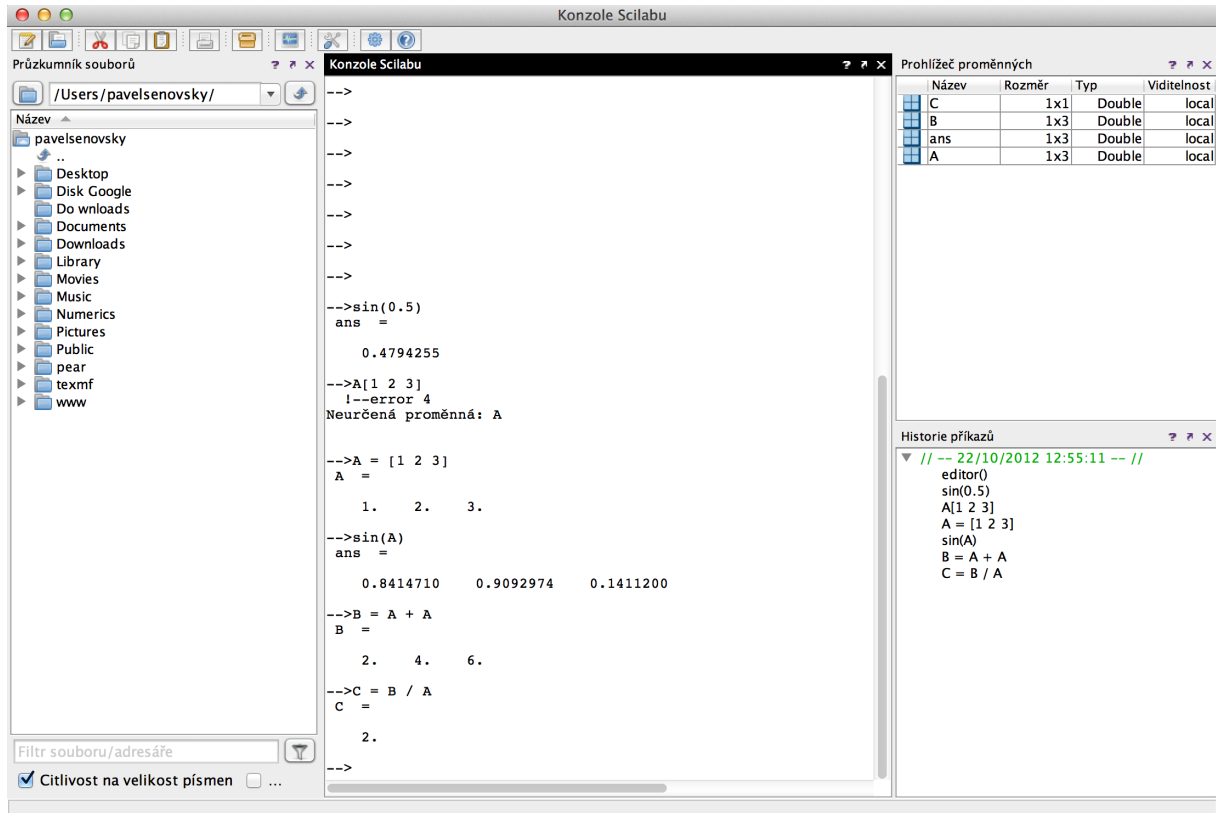


Figure 5.10: Basic console of the SciLab 5.4.0 on Mac OS 10.8

you might see little bit of czech now and then. The majority of the work is done by the programming language, which is independent to the GUI's language, so you won't get lost there.

If you have no prior knowledge of the software like MathLab or SciLab, you will need short introduction to philosophy of such programs. Unfortunately in this textbook, we have no room for such introduction. That's why I will guide you to very nice *A Short Introduction to SciLab* by Terence Leung Ho Yin and Tsing Nam Kiu [26]. Much more complex introduction to SciLab is available from Department of Chemical Engineering by Niket S. Kaisare: *Scilab Primer* [23].

Scilab Primer has some advanced topic covered, so I recommend to start with short introduction or any other introduction. One of the advantages of the SciLab is, that it is quite popular and as such there are many sites with deep coverage of the SciLab's functionality – so if you are not entirely satisfied with the resources I gave you Google out your own. It worth a try, don't you think?



Learning basics of the SciLab

Take your time to get through the basic introductory material for the SciLab. You need minimally to understand variables – normal, vectors and matrices as well as basic operations with them.

Now that we are confident in the usage of the program, we can move to the network optimization algorithms. There is small problem with these – basic SciLab package doesn't support them, so we have to download and install *Numerical* package created by *Fakultat fur Mathematic Ruhr-Universität Bochum*. The Numerical package [16] is available on their web pages for free including basic documentation of the package.

Unzip the package to the directory of your choice. It can be anywhere, just do not forget where, because soon we will need this information. Also we will need the network to work with. For our example we will use network from presentation available from Prometheus demonstrating usage of

algorithms in hand.

Lets use basic network as the one on fig. 5.11.

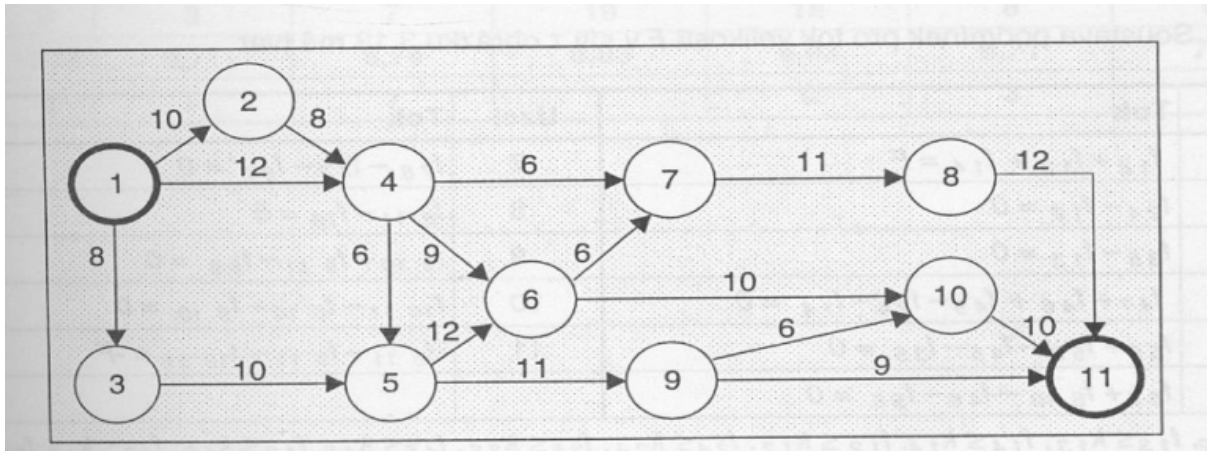


Figure 5.11: Basic network (Source: Gros [22])

In Ford-Fulkerson's algorithm we will search for the highest possible capacity of the network between the nodes 1 and 11.

First we need to create network structure for our computation. Numerics package allows to define it in two ways – using square matrix or list of vectors. Lets start with the matrix, we will use first five nodes:

	1	2	3	4	5	...
1	-%inf	10	8	12	-%inf	
2	10	-%inf	-%inf	8	-%inf	...
3	-%inf	-%inf	-%inf	-%inf	10	...
4	12	8	-%inf	-%inf	6	...
5	-%inf	-%inf	10	6	-%inf	...
...

As you can see most of the connections between nodes do not exist in our network – we fill in -%inf constant in there. For larger networks the matrix would significantly grow, thats why I prefer to use list of the vectors to define the network as is describes only existing paths and exactly once in formate of (star_node, finish_node, capacity). In our case for nodes 1 – 5 it would be:

```
list([1, 2, 10], [2, 4, 8], [1, 4, 12], [1, 3, 8], [3, 5, 10], [4, 5, 6])
```

Much easier, isn't it? Now lets take all components together and compute maximal capacity of our network.

```
mylib=lib("/Users/pavelsenovsky/Numerics")
net = list([1,2,10], [2,4,8], [1,4,12], [1,3,8], [3,5,10], [4,5,6], [4,7,6],
[4,6,9],[5,6,12], [7,8,11], [8,11,12], [6,10,10], [10,11,10], [5,9,11],
[9,10,6], [9,11,9])
first=1
last=11
ford_fulkerson(net,first,last)
```

Actually only mylib, net and ford_fulkerson with net parameters are necessary to perform the computation. So what does our little program do? In mylib row we use lib function to load Numerics package – we need it since, basic SciLab package doesn't have support for the algorithms we need. I computed the example on Mac, so my user directory is in "/Users/pavelsenovsky/" and I unzipped Numerics package in there. You have to assign the path to where you unzipped the package on your computer. Note, that specifying full path is needed.

Variable net hold structure and capacities of our network. I am using list of vectors notation. Variables first and last are necessary if the starting node is not 1 and terminal node has not the highest number (11 in our case).

The function computing Ford-Fulkerson algorithm is `ford_fulkerson`. It has 4 parameters, but only one – network structure in mandatory, second – starting node, third – terminal node a fourth – what should be printed out (0-4) can be omitted.

When we run our program the result will be **25**.

Now that we know the drill, it should be easier for us to solve Dijkstra algorithm to find shortest path. We have net variable holding structure of the network – since we are using same network, we have majority of the work done. In fact, we only have to replace `ford_fulkerson` function call with `dijkstra` function code, see:

```
mylib=lib("/Users/pavelsenovsky/Numerics")
net = list([1,2,10], [2,4,8], [1,4,12], [1,3,8], [3,5,10], [4,5,6], [4,7,6],
[4,6,9], [5,6,12], [7,8,11], [8,11,12], [6,10,10], [10,11,10], [5,9,11],
[9,10,6], [9,11,9])
first=1
last=11
dijkstra(net,first,last)
```

When we run the program, we get shortest path between the **nodes 1-3-5-9-11** with length **38**. Hey, not bad, not bad at all.



Try it yourself

Define network from [6] and compute maximal capacity and shortest path using SciLab.

5.5 GraphViz

What to do in cases, when the structure of the network is known, but there is no visual representation to it. Well we can use general graphic programs as Dia [1] or MS Visio [15] to draw the network by hand. Such task is sometimes very hard if exact location of the nodes in network is not know or we are not interested in it – we are interested in other properties of the network. For such cases we need another tool, which is capable to render the network based on provided information. Such tool is GrapViz [9].

GraphViz is open source software suit used to visualize graphs of various kind. GraphViz is available on Windows, Linux, Solaris and Mac OS (but not on latest version 10.8 – Mountain Lion). The software suit provides variety of command line tools or preparation of the networks and thats why it is favoured by authors of many other software tools using it as the backend for networks renderings.

The structure of the network is prepared using simple language for network description called DOT. We define the structure of the network by writing sequentions of the nodes connected by edges. For the network from fig. 16 one of these sequentions could be 1 -> 2 -> 4 -> 7 -> 8 -> 11.

DOT allows us to construct both oriented and unoriented networks. The difference is in used symbols for oriented edges “->” and unorineted edges “-”.

Whole definition of the network from fig. 5.11 will look in DOT this way:

```
digraph G{
  1 -> 2 -> 4 -> 7 -> 8 -> 11;
  1 -> 4 -> 6 -> 7;
  1 -> 3 -> 5 -> 6 -> 10 -> 11;
  4 -> 5;
  5 -> 9 -> 10;
  9 -> 11;
}
```

In the code higher we are defining graph G as several sequences of the connections between the nodes. The graph is oriented and the connections do not have any significant property to be visualized. The digraph means type of the graph we are going to construct. The type of the graph will have significant influence on the way the graph will look.

You can see, what is possible to generate using GraphViz on its Gallery page [8].

Lets create unoriented variant of the graph with length of the edges defined.

```
graph G{
  1 -- 2 [len=10];
  2 -- 4 [len=8];
  4 -- 7 [len=6];
  7 -- 8 [len=11];
  8 -- 11 [len=12];
  1 -- 4 [len=12];
  4 -- 6 [len=9];
  6 -- 7 [len=6];
  1 -- 3 [len=8];
  3 -- 5 [len=10];
  5 -- 6 [len=12];
  6 -- 10 [len=10];
  10 -- 11 [len=10];
  4 -- 5 [len=6];
  5 -- 9 [len=11];
  9 -- 10 [len=6];
  9 -- 11 [len=9];
}
```

The bulk of the code is slightly larger then in previous example, as we have to define length of each graph's edge separately, but it is not that bad. In fact it is very easy to program small utilities capable to automatically generate such code and then call rendering routines via command line.

The results of the rendering are available on fig. 5.12.

When we compare fig. 5.11 and 5.12 they are both visually different, but the structure itself and the properties of the network remain same. Using different rendering routines will result into different organization of the graph, even if the structure remains the same. Look at the figure 5.13.

Results of neato renderer look much more similar to the original fig. 5.11.

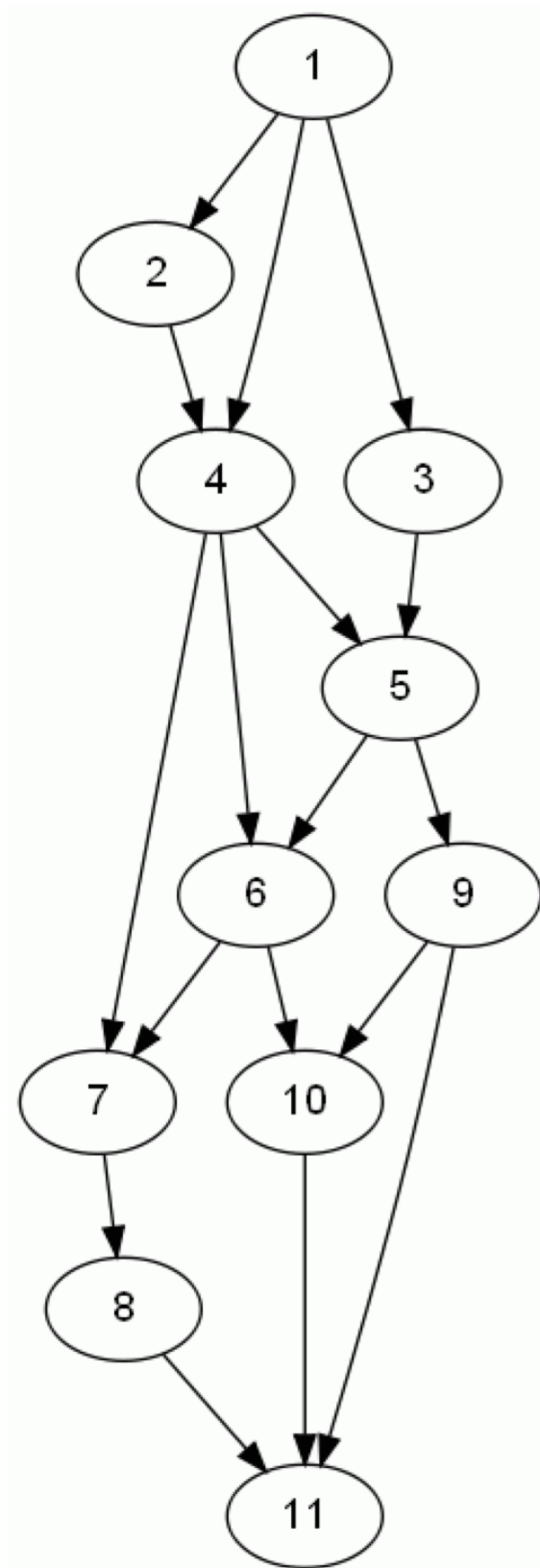


Figure 5.12: DOT automatically generated network based of structure derived from fig. 5.11

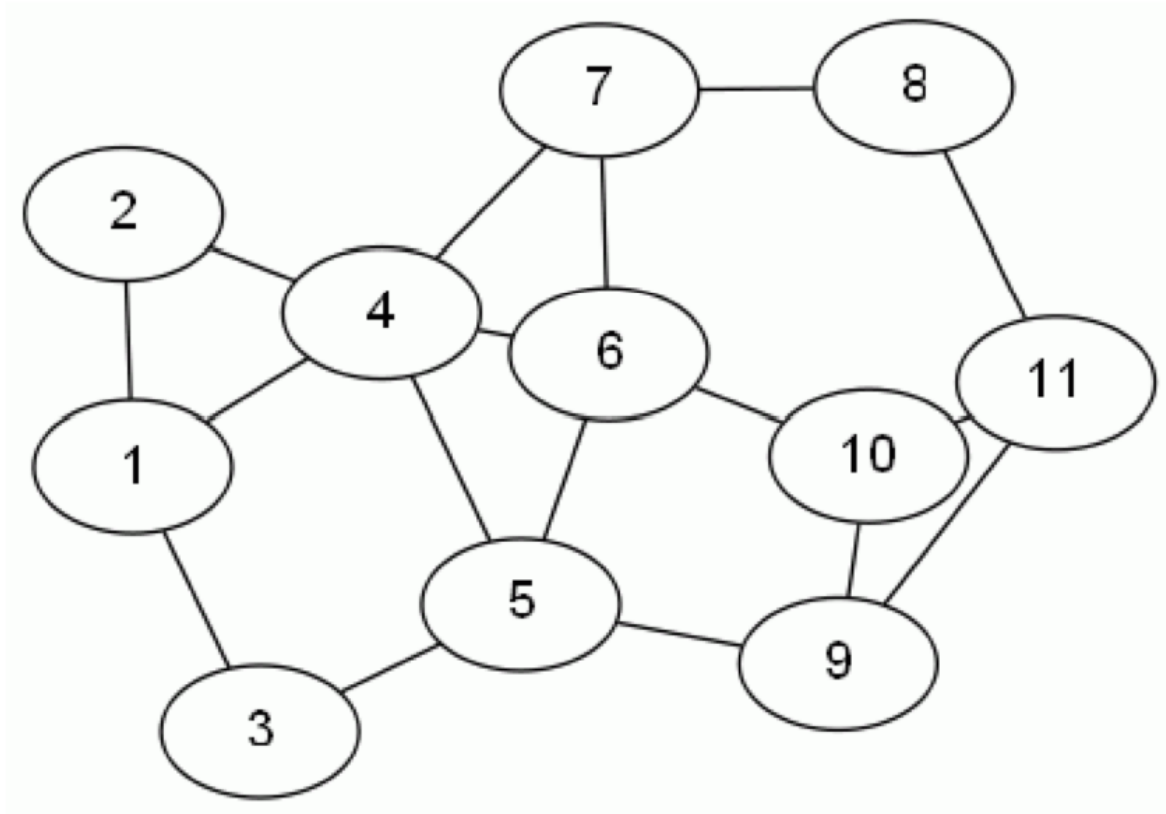


Figure 5.13: Alternative look on network – using neato renderer



Define and render your own network

Define network from [6] and try to render it using different algorithms available from GraphViz. Do not be afraid to experiment.

Chapter 6

Balance models



Study guide

When deciding during various emergencies we should come from deep, in past prepared knowledge/information about the organization we are trying to save. One of basic functions of the company (at least the manufacturing one) is to produce the products for sale.

Balance models, we will deal with in this chapter, are capable to bring us this insight by modeling whole chain of production – allowing us to describe how the raw materials are transformed into the form of half-finished products and these into final products.

Such information is usable for evaluation of emergencies impacts on production capability of the company, so we can develop contingency plans optimizing the post emergency recovery effort.

After reading through this chapter you will

Know

- What the balance models are
- How do we construct one
- And how can we solve one



Time required to study chapter

For getting through this chapter you will need at least 30 minutes.

Balance models are used for exact description of interconnection in production cycle in the company thus allowing us to describe the way the inputs (raw materials, half-finished products) transform on outputs (final products). Balance models are used by economists for long time for planning of production capacity and planning of resources needs of the production.

In emergency planning we can use this type of models for modelling of the emergencies impact on our production capability, especially how will take effect disabling or destruction of part of technology in the company on its ability to produce. Based on these results we may plan way of effective recovery from emergencies.

Lets discuss small example of the problem. On fig. 6.1 we have block schema of the production process of the powder fire extinguishers. The structure of the production process is well known before we usually start with balance models – since the organisation has to know how to produce the goods to survive on the market. Balance models are used to mathematically describe the production process by describing the way the production inputs transform into the goods or half-finished products. It is possible to express interconnection between raw materials, half-finished products and products by following equation 6.1:

$$x = y + Ax \quad (6.1)$$

where

x overall production of the given product

y production expedited outside modelled system

Ax amount of products used for production of different products

A matrix of specific consumption of half-finished products

Equation 6.1 is written in matrix form. If we express the products from figure 6.1, we get following interpretation of the model:

x_1 extinguishing material (solid)

x_2 extinguishing material (powder)

x_3 valve

x_4 metal plate

x_5 body of extinguisher

x_6 fire extinguisher

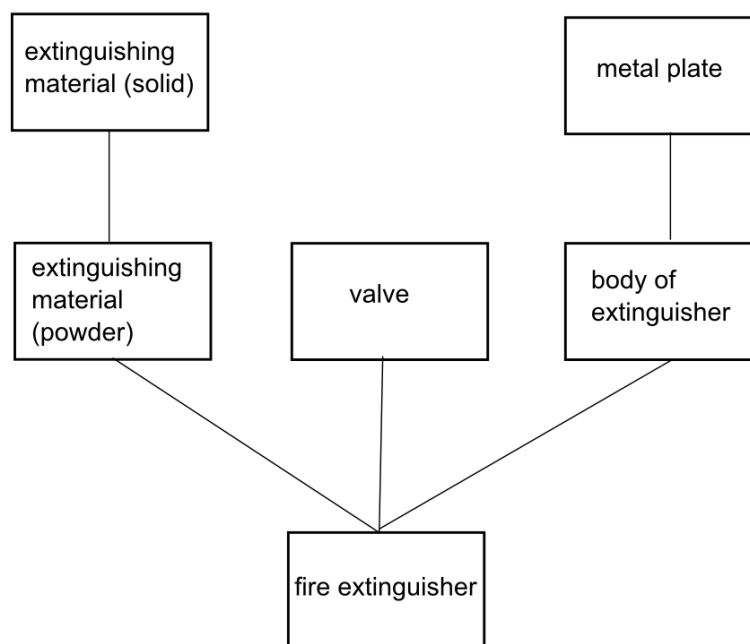


Figure 6.1: Production of the Powder fire extinguisher

We can express equation (6.1) using system of equation (6.2). To specify the balance model, we have to state specific consumption of the half-finished products, see table 6.1.

Table 6.1: Specific consumption

Product	Half-finished product	Specific consumption
Fire extinguisher	Extinguishing material (powder)	10 kg/1 part
	valve	1 part/1 part
	body of extinguisher	1 part/1 part
Extinguishing material (powder)	Extinguishing material (solid)	1 kg/kg
Body of extinguisher	Metal plate	0,25 plate

$$\begin{aligned}
 x_6 &= y_6 + 10x_2 + x_3 + x_5 \\
 x_2 &= y_2 + x_1 \\
 x_5 &= y_5 + 0,25x_4 \\
 x_1 &= y_1 \\
 x_3 &= y_3 \\
 x_4 &= y_4
 \end{aligned}
 \tag{6.2}$$

This equation system is easily solvable, when we understand that the production values are defined by commissions and half finished products may be completely substituted by its balance equations. Simple production systems are even solvable by hand, for more complex scenarios we usually prefer using spreadsheet.

Equation (6.1) may be also expressed in this way (6.3):

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0,25 & 0 & 0 \\ 0 & 10 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}
 \tag{6.3}$$

The balance model higher is very simple. In praxis we forms several models describing different aspects of the production process. Equation (6.1) and its applications (6.3) and (6.2) describe products model. The products model is usually accompanied by the raw materials model. It is possible to compute also energetic balances and technological constrains balances.

Model of usage of raw materials and energetic inputs is demonstrated in equation 6.4.

$$Bx = s \tag{6.4}$$

where x vector of the overall production of the given product

s vector of the overall need of the raw materials to produce all products x

B matrix of the technical coefficients describing raw materials usage to produce product x

Similarly capacity model (6.5) for the production can be used.

$$Cx = f \tag{6.5}$$

where

x vector of the overall production of the given product

f vector of the requirements on production machines, etc.

C matrix of coeff. describing the needed capacity of given machine to produce unit of product x

As you can see the model will not answer the question if the there is enough capacities available to fulfil the production needs of the company. The results have to be interpreted – we get the information on how large the required capacities are. It is up to us to know if we have large enough “real” capacities and if not to solve the problem. Perhaps we can rent capacities elsewhere, the model by itself would not know.

We interpret models described by equation (6.1) and (6.4) in same way.

Balance models

Example of balance model is uploaded to <http://prometheus.vsb.cz> in XLS format (MS Excel). The example is solved in two different ways (equal result wise).



**Questions**

1. What are balance models good for?
2. Explain balance equation of the product.
3. Can you specify parts of the balance model (what can be computed by it)?
4. How do we interpret the model (what information does it compute and how do we work with it)?

Chapter 7

Linear Programming



Study guide

Another type of problems, we have to solve, are those described by system of constraints expressed by equations. For such type of problems we are searching for solution inside the constrained solution space minimizing or maximizing using effectiveness function.

After studying this chapter you will know

How

- to solve linear programming problem using simplex method



Time required to study chapter

For getting through this chapter you will need at least an hour.

We line linear programming into group of algorithms for mathematical solution of optimization problems called *mathematical programming*. Linear programming is called linear, because we solve the problem using system of linear equations.

In linear programming we work with one basic effectiveness function which we optimize. We usually do that by maximizing or minimizing system of the linearly expressed constraints of our problem. Using these constraints we limit the space of possible solutions – so we may actually come to some conclusion.

We may express effectiveness function using following equation (7.1).

$$optimz = \sum_{j=1}^n c_j x_j \quad (7.1)$$

Where z effectiveness function

Optim required optimization max or min

c_j evaluation of variables in effectiveness function (for example price, variable costs etc.)

x_j optimized variable

To limit the solution space of the effectiveness function z we may use three types of the constraints. You can see its mathematical expression in equation (7.2).

$$\begin{aligned} \sum_{j=1}^n a_{ij} x_j &\leq b_i & i = 1, 2, \dots, k \\ \sum_{j=1}^n a_{ij} x_j &= b_i & i = k + 1, k + 2, \dots, k + p \\ \sum_{j=1}^n a_{ij} x_j &\geq b_i & i = k + p + 1, k + p + 2, \dots, k + p + s \\ x_j &\geq 0 & j = 1, 2, \dots, n \end{aligned} \quad (7.2)$$

Where

a_{ij} technical coefficient of the model, these are set independently for each model and they do not change then

x_j optimized variable

b_i right sides of the constrains, these are set in dependence on model (capacity constrains etc.)

The optimized variables must have zero or positive value. This constrain is based on fact, that we usually can't produce negative amount of whatever. Allowing for negative values would lead to non-standard problem, which would not be solvable by linear programming (at least not by method we will use).

First lower then or equal constrain (\leq) notes the situation, that the solution of the presented problem is constrained by availability of some kind of goods or material and the amount of it is limited. The equal sign ($=$) notes the situation, when there are known production processes (or similar), where we know exactly how they do come out. For example we must know exactly how much of raw materials is needed to produce unit of the desired product.

Finally greater then or equal (\geq) describes the situation, when we try to introduce into model some kind of obligation – for example we have already agreed to supply certain amount of the goods – that amount would be the minimal value and we can go only higher from there.

So what are the problems which are solvable by this method? Primary use of this method is by economy for optimization of the manufacturing programme (for sake of profit), but it is general method, which is usable generally – for example for optimization of venting gas from one source using several venting stations, for optimization of transportation of the good using different trucks with different capacity etc.

So how exactly do we formulate the problem and how do we solve it. As first step it is necessary to formulate the model using equations (7.1) and (7.2) – as system of effectiveness function and constrains equations, as in the next example:

$$\max z = 2x_1 - 3x_2 + 4x_3 \quad (7.3)$$

$$\begin{aligned} 4x_1 - 3x_2 + x_3 &\leq 3 \\ x_1 + x_2 + x_3 &\leq 10 \\ 2x_1 + x_2 - x_3 &\leq 10 \end{aligned} \quad (7.4)$$

Such system of equations is solvable using variety of available algorithms as:

1. primary algorithm (simplex method)
2. dual algorithm
3. ...

We will use only the simplex method as most famous representative of linear programming algorithms for solution of this type of problems.

Following text is rather harsh, especially if you heard of linear programming just few moments ago, that's why, you should try nice tutorial by Stephen Waner [27], which is in English and will interactively lead you step by step to learning the method.

So let's solve the problem using simplex method. First step is to transform the constrains to equations. We do that by adding additional variable d_i if the operator was \leq . If the operator was \geq , we have to formally transform it to \leq by multiplying whole equation by -1. Our example would look as follows.

$$\begin{aligned} 4x_1 - 3x_2 + x_3 + d_1 &= 3 \\ x_1 + x_2 + x_3 + d_2 &= 10 \\ 2x_1 + x_2 - x_3 + d_3 &= 10 \\ -2x_1 + 3x_2 - 4x_3 + z &= 0 \end{aligned} \quad (7.5)$$

Additional variables, if computed in optimal solution as not zero represent amount of resource b_i (see (7.1), which will remain, which will not be used. System of equations higher can be transformed to matrix form (see table 7.1).

With each table there is associated just one *basic solution*. Basic solution is one of virtually infinite number of possible solution. To find such solution we search for clean columns, in which there is only

Table 7.1: Working table

x_1	x_2	x_3	d_1	d_2	d_3	z	
4	-3	1	1	0	0	0	3
1	1	1	0	1	0	0	10
2	1	-1	0	0	1	0	10
-2	3	-4	0	0	0	1	0

one nonzero value in the row. Variables in these columns are active, while the remaining variables are not active. Basic solution of our table would be as follows (see table 7.2).

Table 7.2: Iteration 0 – working table – basic solution

x_1	x_2	x_3	d_1	d_2	d_3	z		Basic solution
4	-3	1	1	0	0	0	3	$d_1 = 3/1 = 3$
1	1	1	0	1	0	0	10	$d_2 = 10/1 = 10$
2	1	-1	0	0	1	0	10	$d_3 = 10/1 = 10$
-2	3	-4	0	0	0	1	0	$z = 0/1 = 0$

As you can see in table 7.2, only variables d_1 , d_2 , d_3 and z are active all other variable are inactive (x_1 , x_2 and x_3) and we set them to zero. Active variables form up basic solution for this iteration. For example we compute the value of d_1 dividing right side of equation with value for d_1 in its column so $d_1 = 3/1 = 3$.

Basic solution for this iteration would be:

$$\begin{aligned}
 d_1 &= 3 \\
 d_2, d_3 &= 10 \\
 x_1, x_2, x_3, z &= 0
 \end{aligned}
 \tag{7.6}$$

Well such solution is indeed possible, but it is clearly way too far from optimal solution – which tells us, that our computation will have to continue. We have to transform table after chosen pivot. We search for pivot using two simple criterions:

1. Choosing of the column (for pivot) – we choose the column with most negative number in effectiveness function (last row of the table). The lowest number is -4 in x_3 column.
2. Pivot has to be always positive number with highest testing ratio. Testing ratio is the ratio of right side of the equation and value of the column. We compute testing ratio in table 7.3.

Table 7.3: Testing ratio

x_1	x_2	x_3	d_1	d_2	d_3	z		Testing ratio
4	-3	1	1	0	0	0	3	$3/1 = 3$
1	1	1	0	1	0	0	10	$10/1 = 10$
2	1	-1	0	0	1	0	10	
-2	3	-4	0	0	0	1	0	

Pivot is in the intersect of the chosen column and row. In table 7.3, it is highlighted in bold italic (in the crosssection of the highlighted row and column).

In next step, we have to adjust the table so, that the pivot column is zeroed out, excluding pivot row (see table 21).

Table 7.4: Iteration 2

x_1	x_2	x_3	d_1	d_2	d_3	z		adjustment
3	-4	0	1	-1	0	0	-7	- row 2
1	1	1	0	1	0	0	10	
3	2	0	0	1	1	0	20	+ row 2
2	7	0	0	4	0	1	40	+ 4 * row 2

Because there are no negative numbers in effectiveness function row, the basic solution of this iteration is also the optimal solution of the task. If there was a negative value in last row, we would repeat the whole process over and over again.

Our solution is:

$$\begin{aligned} x_1, x_2 &= 0 \\ x_3 &= 10 \\ d_1 &= -7 \\ d_3 &= 20 \\ d_4 &= 40 \end{aligned} \tag{7.7}$$

Of course, it would be highly ineffective if we had to compute the solution by hand, that's why we usually use linear programming calculator, such as [20]. Usually the programs for mathematic computations such as MathLab or SciLab support linear programming too. There are also smaller single purpose programs specialized on linear programming such as Finite mathematics utility: simplex method tool [4].

The equation system for the problem is written directly into window of the WWW browser of your choice, see fig. 7.1.

Type your linear programming problem below. (Press "Example" to see how to set it up.)

Maximize $p = (1/2)x + 3y + z + 4w$ subject to
 $x + y + z + w \leq 40$
 $2x + y - z - w \geq 10$
 $w - y \geq 10$

Solution:
 Optimal Solution: $p = 115; x = 10, y = 10, z = 0, w = 20$

Solve Example Erase Everything Rounding: 6 significant digits

Mode: Decimal Fraction Integer

The tableaus will appear here.

Tableau #3

x	y	z	w	s1	s2	s3	p
0	2	1.5	0	1	0.5	1.5	20
1	0	-0.5	0	0	-0.5	-0.5	10
0	-1	0	1	0	0	-1	10
0	-7	-1.25	0	0	-0.25	-4.25	45

Tableau #4

x	y	z	w	s1	s2	s3	p
0	1	0.75	0	0.5	0.25	0.75	10
1	0	-0.5	0	0	-0.5	-0.5	10
0	0	0.75	1	0.5	0.25	-0.25	20
0	0	4	0	3.5	1.5	1	115

Figure 7.1: Finite mathematics utility: simplex method tool

As matter of fact it is not important what software is used to compute the solution, it is important to put together correctly the effectiveness function, the constrains and correctly interpret the results.

What if we wanted to use general numerical program such as SciLab, can it help us? The answer is yes. SciLab has implemented karmarkar algorithm for this kind of problem, which has been discovered by Dikin 1967 and later rediscovered by Barnes and Vanderbei et al in 1986. For detail see [11]. Also there is available toolkit for linear and linear quadratic programming called Quapro, see [19].

Lets compute example from [11] using karmarkar and Simplex method tool [4].

$$\begin{aligned} \text{minimize } z &= -x_1 - x_2 \\ x_1 \vee x_2 &= 0 \\ x_1 + x_2 + x_3 &= 2 \\ x_{1,2,3} &\geq 0 \end{aligned} \tag{7.8}$$

Simplex method tool says that the result is for effective function is -2 with $x_{1,2} = 1$.

The script in SciLab for karmarkar algorithm would look like this (as per [11]):

```
Aeq = [
    1 -1 0
    1 1 1
];
beq = [0;2];
c = [-1;-1;0];
[xopt,fopt,exitflag,iter,yopt]=karmarkar(Aeq,beq,c)
```

What is our interpretation of the script? The variable Aeq holds matrix of the coefficients of the left sides of the constrains, each row for each constrain. We have two constrains that's why we have 2 rows in Aeq matrix.

```
Aeq = [
    1 -1      0
    x1 - x2      = 0
    1 1      1
]
x1 + x2 + x3 = 2
```

Variable beq is column vector of right sides of the constrains. Finally c is column vector of coefficients of the effective function.

Function karmarkar takes three parameters Aeq, beq and c but returns no less than five results which are available to us using row vector. The last row of our script returns the result, which is unrounded $z = -1,999989$ for effective function and the variables $x_{1,2} = 0,9999948$ and $x_3 = 0,00001$. When we round it, we get exactly the same result as by using Simplex method tool.

On the first sight the usage of the Simplex method tool is much easier than using the SciLab, but easiness of the use comes at the expense of universality. In SciLab you can incorporate optimization into larger problem solving. Also when we want to solve large amount of such problems at once – usage of the SciLab is much better as it lets you to customize the scripts to your needs, reading and writing to disc and more.



Linear programming task

Two students Ann and Charles work X and Y hours weekly. Together they may work only 40 hours a week. Considering rules for the brigade-workers, Ann may work maximally 8 hours longer than Charles, but Charles may only work 6 hours longer than Ann. Additional constraint is $18 \leq 2y + x$. Try to find optimal solution for situations:

1. If hour pay of Ann is 15 EUR and Charles's 17 EUR - find maximal combined pay
2. If hour pay of Ann is 17 EUR and Charles's 15 EUR - find maximal combined pay
3. If both of them get 16 EUR - find maximal combined pay



Linear programming task - solution

Ann 15 EUR (y) and Charles 17 EUR (x)

Variable represents hours each of them will work.

Effectiveness function: $\max z = 15x + 17y$

Both may not exceed 40 hours a week: $x + y \leq 40$

Ann may work 8 hours longer then Charles: $x - y \leq 8$

Charles may work 6 hours longer then Ann: $-x + y \leq 6$

Additional constrain: $x + 2y \geq 18$

Optimal solution: Ann 17 hours, Charles 23 hours, they will earn 646 EUR

Solution second sub task:

Try to construct the constrains yourself.

Result: Ann 24 hours, Charles 16 hours, they will earn 648 EUR

Solution third sub task:

Result: Ann 17 hours, Charles 23 hours, they will earn 640 EUR



Linear programming task 2

The company produces two products (X and Y) and for this, it is using two machines (A and B). For production of each unit of product X, it is necessary to use 50 minutes of machine A and 30 minutes of machine B. For producing unit of product Y we need 24 minutes of machine A and 33 minutes of machine 2

At the beginning of the week, there are 30 units of product X and 90 units of product 90 B in the store. Available time of machine A is 40 hours and B 35 hours.

Demand for product X this week will be 75 units and Y 95 units. Company's politic states that we maximize sum of products X and Y in store at the end of the week.



Linear programming task 2 - solution

Produce 45 units of product X and 6 units of product Y (in store at the end of the week 51 units)



Not sure if you can solve the linear programing problems?

Good news, the Internet is virtually inexhaustible source of the examples, you can practice on. Solve them by both SciLab and Simplex method tool.

You can also install Quapro toolkit into your SciLab and see for yourself the differences.

Chapter 8

Basics of the Game Theory



Study guide

So far we focused on the decision making based on “hard”, measurable data, but most decisions take place in dynamic environment, where other people are trying to achieve their goals. Game theory provides us with understanding of how these interactions between the “players” actually work.

After studying this chapter you will know

- what is game theory a what is it good for
- how to solve simple problem
- various model situations and its specifics



Time required to study chapter

For getting through this chapter you will need at least 30 minutes.

Up to now we described the decision or preparation to decision as process using objective “hard” characteristic, when there was no opponent to the decision with different goals than we. The decision simply hasn’t been made against human being. Such situation is not entirely common. So we need a tool to deal with this type of situation too.

Theory describing these decision situations is called game theory. Decision situation is described here as a game, where all decision makers are players. For demonstration of the theory, we usually use theoretical problem called prisoners dilemma.

The prisoner dilemma is following: *Two prisoners are awaiting their trial and they can’t speak with each other. So they think about what to say, when the trial comes. If the prisoner A frames prisoner B and prisoner B will say nothing, prisoner A will get free sentence while prisoner B gets hard sentence. If both prisoners will frame each other, both of them will get hard sentence. If both prisoners will not speak at all, they will both get low sentence. Finally if prisoner B frames prisoner A and prisoner A will not speak, prisoner B will get free while prisoner A will get hard sentence. What should both prisoners do?*

We can describe this situation in tabular form.

Table 8.1: Decision strategies – prisoner A

		Prisoner A	
		Speak	Not speak
Prisoner B	Not speak	normal sentence	low sentence
	Speak	hard sentence	hard sentence

We can formulate inverse table for prisoner B, but for this type of the decision it is not necessary.

The goal of both prisoners is to maximize their profit in situation, when they do not have information on which strategy the opponent chooses. So we choose one based on what we think the opponent will do. Such decision is not entirely rational.

The goal of the game theory is to cultivate the game so the players do not choose for the strategy maximizing their profits but the one which is optimizing their profits. We are searching for optimal strategy.

If we expressed the evaluation in numerical form, the numbers would form up matrix of the profits A about which we could write the following (8.1):

$$\max_i \min_j a_{ij} \leq \min_i \max_j a_{ij} \quad (8.1)$$

Gros writes about the situation following [22]: ... highest from minimal profits, which can first player secure can't be larger than highest profit, which the second player tries to minimize.

In situation when there is a point representing equality of both sides of (8.1) we say that the matrix of the profits has a saddle point. This point represents optimal strategy, the player should choose if he wants to optimize his profit.

Let's demonstrate this on our prisoners dilemma. First we have to evaluate the profit of the strategies in numerical form. In our case it would be low sentence 50 points, freedom 100 points, hard sentence 0 points.

Table 8.2: Searching for saddle point

		Prisoner A		\min_j
		Not speak	Speak	
Prisoner B	Not speak	50	100	50
	Speak	0	0	0
\max_i		50	100	

Saddle point (in table bold) we got this way recommends prisoner A not to speak. If the prisoner B knows game theory, he will choose this strategy and both will optimize their profits.

Such simple games are called games of *two players in normal state*. Unfortunately not all games have such simple solution (the saddle point) – in such case we talk about *mixed extension of two players game*. Under these difficult circumstances we do not search for single optimal strategy, but for mix of strategies and probabilities by which we should change them. We search for mixed strategies. All games expressible by matrix have solution in **mixed strategies**.

Such game is transformable into form solvable using methods of linear programming.

If all decision situations have had saddle point we would see the benefits of its usage more clearly. In situation, when we have problems to quantify possible reactions on “multiple fronts” as we are rarely in situation when we have single, clearly defined, “enemy”. In reality there may dozens of them all pursuing their agendas. In such situations using game theory is much harder and its benefits much harder to see. So why are we even discussing this problematic as we clearly have no room for in depth analysis of all aspects of the game theory.

The short answer would be – to change a mind set of you as a student. And to do that we will introduce model types of the games with known outcomes. There are dozens of games, but we will focus just on two of them.

1. game of chicken
2. stag hunt

Game of chicken in games theory is based on real-world, dangerous game, favored by less responsible youth, which involves two strong cars and racing against each other. The one who swerve “chickens out”, letting the other to win. Not swerving means frontal crash against each other in full speed, which is basically death sentence for both of them.

In the model game both player have same car and are have also same capabilities as drivers and of course, they both want to win the race. Lets quantify the outcomes of the decision situation:

- win race = 10
- loose race = -10
- no win situation = 1

- crash = -100

With limited strategies as of swerve or race we can construct the matrix of the outcomes, see tab. 8.3. Since there are two players I will put in matrix outcomes from point of view of both players, so we can better understand the problem.

Table 8.3: Game of chicken

		Racer A	
		swerve	race
Racer B	swerve	1, 1	-10, 10
	race	10, -10	-100, -100

It is obvious, that both players can not win. They can both die, but that's usually something we try to avoid. Optimal strategies lie in forcing other player to chicken out. Unfortunately the players can't guarantee that they will be able to force other one to swerve there is no optimal strategy.

Logical choice would be to swerve under such circumstances for both players. Also there is a solution in symmetric Nash equilibrium – provided that both players change their strategies randomly.

In this case the negative results are really negative (we can hardly imagine more adverse negative effect than death), but we may come to different scenario using game of chicken type of the game.

Lets think about the scenario of collective bargaining in some kind of company. Unions bargain against the management for higher salaries. If they both push – the agreement will not be possible and the worker will go on strike, the company will not produce (and will not pay for worker on strike). If it goes on for long, the company will go bankrupt and the workers will be without the work – everybody loses.

When both sides give something away from their demands the workers will gain something while management has to pay something. Other options lead to the victory of one side and defeat of the other side (either management or union).

Another type of the game is **stag hunt**. This game is derived from hypothetical situation of stag hunt, where the hunters wait in hiding for the game. But it doesn't show up for long time. If the stag hunt is successful, then all of the hunters will eat, but the game is not guaranteed to come. Then a hare comes instead. If the hunter guns it down, he can eat it, but it is not large enough to fill stomachs of all the hunters. So what should the hunter choose – be selfish or cooperate with others?

Lets construct the matrix of the outcomes. We will simplify the situation for two hunters and again we will bring the point of the view of both players in the matrix, see table 8.4.

Table 8.4: Stag hunt

		Hunter A	
		cooperate	defect
Hunter B	cooperate	2, 2	0, 3
	defect	3, 0	, 1

If the hunters cooperate, they will both eat, if one of them defects, then the one defecting gets all benefits of the betrayal but his fellow comrade in hunting will get nothing. If both of them decide not to cooperate (like continuing the hunt on their own), both of get some benefit from the decision, but not as large as in case they cooperate.

Basically the stag hunt situation in real world is the situation when two (or more) parties have to cooperate to achieve common goal. For example when protecting an endangered species two neighborhood countries arrange project to save the species from dying out. If both of them fulfill their obligations, the species will be saved. If one of them withdraws its support it will get some benefits as the money, it saved by withdrawing, can be used elsewhere to further other goals of the country. The other country will sink its money in now futile attempt to save the species and fails. If both countries withdraw their support, they may spend money in different areas, but the species dies out.

Also there are many more types of the games. As a good start in studying of its basics you can use *Wikipedia's List of games in Game Theory* [12] page.

**So what do you thing?**

What type of the game would be the situation of financial crisis of many countries in the eurozone and European Stability Mechanism? Is it stag hunt or game of chicken? What would be the most important moments of such decision? *Please note that this question has no “right” answer. It is here for you take perhaps slightly different approach to it.*

**Questions**

1. What is difference between stag hunt game and the game of chicken?
2. What is saddle point and how do we find it?
3. Do all games have the solution in saddle point and if not what type of the solution do we look for in them?
4. Describe prisoner’s dilemma problem.

Chapter 9

Localization models



Study guide

Another optimization problem is the problem of physical localization of the object in the space.

After studying this chapter you will know

- what are localization models
- how to solve simple localization problems
- how to approach the solution of more difficult problems



Time required to study chapter

You will need at least 30 minutes to study basic theory of localization models.

Localization models are here to help us solve problems requiring us to place something somewhere under some constraints we are placing on optimality of searched for solution. The models are therefore clearly optimization models.

The problems solved by this type of models may significantly differ based on the problem definition – are we placing one object or are there more of them, are we working in 2D or is 3D required, can we choose anyplace or are we choosing between limited amount of places, do we know how many objects do we want to place or is their number also part of our considerations?

What types of the problem are we talking here about:

1. In the area we have already three manufacturing facilities, company's **Chief Executive Officer (CEO)** wants us to find suitable place for storage where the company will temporarily store its production from above mentioned production facilities.
2. After the latest floods the area government found out that there are parts of the area, which are not covered by the wireless warning system. The task is to find suitable placement for new units of the warning system. Also the proposed investment should be cost effective (largest coverage with minimum of the warning system units).
3. ...

To find a solution, we have to specify optimization criterion. Such criterion is usually connected to the cost of connection between the already localized objects and the new (proposed) objects. We will compute such cost N_{ij} for each connection between objects i and j .

The costs themselves are usually function transportation distance d_{ij} . Based on presumption that to transport the goods on longer distances costs more than to do the same on short distances. The amount of the goods transported x_{ij} is also important for costs estimation. Third and last value, which

affects costs is the type of the vehicle used for transportation c_{ij} . Different types of the transport have different costs of usage.

Mathematically we can express the costs between the existing and new object as in equation (9.1).

$$N_{ij} = x_{ij}d_{ij}c_{ij} = d_{ij}w_{ij} \quad (9.1)$$

Since x and c values are from computational point of view constants, we can substitute them by value w , representing weight of the connection. To solve the problem we will have to optimize d value only.

Connection between new objects, if we have to place more then one object can be expressed as on equation (9.2).

$$N_{ik} = x_{ik}d_{ik}c_{ik} = d_{ik}w_{ik} \quad (9.2)$$

Cost function then may be expressed as a sum of the costs, see (9.3).

$$\min z = \sum_{i=1}^m \sum_{j=1}^n N_{ij} + \sum_{i=1}^n \sum_{k=1}^n N_{ik} \quad (9.3)$$

If we are localizing one object only, whole sum of N_{ik} equals to zero simplifying greatly the optimization problem.

As we stated higher, the crucial point of the optimization is distance value. There are three basic types of distances. We choose between them based on problem characteristic, or more specifically characteristic of transportation. Lets visualize the various distances, see fig. 9.1.

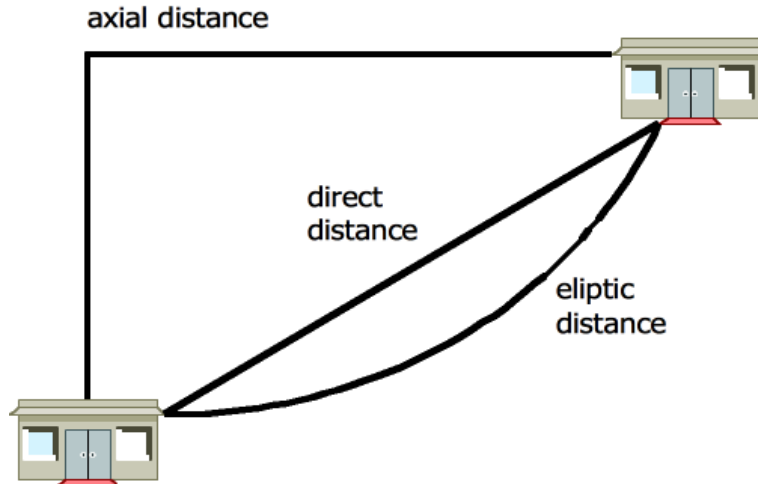


Figure 9.1: Three basic types of the distances

Mathematically we can express them using equations (9.4 – 9.7).

The first distance we usually imagine it direct distance. In localization models this distance is not so common, at least if the problem is to localize the object which should be reachable on the ground. Only rarely can we use direct route under such conditions. Perhaps if we are working on “macro” scale, for example we could be choosing city to place new logistic centre, then the road network could be perhaps approximated by direct lines.

Usage of direct distance is common for wireless communications, where we are not limited by terrain segmentations.

The equation to compute the direct distance is notoriously well known, see equation (9.4). In localization models we also use its slight modification adding k constant to compensate the fact that in reality the direct route is not direct entirely (small curves, it goes over the hill), which lengthens it, see equation (9.5).

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (9.4)$$

$$d_{ij} = k\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (9.5)$$

Axis distance can be used in situation where we have orderly road system. That is typical for newly build cities or its parts. Look for example on the road systems of the New York. Such distance can be computed using equation (9.6).

$$d_{ij} = |x_i - x_j| + |y_i - y_j| \quad (9.6)$$

Lastly we can use quadratic distance as computed in equation (9.7).

$$d_{ij} = (x_i - x_j)^2 + (y_i - y_j)^2 \quad (9.7)$$

The most simple example we can demonstrate the problem solution would be localizing one object in 2D using axial distances. We would be optimizing following expression (9.8).

$$\min z = \sum_{j=1}^n |x - x_j| + |y - y_j| \quad (9.8)$$

Sum of additions can be easily expressed as an addition of the sums allowing us to separate x and y coordinate, see (9.9).

$$\min z = \sum_{j=1}^n w_j |x - x_j| + \sum_{j=1}^n w_j |y - y_j| \quad (9.9)$$

The solution, geometrically, would be in the middle. As we are working with existing road system we are trying to find coordinate for which the sum of the weights w on the right and left (for x coordinate) and higher and lower (for the y coordinate) approximately equals.

We can do that easily in tabular form. Lets find optimal location for new object, so that it may communicate effectively with four other existing objects A-D, see table 9.1 for their coordinates and weights connected to them.

Table 9.1: Localization of existing objects and estimation of the weights

Object	x	y	w
A	1	8	50
B	4	2	100
C	9	2	80
D	8	5	70

Now if we want to perform the computation, we have to order the object by X and Y coordinate and choose the coordinate which exceeds half of the sum of the weights, see tab. 9.2.

Table 9.2: Localization of existing objects and estimation of the weights

Order by X column				
Object	x	y	w	$\sum w$
A	1	8	50	50
B	4	2	100	150
D	8	5	70	220
C	9	2	80	300
Order by Y column				
Object	x	y	w	$\sum w$
B	4	2	100	100
C	9	2	80	180
D	8	5	70	250
A	1	8	50	300

Under given circumstances the new object shall be localized on coordinated X=8 and Y=2.

When using quadratic distance then the objective function we are optimizing looks as in equation (9.10).

$$\min z = \sum_{j=1}^n w_j ((x - x_j)^2 + (y - y_j)^2) \quad (9.10)$$

We can search for local minima of the function by deriving the equation (9.10) by coordinate variables, see (9.11) and putting them equal to zero. The equation (9.11) has analytical solution (9.12), which is then easily computable using any spreadsheet program.

$$\begin{aligned} \frac{\partial z}{\partial x} &= -2 \sum_{j=1}^n w_j (x - x_j) = 0 \\ \frac{\partial z}{\partial y} &= -2 \sum_{j=1}^n w_j (y - y_j) = 0 \end{aligned} \quad (9.11)$$

$$\begin{aligned} x &= \frac{\sum_{j=1}^n w_j x_j}{\sum_{j=1}^n w_j} \\ y &= \frac{\sum_{j=1}^n w_j y_j}{\sum_{j=1}^n w_j} \end{aligned} \quad (9.12)$$

Finally for direct distance we are optimizing the equation (9.13). Unfortunately partial derivations (9.14) do not have analytical solution, so we have to compute them using numerical methods.

$$\min z = \sum_{j=1}^n w_j \sqrt{(x - x_j)^2 + (y - y_j)^2} \quad (9.13)$$

$$\begin{aligned} \frac{\partial z}{\partial x} &= \sum_{j=1}^n \frac{w_j (x - x_j)}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \\ \frac{\partial z}{\partial y} &= \sum_{j=1}^n \frac{w_j (y - y_j)}{\sqrt{(x - x_j)^2 + (y - y_j)^2}} = 0 \end{aligned} \quad (9.14)$$

To effectively compute the coordinates we will use the substitution (9.15).

$$f_j(x, y) = \frac{w_j}{\sqrt{(x - x_j)^2 + (y - y_j)^2 + \varepsilon}} \quad (9.15)$$

Epsilon in equation (9.15) should allow us to go around the possible equality of the coordinates, which would lead to situation, when we would divide by zero.

In iterations we go from the point of optimal solution for quadratic distance and compute the next set the coordinates using equation (9.16).

$$x^{(k+1)} = \frac{\sum_{j=1}^n x_j f_j(x^{(k)}, y^{(k)})}{\sum_{j=1}^n f_j(x^{(k)}, y^{(k)})}, y^{(k+1)} = \frac{\sum_{j=1}^n y_j f_j(x^{(k)}, y^{(k)})}{\sum_{j=1}^n f_j(x^{(k)}, y^{(k)})} \quad (9.16)$$

Using the old and new coordinated we compute how much the iteration contributed to the improvement of the solution (25).

$$\Delta z = z^{(k)} - z^{k+1} \quad (9.17)$$

With each iteration, the improvement is smaller and smaller. So we are basically optimizing in kilometres, then hundreds of meters, meters, decimetres . . . It is usually not effective to reach higher precision, as the benefits of such improvement would be marginal.

Last set of coordinates is as solution.

When computing the localization of multiple new objects, doing se in hand is practically impossible, well maybe it is more impractical then impossible, so we try to use software tools to help us.



Questions

1. What are localization models?
2. What type of the problems do we solve by it?
3. Are there differences in look on the distances in these models?
4. How do we in general approach the solution of the localization model when localizing single object?
5. Can you specify how iterative method for computation of optimal placement of the object works?

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Glossary

AI artificial intelligence.

CEO Chief Executive Officer.

CPM critical path method.

GPS Global Positioning System.

GUI graphical user interface.

JRE Java Runtime Environment.

MCA multicriterial analysis.

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